LABORATORIO 14/11/2023

Esercizio 1

$\vdash A \vee \neg A$

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{[\neg(A\vee\neg A)]^1 \quad \dfrac{[A]^2}{A\vee\neg A}\,\vee i_1}{\bot}\,\neg e
}{\neg A}\,\neg i_2
\quad
\dfrac{[\neg(A\vee\neg A)] \quad \dfrac{\neg A}{A\vee\neg A}\,\vee i_2}{\bot}\,\neg e
}{A\vee\neg A}\,RAA_1
}{}
$$

$\vdash \neg(A\wedge B)\to(\neg A\vee\neg B)$

$[\neg(A\wedge B)]^1$
$[\neg(\neg A\vee\neg B)]^2$
$[A]^3$
$[B]^4$

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{[\neg(A\wedge B)] \quad \dfrac{[A]\ [B]}{A\wedge B}}{\bot}\,\neg e
}{\neg B}\,\neg i_4
}{\neg A\vee\neg B}\,\vee i_2
\quad [\neg(\neg A\vee\neg B)]
}{\bot}\,\neg e
}{\neg A}\,\neg i_3
}{
\dfrac{
\dfrac{[\neg(\neg A\vee\neg B)] \quad \neg A\vee\neg B}{\bot}\,\neg e
}{\neg A\vee\neg B}\,RAA_2
}{\neg(A\wedge B)\to(\neg A\vee\neg B)}\,\to i_1
$$

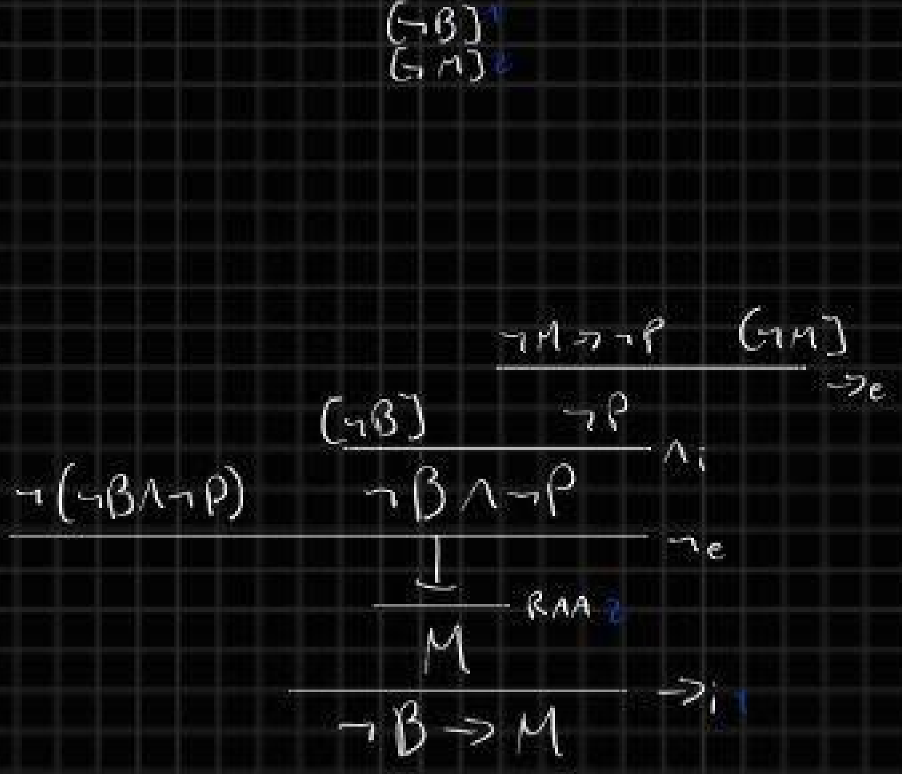
$\neg(A\vee B)\vdash(\neg A\wedge\neg B)$

$[A]^1 \qquad [B]^2$

$$
\dfrac{
\dfrac{
\dfrac{\neg(A\vee B) \quad \dfrac{[A]}{A\vee B}\,\vee i_1}{\bot}\,\neg e
}{\neg A}\,\neg i_1
\quad
\dfrac{\neg(A\vee B) \quad \dfrac{[B]}{A\vee B}\,\vee i_2}{\bot}\,\neg e
}{\neg A\wedge\neg B}\,\wedge i
$$

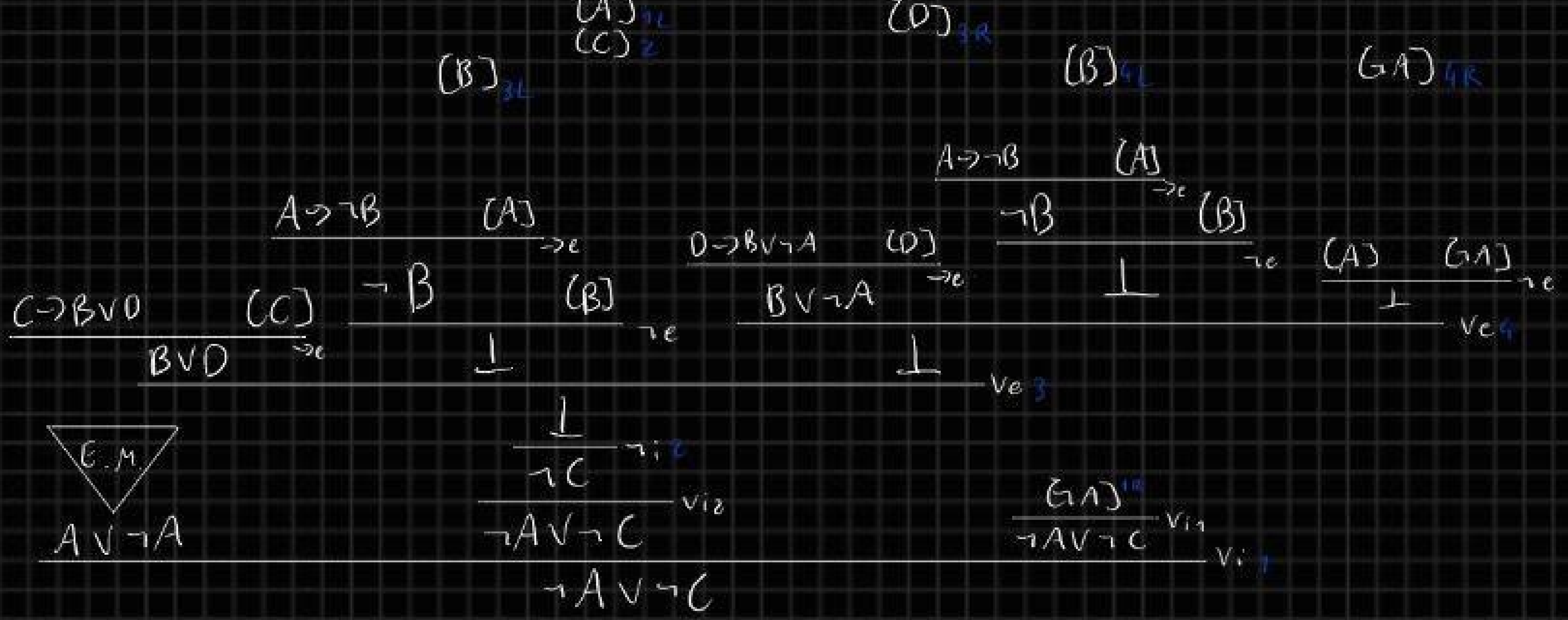
Esercizio 2

$\neg(\neg B \wedge \neg P);\quad \neg M \to \neg P \quad \vdash \neg B \to M$



Esercizio 3

$A \to \neg B;\quad C \to B \vee D;\quad D \to B \vee \neg A \quad \vdash \neg A \vee \neg C$



Esercizio 4

$(A \to B \wedge C) \to C;\quad \neg B \to A \wedge C \;\vdash\; A \vee C$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\neg A]^{1R} \quad [A]^{2}}{\bot}\,\neg e}{B \wedge C}\,\bot e}{A \to B \wedge C}\,\to i^{2} \quad (A \to B \wedge C) \to C}{C}\,\to e}{A \vee C}\,\vee i_2 \qquad \dfrac{[A]^{1L}}{A \vee C}\,\vee i_1 \qquad \dfrac{EM}{A \vee \neg A}\,\vee e^{1}$$

Esercizio 5

$C \vee D \to A \wedge B;\quad \neg A \to C;\quad B \wedge \neg D \;\vdash\; A \wedge B$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\neg A \to C \quad [\neg A]}{C}\,\neg e}{C \vee D}\,\vee i_1 \quad C \vee D \to A \wedge B}{A \wedge B}\,\to e}{A}\,\wedge e_1 \quad [\neg A]^{2}}{A}\,RAA^{2} \quad [B]^{1}}{A \wedge B}\,\wedge i \quad B \wedge \neg D}{A \wedge B}\,\wedge e^{1}$$

Esercizio 6

Vita: V, Speranza: S, Bene: B, Accontenta: A

$V \to S$; $(V \to B) \to A$; $\neg S \vee B$ $\vdash A$

$[\neg S]^{1L}$ $[V]^{2}$ $[B]^{1R}$ $[V]^{3}$

$$\dfrac{\neg S \vee B \qquad \dfrac{(V \to B) \to A \qquad \dfrac{\dfrac{\dfrac{\dfrac{V \to S \quad [V]}{S}\,{\to}e \quad [\neg S]}{\bot}\,{\neg}e}{B}\,{\bot}e}{V \to B}\,{\to}i\,2}{A}\,{\to}e \qquad \dfrac{(V \to B) \to A \qquad \dfrac{[B]}{V \to B}\,{\to}i\,3}{A}}{A}\,{\vee}e\,1$$