

Esercizio 1

$\vdash A \rightarrow \neg(\neg A)$

$[A]$
 $(\neg A)$

$$\frac{(\neg A) \quad [A] \neg e}{\perp} \neg e$$

$$\frac{\perp}{\neg(\neg A)} \neg i$$

$$\frac{\neg(\neg A)}{A \rightarrow \neg(\neg A)} \rightarrow i$$

$\neg A \vdash A \rightarrow \perp$

$[A]$

$$\frac{\neg A \quad [A] \neg e}{\perp} \neg e$$

$$\frac{\perp}{A \rightarrow \perp} \rightarrow i$$

$A \rightarrow \perp \vdash \neg A$

$[A]$

$$\frac{A \rightarrow \perp \quad [A] \rightarrow e}{\perp} \rightarrow e$$

$$\frac{\perp}{\neg A} \neg i$$

Esercizio 2

$$\vdash (\neg A \vee \neg B) \rightarrow \neg(A \wedge B)$$

$$\begin{array}{c} \frac{\frac{\frac{\frac{[A \vee \neg B]}{[A]} \quad [A]}{\perp} \quad \frac{\frac{[A \vee \neg B]}{[\neg B]} \quad [\neg B]}{\perp}}{\vee e} \quad \frac{[A \wedge B]}{\perp} \quad \frac{[A \wedge B]}{\perp}}{\wedge c} \quad \frac{\perp}{\neg(A \wedge B)} \neg i}{(\neg A \vee \neg B) \rightarrow \neg(A \wedge B)} \rightarrow i \end{array}$$

$$\vdash A \wedge \neg B \vdash \neg(A \vee B)$$

$$\begin{array}{c} [A \vee B] \\ [A] \quad [\neg B] \\ [A] \quad [\neg B] \end{array}$$

$$\begin{array}{c} \frac{\frac{[A]}{[A]} \quad [A]}{\perp} \quad \frac{\frac{[\neg B]}{[\neg B]} \quad [\neg B]}{\perp}}{\vee e} \quad \frac{[A \wedge \neg B]}{\perp} \quad \frac{[A \wedge \neg B]}{\perp}}{\wedge c} \quad \frac{\perp}{\neg(A \vee B)} \neg i \end{array}$$

$$\vdash (A \rightarrow B) \rightarrow \neg B \rightarrow \neg A$$

$$\begin{array}{c} [A \rightarrow B] \\ [\neg B] \\ [A] \end{array}$$

$$\begin{array}{c} \frac{\frac{\frac{[A \rightarrow B]}{[A \rightarrow B]} \quad [A]}{B} \rightarrow e \quad [\neg B]}{\perp} \neg e \\ \frac{\perp}{\neg A} \neg i \\ \frac{\neg A}{\neg B \rightarrow \neg A} \rightarrow i \\ \frac{\neg B \rightarrow \neg A}{(A \rightarrow B) \rightarrow \neg B \rightarrow \neg A} \rightarrow i \end{array}$$

$$\vdash A \wedge B \vdash \neg(\neg A \vee \neg B)$$

$$\begin{array}{c} [A \vee \neg B] \\ [A] \quad [\neg B] \\ [A] \quad [\neg B] \end{array}$$

$$\begin{array}{c} \frac{\frac{[A]}{[A]} \quad [A]}{\perp} \quad \frac{\frac{[\neg B]}{[\neg B]} \quad [\neg B]}{\perp}}{\vee e} \quad \frac{[A \wedge B]}{\perp} \quad \frac{[A \wedge B]}{\perp}}{\wedge c} \quad \frac{\perp}{\neg(\neg A \vee \neg B)} \neg i \end{array}$$

Esercizio 3

$$\vdash (A \rightarrow B \rightarrow C) \rightarrow (A \wedge B) \rightarrow C$$

$$\begin{array}{l} [A \rightarrow B \rightarrow C] \\ [A \wedge B] \\ [A] [B] \end{array}$$

$$\begin{array}{l} \frac{[A \rightarrow B \rightarrow C] \quad [A] \rightarrow_e}{B \rightarrow C} \quad \frac{[B]}{\rightarrow_e} \\ \frac{[A \wedge B] \quad C}{C} \wedge_e \\ \frac{C}{(A \wedge B) \rightarrow C} \rightarrow_i \\ \frac{(A \rightarrow B \rightarrow C) \rightarrow (A \wedge B) \rightarrow C}{(A \rightarrow B \rightarrow C) \rightarrow (A \wedge B) \rightarrow C} \rightarrow_i \end{array}$$

$$\vdash ((A \wedge B) \rightarrow C) \rightarrow A \rightarrow B \rightarrow C$$

$$\begin{array}{l} [(A \wedge B) \rightarrow C] \\ [A] \\ [B] \end{array}$$

$$\begin{array}{l} \frac{[(A \wedge B) \rightarrow C] \quad \frac{[A] \quad [B]}{A \wedge B} \wedge_i}{C} \rightarrow_e \\ \frac{C}{B \rightarrow C} \rightarrow_i \\ \frac{B \rightarrow C}{A \rightarrow B \rightarrow C} \rightarrow_i \\ \frac{(A \wedge B) \rightarrow C}{(A \wedge B) \rightarrow C} \rightarrow_i \end{array}$$

$$A \rightarrow B \vdash \neg C \rightarrow (A \vee C) \rightarrow \neg B \rightarrow \perp$$

$$\begin{array}{l} [C] \\ [A \vee C] \\ [\neg B] \end{array}$$

(A)

(C)

$$\begin{array}{l} \frac{A \rightarrow B \quad [A] \rightarrow_e}{B} \quad \frac{[\neg B]}{\rightarrow_e} \quad \frac{[C] \quad [C]}{\rightarrow_e} \\ \frac{[A \vee C] \quad B}{\perp} \vee_e \quad \frac{[C] \quad [C]}{\perp} \vee_e \\ \frac{\perp}{\neg B \rightarrow \perp} \rightarrow_i \\ \frac{\neg B \rightarrow \perp}{(A \vee C) \rightarrow \neg B \rightarrow \perp} \rightarrow_i \\ \frac{(A \vee C) \rightarrow \neg B \rightarrow \perp}{\neg C \rightarrow (A \vee C) \rightarrow \neg B \rightarrow \perp} \rightarrow_i \end{array}$$

Esercizio 4

$$\begin{aligned}
 & [C \wedge G \rightarrow E] \\
 & [\neg L \rightarrow E \vee C] \\
 & [G \vee L] \\
 & [G]
 \end{aligned}$$

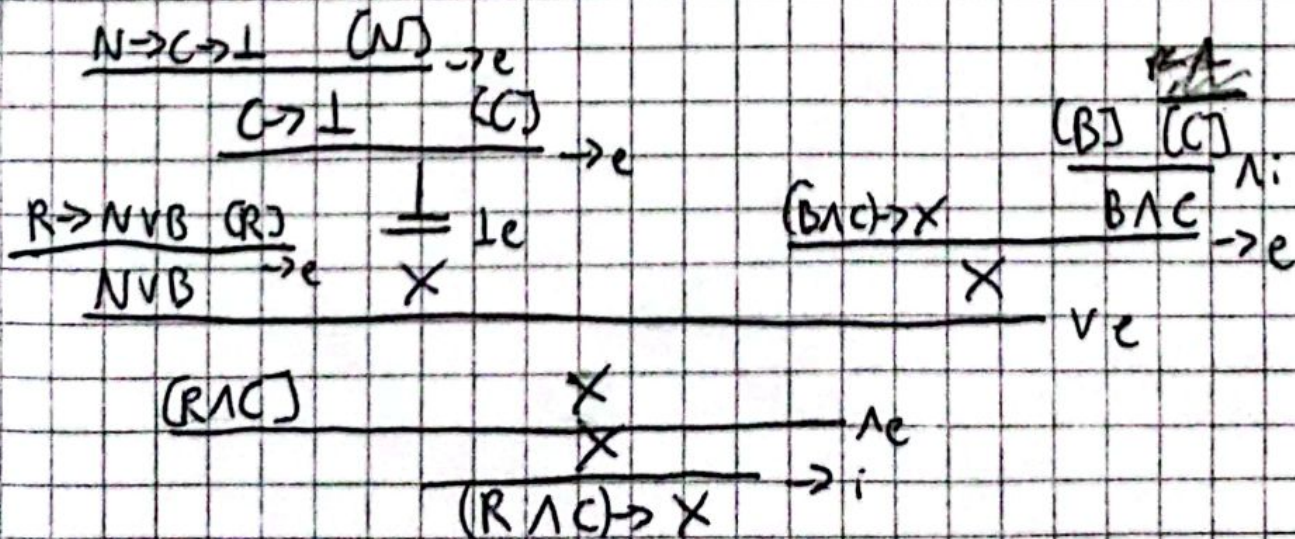
$$\begin{array}{cccc}
 & [G] & & [L] \\
 [E] & & [G] &
 \end{array}$$

$$\begin{array}{l}
 \frac{[\neg L \rightarrow E \vee C] \quad [G]}{E \vee C} \rightarrow_e \quad \frac{[C] \quad [G]}{C \wedge G} \rightarrow_i \quad \frac{[G] \quad [L]}{\frac{1}{E} e} \rightarrow_e \\
 \frac{[G \vee L]}{E} \quad \frac{E}{\neg L \rightarrow E} \rightarrow_i \\
 \frac{[G \vee L] \quad \neg L \rightarrow E}{G \vee L \rightarrow \neg L \rightarrow E} \rightarrow_i \\
 \frac{[\neg L \rightarrow E \vee C] \rightarrow G \vee L \rightarrow \neg L \rightarrow E}{[C \wedge G \rightarrow E] \rightarrow [\neg L \rightarrow E \vee C] \rightarrow G \vee L \rightarrow \neg L \rightarrow E} \rightarrow_i
 \end{array}$$

Esercizio 5

$R \rightarrow (N \vee B), N \rightarrow C \rightarrow L, (B \wedge C) \rightarrow X \vdash (R \wedge C) \rightarrow X$

$(R \wedge C)$
 $(R) (C)$
 (N) (B)



Esercizio 6

Sopra : S ; Sotto : $\neg S$
 Campa : C ; Crepa : $\neg C$
 Strega nell'armadio : A

$S \rightarrow C$; $\neg S \rightarrow \neg C$; ~~A~~ $A \rightarrow \neg C$; $\neg S \rightarrow \neg A \vdash A \rightarrow S$

[A]

[S]

$$\frac{A \rightarrow \neg C \quad [A]}{\neg C} \rightarrow_e \quad \frac{S \rightarrow C \quad [S]}{C} \rightarrow_e$$

$$\frac{\neg S \rightarrow \neg A}{\neg S} \rightarrow_e$$

$$\frac{\neg A \quad [A]}{\neg A} \rightarrow_e$$

$$\frac{\neg S}{S} \rightarrow_e$$

$$\frac{\neg S}{A \rightarrow S} \rightarrow_i$$