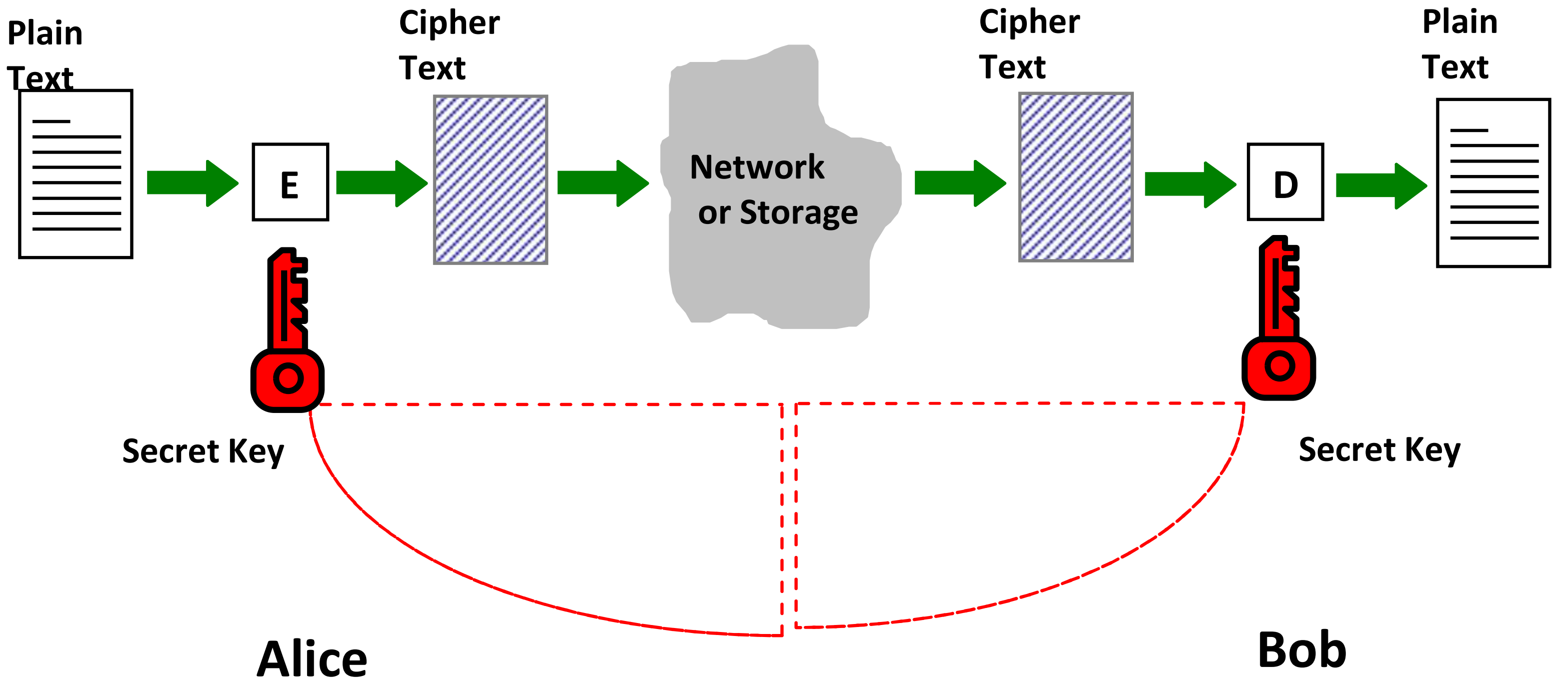
# **Asymmetric Cryptography**

Public key encryption:
definitions and security

# Symmetric Cipher



# Problems with Symmetric Ciphers

- In order for Alice & Bob to be able to communicate securely using a symmetric cipher, such as AES, they must have a **shared key** in the first place.
  - What if they have never met before?

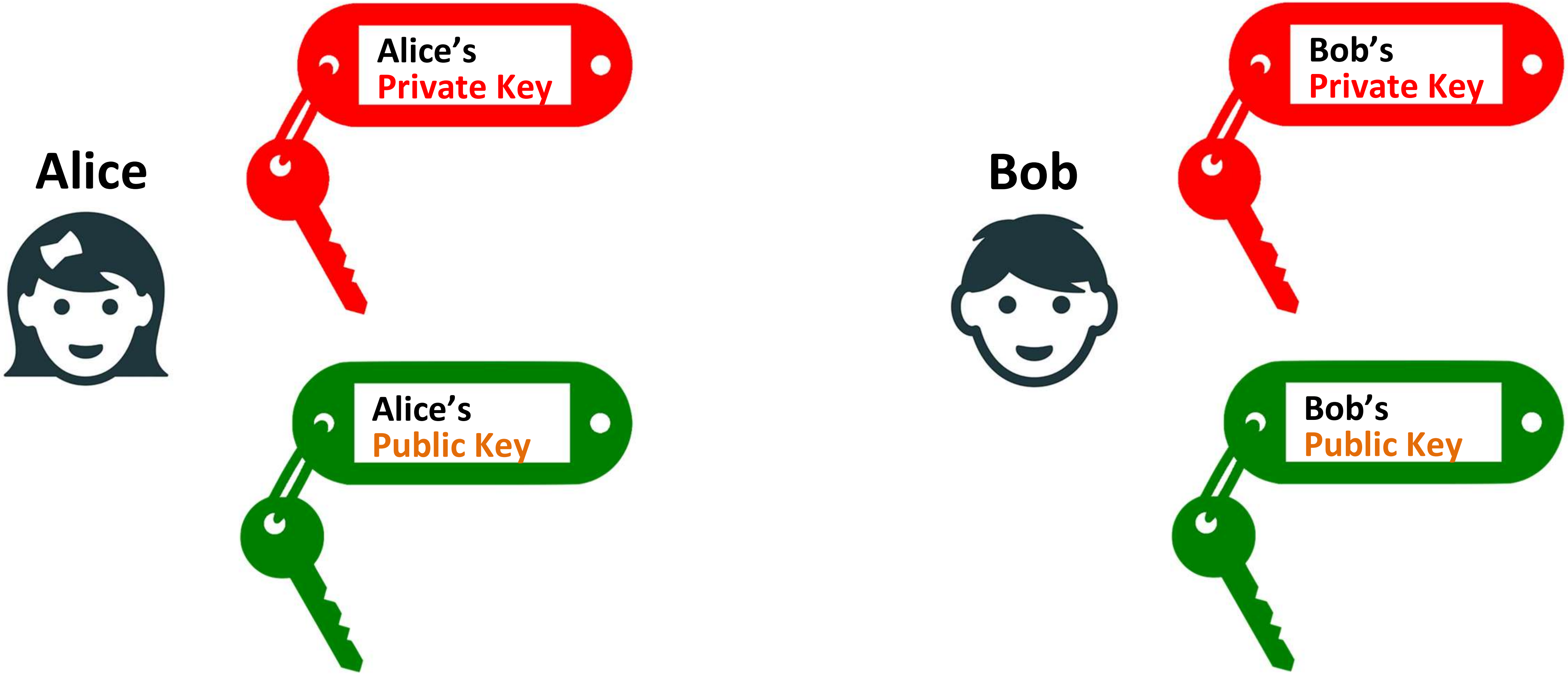
- Alice needs to keep 100 different keys if she wishes to communicate with 100 different people

# Motivation of Asymmetric Cryptography

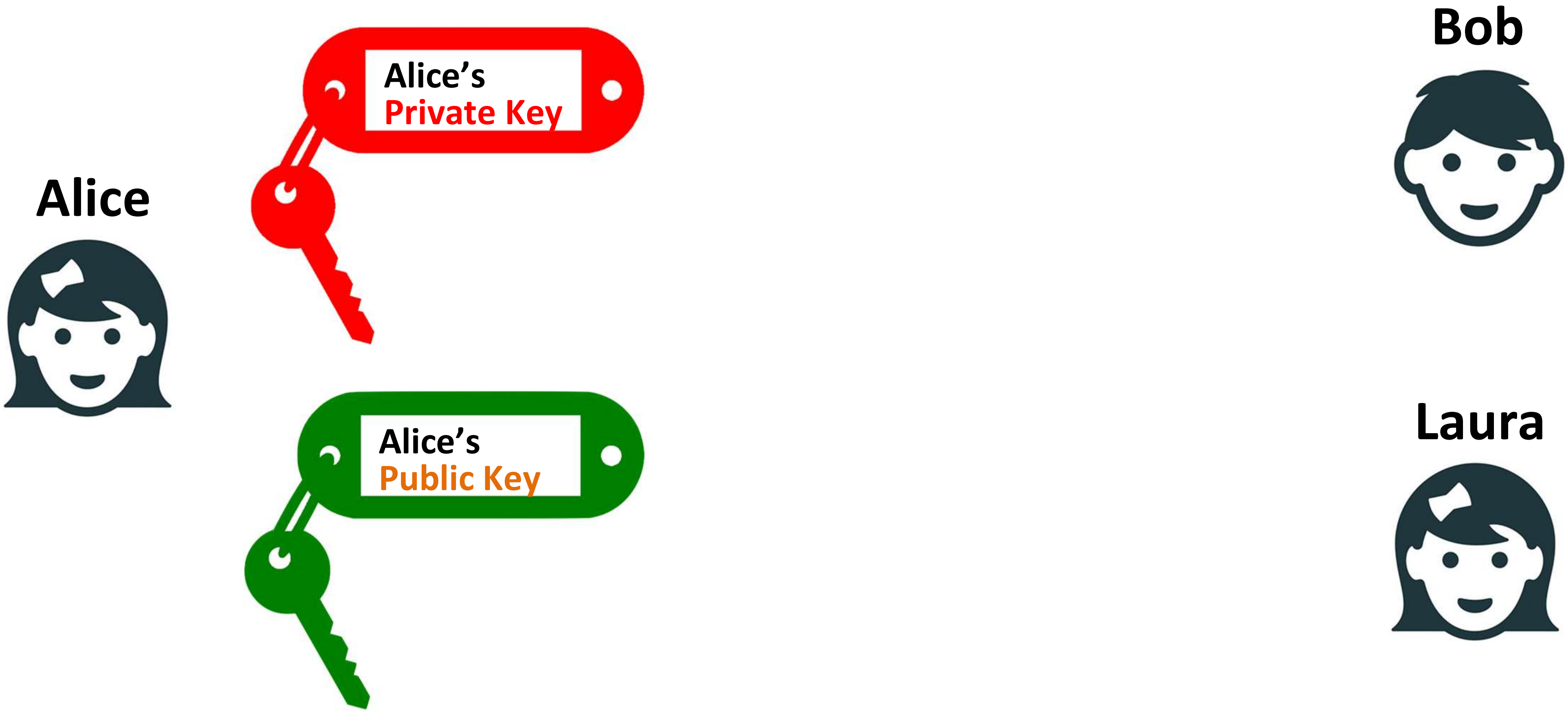
- Is it possible for Alice & Bob, who have no shared secret key, to communicate securely?

- This led to **Asymmetric Cryptography**

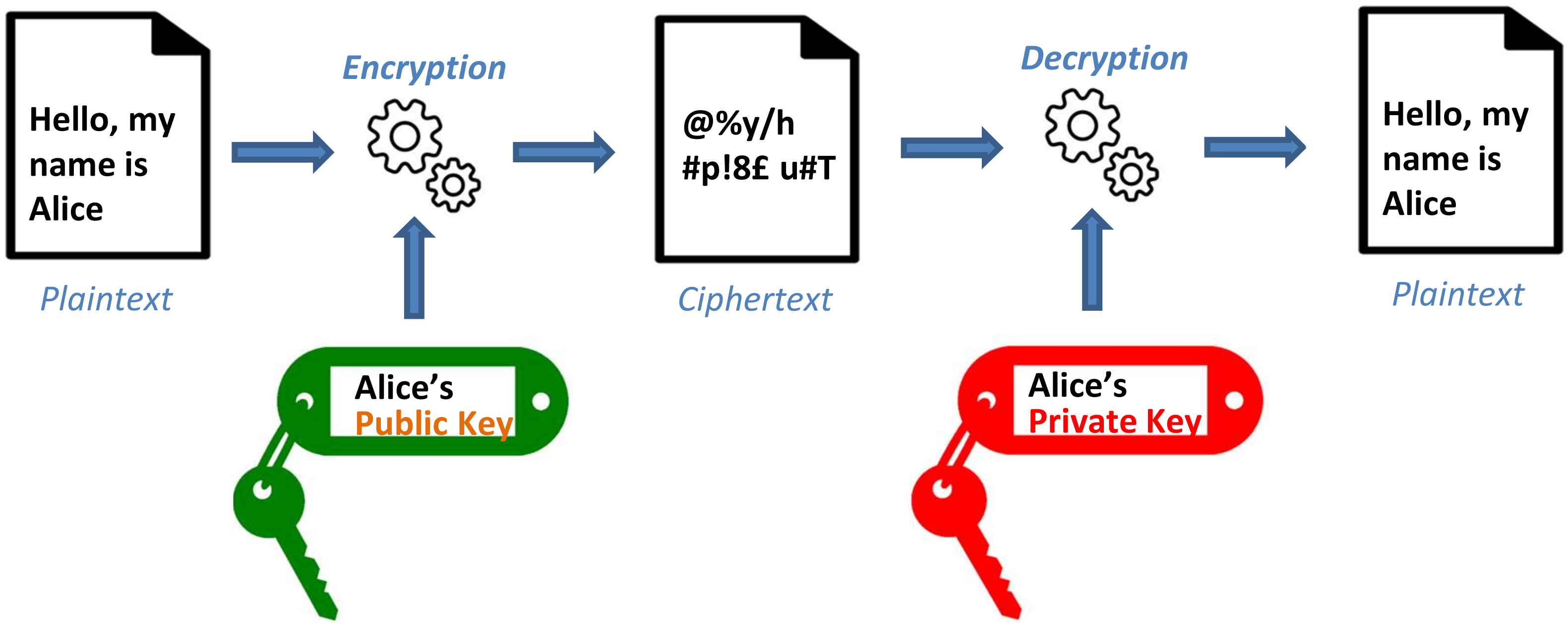
# Asymmetric Cryptography



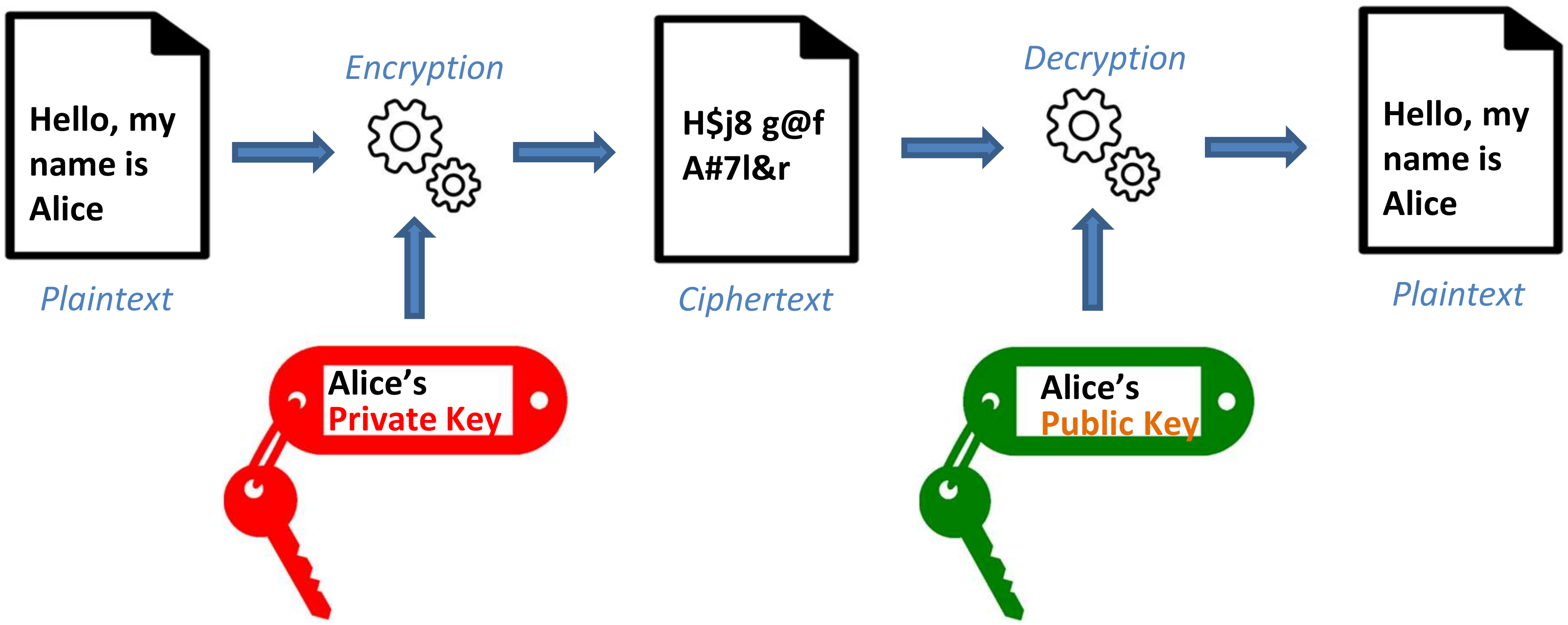
# Asymmetric Cryptography



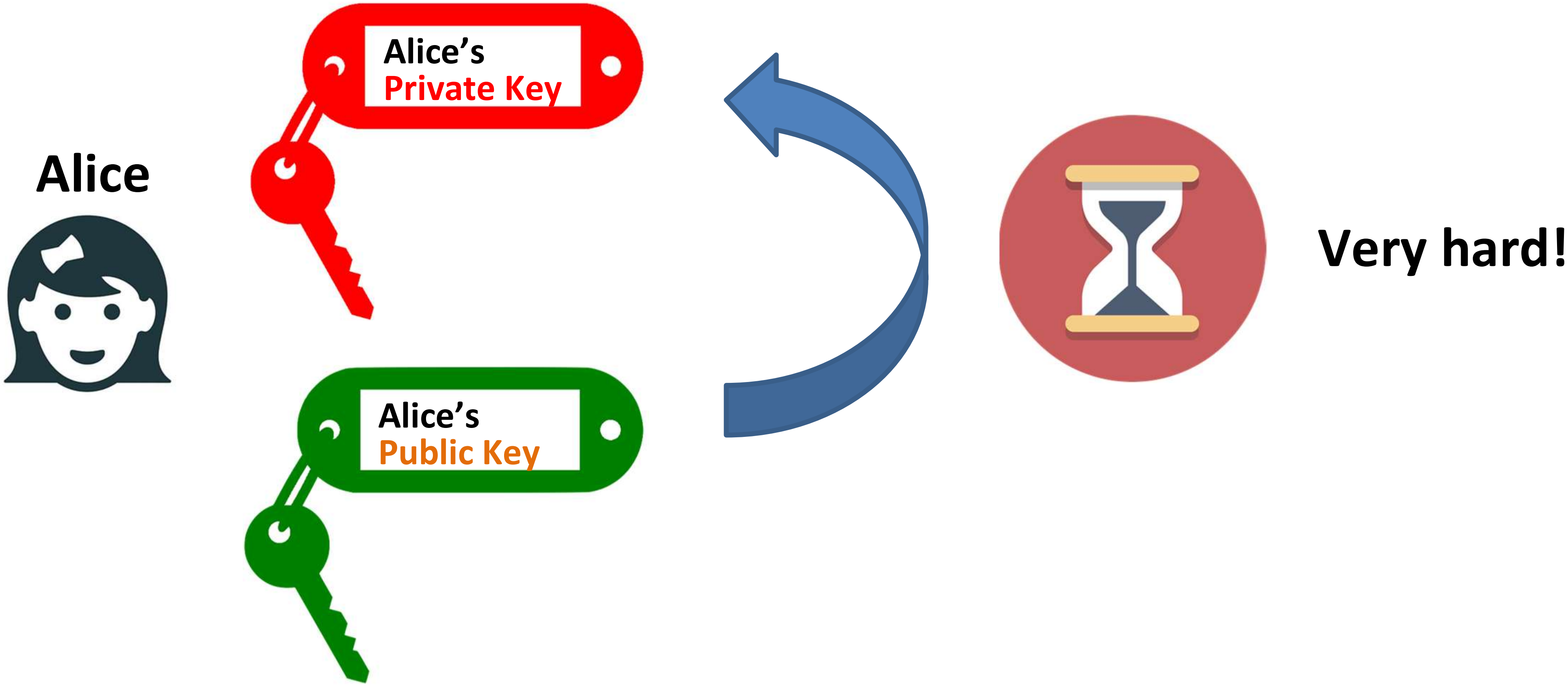
# Public and private keys



# Public and private keys



# Public and private keys



# Asymmetric Cryptography

- **Public key**
- **Private key**

- **$E(\text{private-key}_{Alice,}\ m) = c$**
- **$D(\text{public-key}_{Alice,}\ c) = m$**

- **$E(\text{public-key}_{Alice,}\ m) = c$**
- **$D(\text{private-key}_{Alice,}\ c) = m$**

# Main ideas

- Bob:
  - **publishes**, say in Yellow/White pages, his **public key**, and
  - **keeps** to himself the **matching private key**.

# Main ideas (Confidentiality)

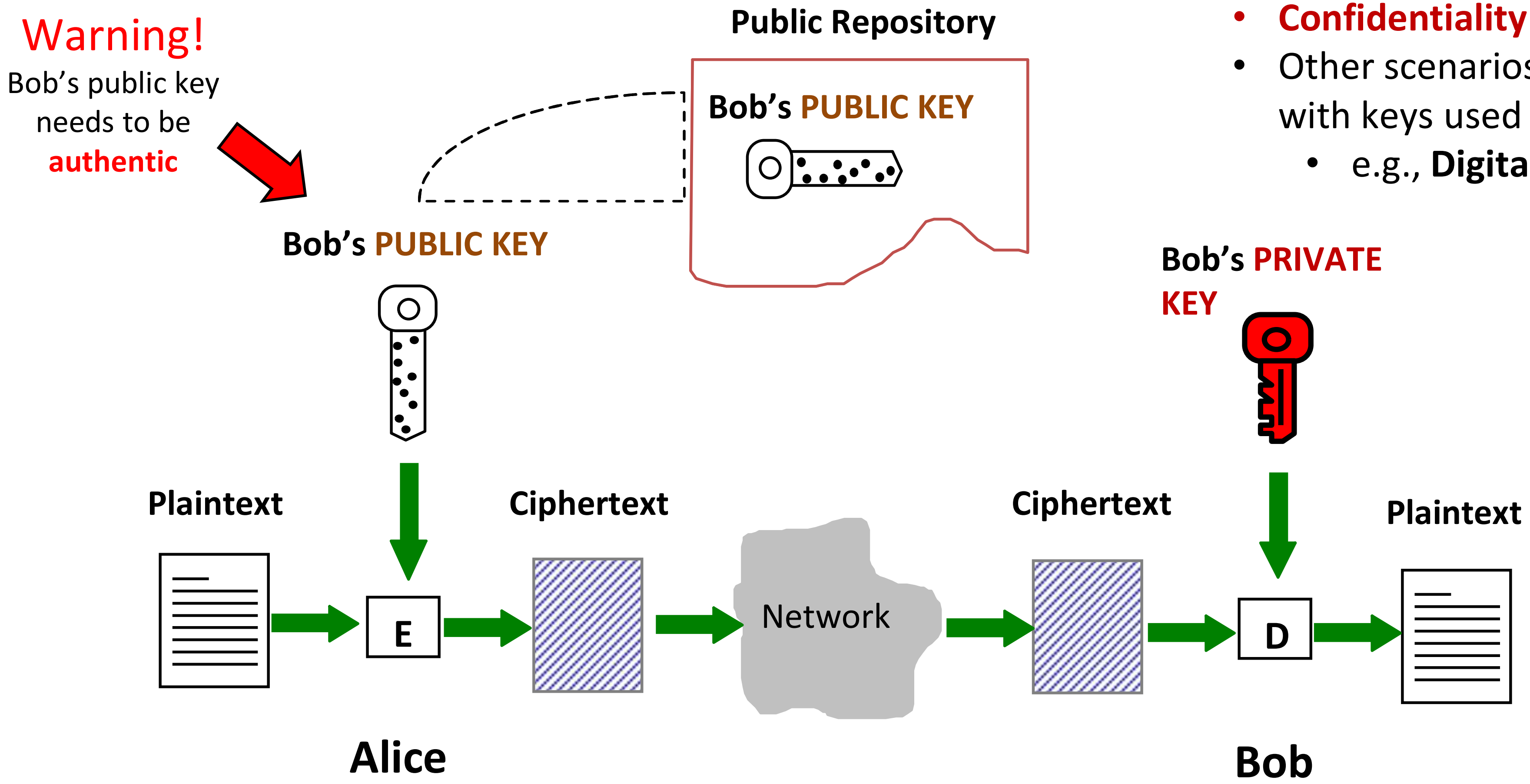
- Alice:
  - Looks up the phone book, and **finds out** **Bob's public key**
  - **Encrypts** **a message using** **Bob's public key** and the encryption algorithm.
  - **Sends the ciphertext** to Bob.

# Main ideas (Confidentiality)

- Bob:
  - **Receives the ciphertext** from Alice.
  - **Decrypts** the ciphertext **using his private key**, together with the decryption algorithm

# Asymmetric Encryption

- **Confidentiality scenario**
- Other scenarios are possible, with keys used differently…
  - e.g., **Digital signatures**



# Main differences with Symmetric Crypto

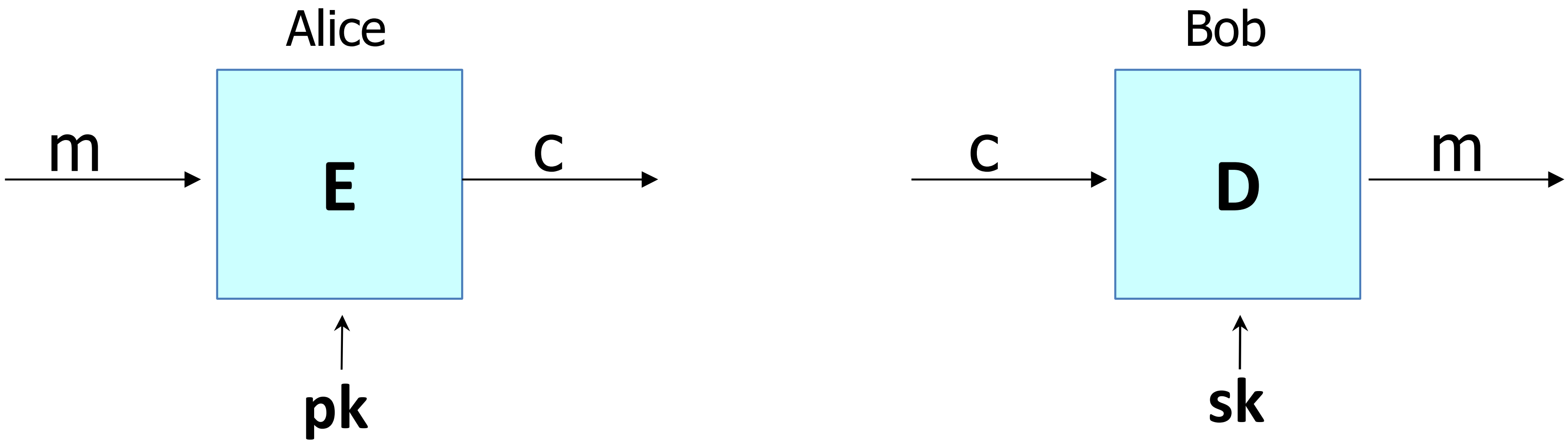
- The ***public key*** is different from the ***private key***.
- Infeasible for an attacker to find out the private key from the public key.
- No need for Alice and Bob to distribute a shared secret key beforehand!
- Only one pair of public and private keys is required for each user!

# Let’s start seriously

- Define what is public key encryption
- What it means for public key encryption to be secure

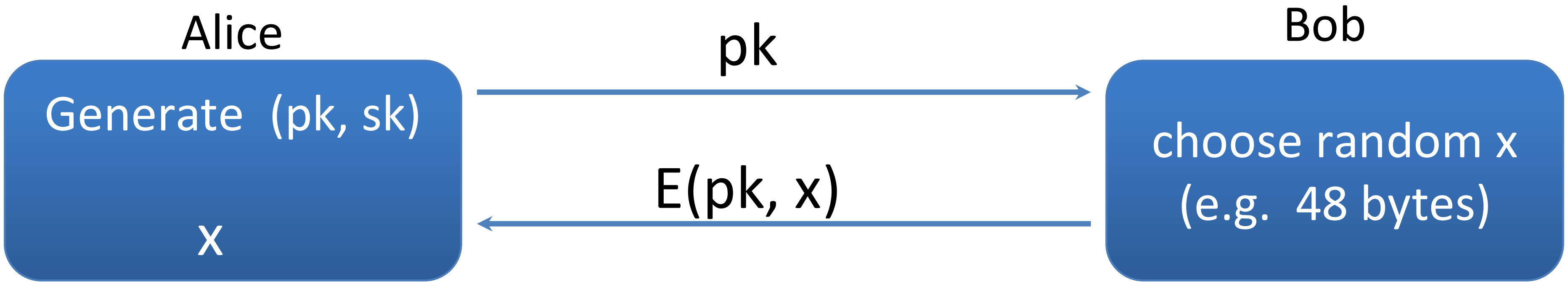
# Public key encryption

Bob: generates $(p_k, s_k)$ and gives $p_k$ to Alice



# Applications

**Session setup** (for now, only eavesdropping security)




**Non-interactive applications**: (e.g. Email)

- Bob sends email to Alice encrypted using $pk_{alice}$
- Note: Bob needs $pk_{alice}$ (public key management)

# Public key encryption

**<u>Def</u>**: a public-key encryption system is a triple of algs. (G, E, D)

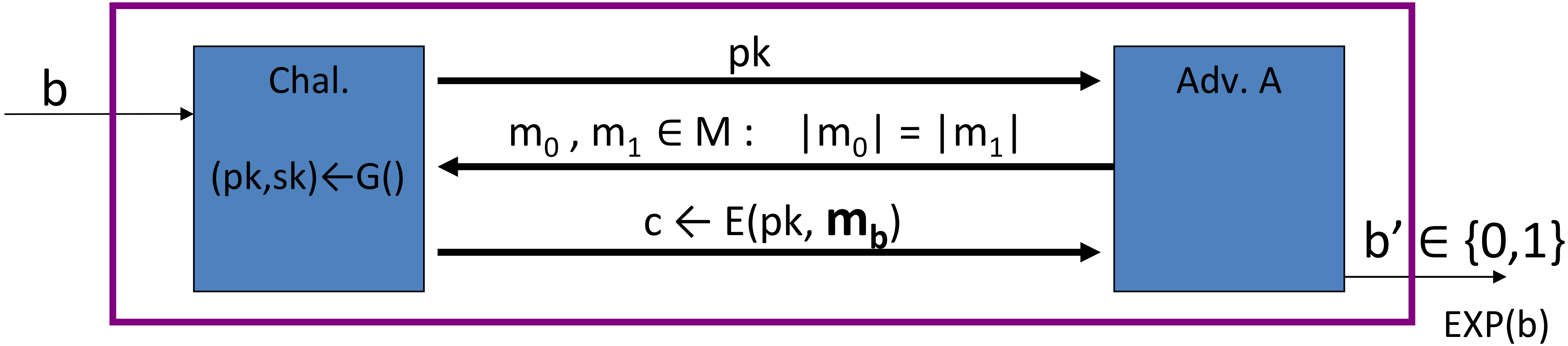
- **G**(): randomized alg. outputs a key pair (pk, sk)
- **E**(pk, m): randomized alg. that takes $m \in M$ and outputs $c \in C$
- **D**(sk,c): det. alg. that takes $c \in C$ and outputs $m \in M$ or $\perp$

Consistency: $\forall$(pk, sk) output by G :

$$\forall m \in M: \quad D(sk, E(pk, m)) = m$$

# Security: eavesdropping

For $b$=0,1 define experiments EXP(0) and EXP(1) as:




Def: $\mathbb{E}$ =(G,E,D) is sem. secure (a.k.a IND-CPA) if for all efficient A:

$$\mathrm{Adv}_{SS}[A,\mathbb{E}] = \Big| \Pr[\mathrm{EXP}(0)=1] - \Pr[\mathrm{EXP}(1)=1] \Big| < \text{negligible}$$

# Relation to symmetric cipher security

Recall: for symmetric ciphers we had two security notions:

- One-time security and many-time security (CPA)
- We showed that one-time security $\not\Rightarrow$ many-time security

For public key encryption:

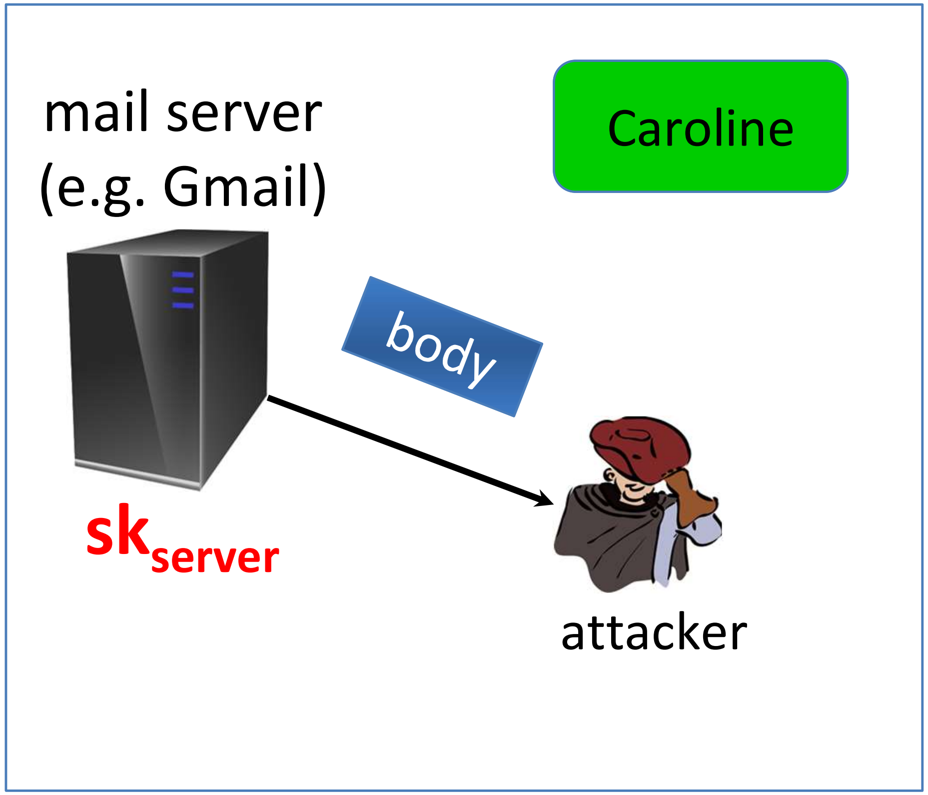
- One-time security $\Rightarrow$ many-time security (CPA)

  (follows from the fact that attacker can encrypt by himself)

- Public key encryption **must** be randomized

# Security against active attacks

What if attacker can tamper with ciphertext?

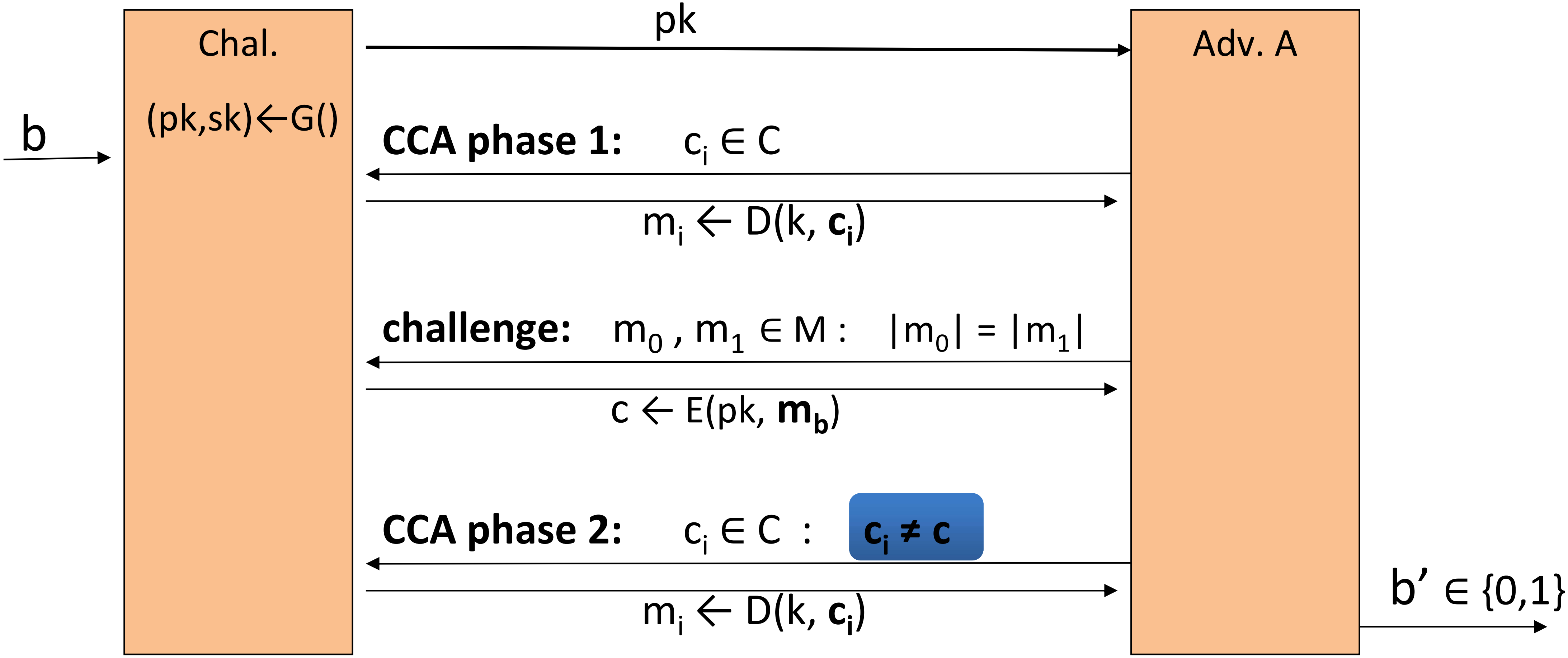

Attacker is given decryption of msgs

that start with **"to: attacker"**

# (pub-key) Chosen Ciphertext Security: definition

$\mathbb{E}$ = (G,E,D) public-key enc. over (M,C)

For b=0,1 define EXP(b):

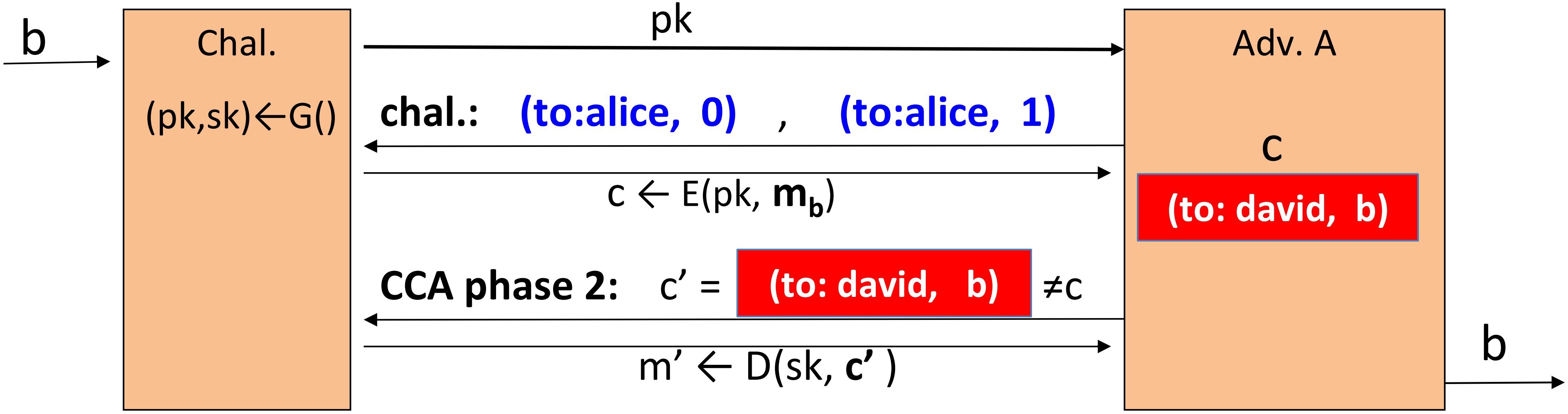


# Chosen ciphertext security: definition

**Def**: $\mathbb{E}$ is CCA secure (a.k.a IND-CCA) if for all efficient A:

$\mathrm{Adv}_{\mathrm{CCA}}[A,\mathbb{E}] = \left| \Pr[\mathrm{EXP}(0)=1] - \Pr[\mathrm{EXP}(1)=1] \right|$ is negligible.

Example: Suppose **(to: alice, body)** $\longrightarrow$ **(to: david, body)**



# Active attacks: symmetric vs. pub-key

Recall: secure symmetric cipher provides **authenticated encryption**

**[** chosen plaintext security & ciphertext integrity **]**

- Roughly speaking: **attacker cannot create new ciphertexts**
- Implies security against chosen ciphertext attacks

In public-key settings:

- Attacker **can** create new ciphertexts using pk !!
- So instead: we directly require chosen ciphertext security

# Trapdoor Permutations

# Trapdoor functions (TDF)

**<u>Def</u>**: a trapdoor func. $X \longrightarrow Y$ is a triple of efficient algs. $(G, F, F^{-1})$

- $G()$: randomized alg. outputs a key pair $(pk, sk)$
- $F(pk,\cdot)$: det. alg. that defines a function $X \longrightarrow Y$
- $F^{-1}(sk,\cdot)$: defines a function $Y \longrightarrow X$ that inverts $F(pk,\cdot)$

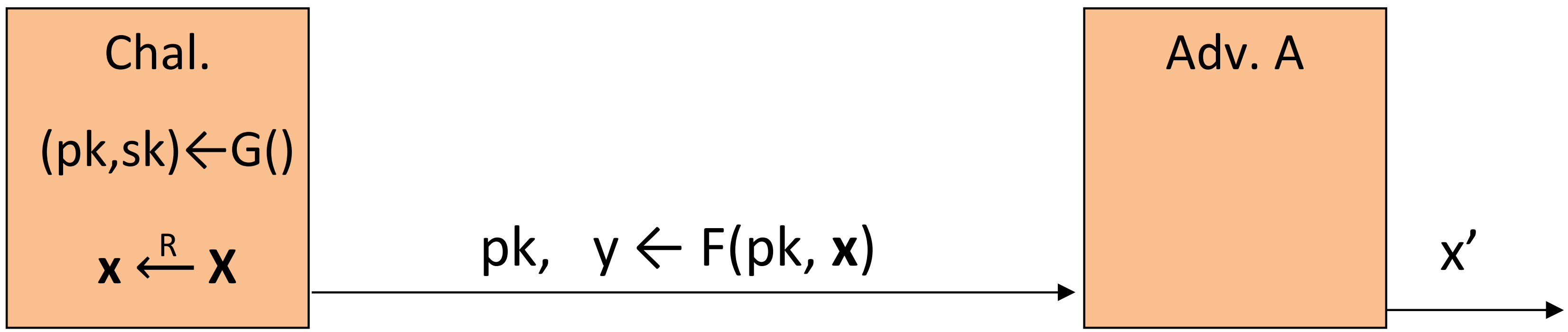
More precisely: $\forall (pk, sk)$ output by G

$$\forall x \in X: \quad F^{-1}(sk, F(pk, x)) = x$$

# Secure Trapdoor Functions (TDFs)

$(G, F, F^{-1})$ is secure if $F(pk, \cdot)$ is a "one-way" function:

can be evaluated, but cannot be inverted <u>without sk</u>

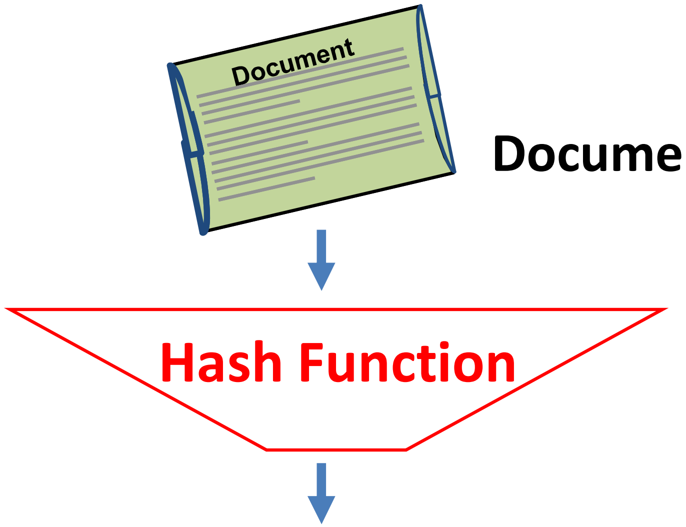



**Def**: $(G, F, F^{-1})$ is a secure TDF if for all efficient A:

$$\mathrm{Adv}_{OW}[A,F] = \Pr[\, x = x' \,] < \text{negligible}$$

# Hash Functions

- **Hash functions:**
  - **Input:** arbitrary length
  - **Output:** fixed length (generally much shorter than the input)




**Document with arbitrary length**

**Hash value** for the document
(fixed length, e.g. 256 bits)

# One-Way Hash Algorithm

- A one-way hash algorithm hashes an input document into a condensed short output (say of 256 bits)
  - Denoting a one-way hash algorithm by **H(.)**, we have:
    - Input: **m** - a binary string of any length
    - Output: **H(m)** - a binary string of **L** bits, called the "hash of m under H".
    - The output length parameter **L** is fixed for a given one-way hash function **H**,
    - Examples:
      - The one-way hash function "MD5" has **L** = 128 bits
      - The one-way hash function "SHA-1" has **L** = 160 bits

# Properties of One-Way Hash Algorithm

A good one-way hash algorithm **H** needs to have the following properties:

1. **Easy to Evaluate**:

   The hashing algorithm should be fast

2. **Hard to Reverse**:

   There is no feasible algorithm to “**reverse**” a hash value,

   That is, given any hash value **h**, it is computationally infeasible to find any document **m** such that **H(m) = h**.

3. **Hard to find Collisions**:

   There is no feasible algorithm to find **two** or **more** input documents which are hashed into the **same** condensed output,

   That is, it is computationally infeasible to find any two documents **m1**, **m2** such that **H(m1)= H(m2)**.

4. **A small change** to a message **should change the hash value so extensively** that the new hash value appears uncorrelated with the old hash value

# Public-key encryption from TDFs

- $(G, F, F^{-1})$: secure TDF $X \longrightarrow Y$
- $(E_s, D_s)$ : symmetric auth. encryption defined over $(K,M,C)$
- $H: X \longrightarrow K$ a hash function

We construct a pub-key enc. system $(G, E, D)$:

Key generation G: same as G for TDF

# Public-key encryption from TDFs

- $(G, F, F^{-1})$: secure TDF $X \longrightarrow Y$
- $(E_s, D_s)$ : symmetric auth. encryption defined over $(K,M,C)$
- $H: X \longrightarrow K$ a hash function

**<u>E( pk, m )</u> :**

$x \xleftarrow{R} X, \quad y \longleftarrow F(pk, x)$

$k \longleftarrow H(x), \; c \longleftarrow E_s(k, m)$

output $(y, c)$

**<u>D( sk, (y,c) )</u> :**

$x \longleftarrow F^{-1}(sk, y),$

$k \longleftarrow H(x), \; m \longleftarrow D_s(k, c)$

output $m$

In pictures:

$$\underbrace{F(pk, x)}_{\text{header}}\ \underbrace{E_s(\,H(x),\ m\,)}_{\text{body}}$$

**<u>Security Theorem</u>**:

If **$(G, F, F^{-1})$** is a secure TDF, **$(E_s, D_s)$** provides auth. enc.

and **H:** $X \longrightarrow K$ is a "random oracle"

then **(G,E,D)** is $CCA^{ro}$ secure.

# Incorrect use of a Trapdoor Function (TDF)

**Never** encrypt by applying **F** directly to plaintext:

**E( pk, m) :**

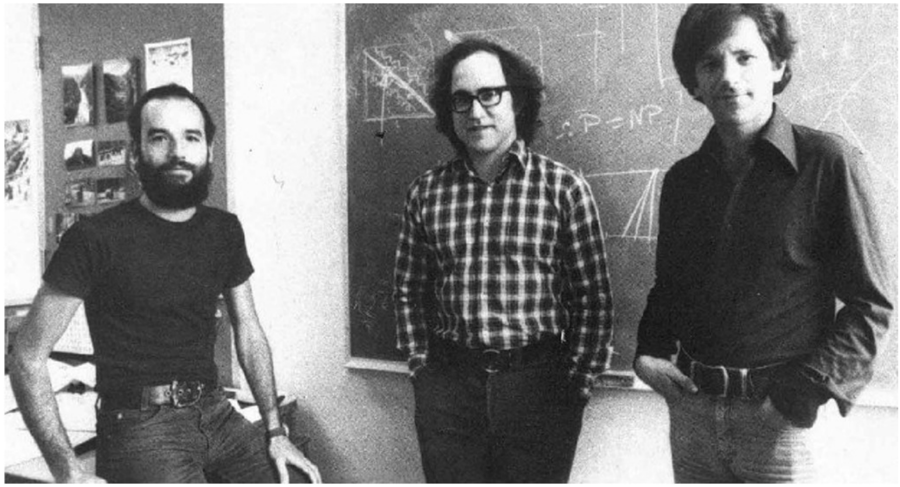
output $c \longleftarrow F(pk, m)$

**D( sk, c ) :**

output $F^{-1}(sk, c)$

Problems:

- Deterministic: cannot be semantically secure !!
- Many attacks exist (next segment)



# The **RSA** trapdoor permutation

- One of the first practical responses to the challenge posed by Diffie-Hellman was developed by ***R****on Rivest*, *Adi* ***S****hamir*, and *Len* ***A****dleman* of MIT in 1977
- Resulting algorithm is known as ***RSA***
- Based on properties of *prime numbers* and results from *number theory*

# Review: trapdoor permutations

Three algorithms: $(G, F, F^{-1})$

- G: outputs pk, sk. pk defines a function $F(pk, \cdot): X \to X$
- $F(pk, x)$: evaluates the function at $x$
- $F^{-1}(sk, y)$: inverts the function at $y$ using sk

**Secure** trapdoor permutation:

The function $F(pk, \cdot)$ is one-way without the trapdoor sk

# Review: arithmetic mod composites

Let $N = p\cdot q$ where $p,q$ are prime where $p,q \approx N^{1/2}$

$Z_N = \{0,1,2,\ldots,N-1\}$ ; $(Z_N)^*$ = {invertible elements in $Z_N$}

<u>Facts</u>: $x \in Z_N$ is invertible $\Leftrightarrow$ $\gcd(x,N) = 1$

– Number of elements in $(Z_N)^*$ is $\varphi(N) = (p-1)(q-1) = N-p-q+1$

<u>Euler's thm</u>: $\boxed{\forall\, x \in (Z_N)^* \;:\; x^{\varphi(N)} = 1}$

# The RSA trapdoor permutation

First published: Scientific American, Aug. 1977.

Very widely used:

- SSL/TLS: certificates and key-exchange
- Secure e-mail and file systems

  … many others

# The RSA trapdoor permutation

**G**(): choose random primes **p**,**q** ≈1024 bits. Set **N=pq**.

choose integers **e , d** s.t. **e·d = 1 (mod φ(N) )**

output pk = (N, e) , sk = (N, d)

---

**F( pk, x )**: $\mathbb{Z}_N^* \rightarrow \mathbb{Z}_N^*$ ; **RSA(x) =** $\mathbf{x^e}$ (in $Z_N$)

---

$\mathbf{F^{-1}}$**( sk, y)** $= y^d$ ; $y^d = \mathbf{RSA(x)^d} = x^{ed} = x^{k\varphi(N)+1} = \left(x^{\varphi(N)}\right)^k \cdot \mathbf{x} = x$

# RSA - small example

- Bob (**keys generation**):
  - chooses 2 primes: **p=5, q=11**
  - multiplies p and q: **n = p×q = 55**
  - chooses a number **e=3** s.t. **gcd(e, 40) = 1; (40 = 55-5-11+1)**
  - compute **d=27** that satisfy **(3 × d) mod 40 = 1**

  - **Bob's public key: (3, 55)**
  - **Bob's private key: 27**

# RSA - small example

- Alice (**encryption**):
  - has a message **m=13** to be sent to Bob
  - finds out **Bob's public encryption key (3, 55)**
  - calculates **c** as follows:

$$\begin{aligned} c &= m^e \bmod n \\ &= 13^3 \bmod 55 \\ &= 2197 \bmod 55 \\ &= 52 \end{aligned}$$

  - sends the ciphertext **c=52** to Bob

# RSA - small example

- Bob (**decryption**):
  - receives the ciphertext **c=52** from Alice

  - uses his matching private decryption key **27** to calculate **m**:

$$m = 52^{27} \bmod 55$$
$$= 13 \text{ (Alice's message)}$$

# The RSA assumption

RSA assumption: RSA is a one-way permutation

For all efficient algs. A:

$$\Pr\left[\ A(N,e,y) = y^{1/e}\ \right] < \text{negligible}$$

where $p,q \xleftarrow{R}$ n-bit primes, $N \leftarrow pq$, $y \xleftarrow{R} Z_N^*$

# Review: RSA pub-key encryption (ISO std)

$(E_s, D_s)$: symmetric enc. scheme providing auth. encryption.

H: $Z_N \rightarrow K$ where K is key space of $(E_s, D_s)$

- **G**(): generate RSA params: pk = (N,e), sk = (N,d)
- **E**(pk, m): (1) choose random x in $Z_N$

  (2) $y \leftarrow RSA(x) = x^e$ , $k \leftarrow H(x)$

  (3) output $(y, E_s(k,m))$
- **D**(sk, (y, c) ): output $D_s( H(RSA^{-1}(y)), c) \rightarrow m$

# Plain/Textbook RSA is insecure

Textbook RSA encryption:

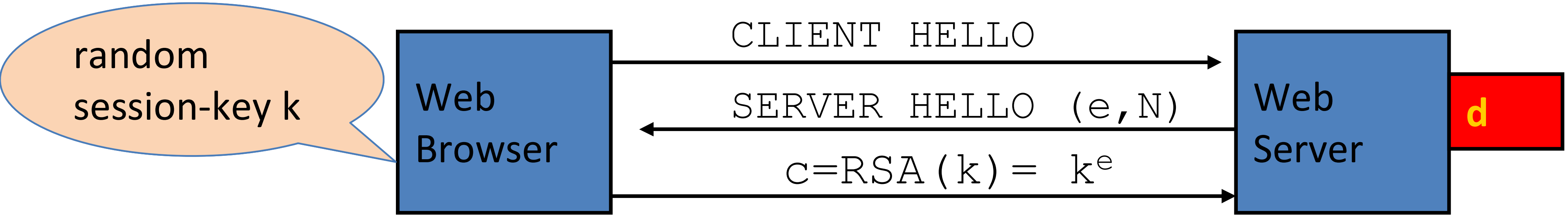
- public key: **(N,e)** Encrypt: $\mathbf{c \longleftarrow m^e}$ (in $Z_N$)
- secret key: **(N,d)** Decrypt: $\mathbf{c^d \longrightarrow m}$

Insecure cryptosystem !!

- Is not semantically secure and many attacks exist

⇒ The RSA trapdoor permutation is not an encryption scheme !

# A simple attack on textbook RSA




Suppose k is 64 bits: $k \in \{0,\ldots,2^{64}\}$. Eve sees: $c = k^e$ in $Z_N$

If $\mathbf{k = k_1 \cdot k_2}$ where $k_1, k_2 < 2^{34}$ (prob. ≈20%) then $\mathbf{c/k_1^e = k_2^e}$ in $Z_N$

Meet-in-the-middle attack:

Step 1: build table: $c/1^e, c/2^e, c/3^e, \ldots, c/2^{34e}$ . time: $2^{34}$

Step 2: for $k_2 = 0,\ldots, 2^{34}$ test if $k_2^e$ is in table. time: $2^{34}$

Output matching $(k_1, k_2)$. Total attack time: $\approx 2^{40} \ll 2^{64}$

# Is RSA a one-way function?

Is it really hard to invert RSA without knowing the trapdoor?

# Is RSA a one-way permutation?

To invert the RSA one-way func. (without d) attacker must compute:

x from $c = x^e \pmod{N}$.

How hard is computing e'th roots modulo N ($c^{1/e}$ / $\sqrt[e]{c}$ modulo N) ??

Best known algorithm:

- Step 1: factor N (**hard**)
- Step 2: compute e'th roots modulo p and q (**easy**)

# Shortcuts?

Must one factor N in order to compute e’th roots?

To prove no shortcut exists show a reduction:

- Efficient algorithm for e’th roots mod N

  $\Rightarrow$ efficient algorithm for factoring N.
- Oldest problem in public key cryptography.

Some evidence no reduction exists: (BV’98)

- “Algebraic” reduction $\Rightarrow$ factoring is easy.

# How **not** to improve RSA's performance

To speed up RSA decryption use small private key **d**  ( $d \approx 2^{128}$ )

$$c^d = m \pmod{N}$$

Wiener'87: if $d < N^{0.25}$ then RSA is insecure.

BD'98: if $d < N^{0.292}$ then RSA is insecure (open: $d < N^{0.5}$ )

<u>Insecure:</u> priv. key d can be found from (N,e)

# Wiener's attack (at home)

(N,e) => d and d < $N^{0.25}/3$

Recall: $e \cdot d = 1 \pmod{\varphi(N)} \Rightarrow \exists\, k \in Z: \quad e \cdot d = k \cdot \varphi(N) + 1$

$$\left|\frac{e}{\psi(N)} - \frac{k}{d}\right| = \frac{1}{d \cdot \varphi(N)} \leq \frac{1}{\sqrt{N}}$$

---

$\varphi(N) = N\text{-}p\text{-}q+1 \Rightarrow |N - \varphi(N)| \leq p+q \leq 3\sqrt{N}$

$d \leq N^{0.25}/3 \Rightarrow \frac{1}{2d^2} - \frac{1}{\sqrt{N}} \geq \frac{3}{\sqrt{N}}$

$$\left|\frac{e}{N} - \frac{k}{d}\right| \leq \left|\frac{e}{N} - \frac{e}{\varphi(N)}\right| + \left|\frac{e}{\varphi(N)} - \frac{k}{d}\right| \leq \frac{1}{2d^2}$$

---

Continued fraction expansion of e/N gives k/d.

$e \cdot d = 1 \pmod{k} \Rightarrow \gcd(d,k)=1 \Rightarrow$ can find d from k/d

# RSA in Practice

# RSA With Low public exponent

To speed up RSA encryption use a small **e**: $c = m^e \pmod N$

- Minimum value: **$e=3$** ( $\gcd(e, \varphi(N)) = 1$) (Q: why not 2?)
- Recommended value: **$e=65537=2^{16}+1$**

Encryption: 17 multiplications

<u>Asymmetry of RSA:</u> fast enc. / slow dec.

– ElGamal: approx. same time for both.

# Key lengths

Security of public key system should be comparable to security of symmetric cipher:

| Cipher key-size | RSA Modulus size |
|---|---|
| 80 bits | 1024 bits |
| 128 bits | 3072 bits |
| 256 bits (AES) | **15360** bits |

# Implementation attacks

**Timing attack**: [Kocher et al. 1997] , [BB'04]

The time it takes to compute $c^d \pmod N$ can expose d

**Power attack**: [Kocher et al. 1999)

The power consumption of a smartcard while it is computing $c^d \pmod N$ can expose d.

**Faults attack**: [BDL'97]

A computer error during $c^d \pmod N$ can expose d.

A common defense: check output. 10% slowdown.

# An Example Fault Attack on RSA (CRT)

A common implementation of RSA decryption: $x = c^d$ in $Z_N$

$$\left.\begin{array}{ll} \text{decrypt mod p:} & x_p = c^d \text{ in } Z_p \\ \text{decrypt mod q:} & x_q = c^d \text{ in } Z_q \end{array}\right\} \text{combine to get } x = c^d \text{ in } Z_N$$

Suppose error occurs when computing $x_q$, but no error in $x_p$. Then:

output is $x'$ where $x' = c^d$ in $Z_p$ but $x' \neq c^d$ in $Z_q$

$\Rightarrow$ $(x')^e = c$ in $Z_p$ but $(x')^e \neq c$ in $Z_q$ $\Rightarrow$ $\gcd\big( (x')^e - c\,,\, N \big) =$

# RSA Key Generation Trouble [Heninger et al./Lenstra et al.]

OpenSSL RSA key generation (abstract):

```
prng.seed(seed)
p = prng.generate_random_prime()
prng.add_randomness(bits)
q = prng.generate_random_prime()
N = p*q
```

Suppose poor entropy at startup:

- Same p will be generated by multiple devices, but different q
- $N_1$ , $N_2$ : RSA keys from different devices $\Rightarrow$ $\gcd(N_1,N_2) = p$

# RSA Key Generation Trouble [Heninger et al./Lenstra et al.]

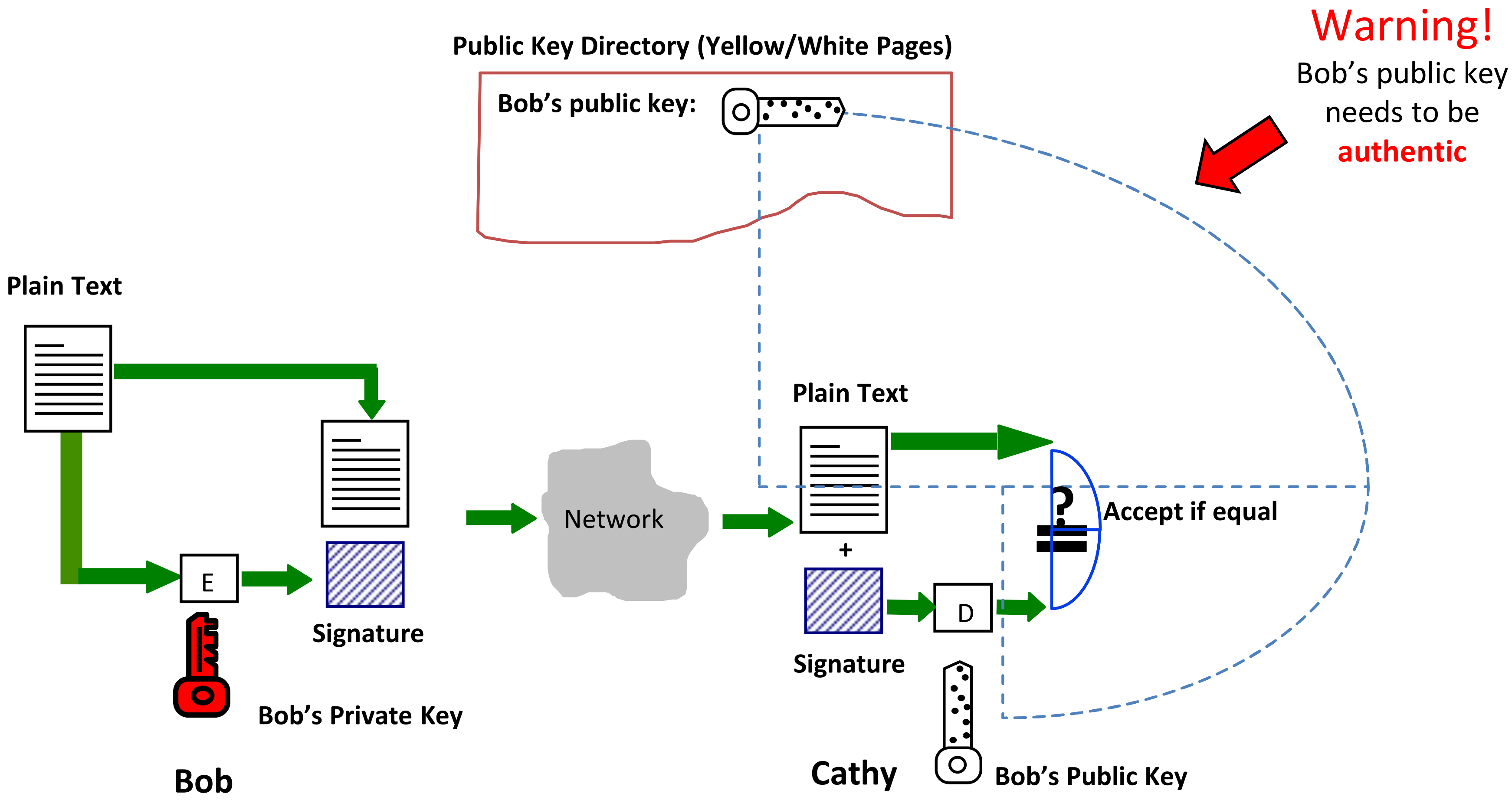
Experiment: factors 0.4% of public HTTPS keys !!

Lesson:

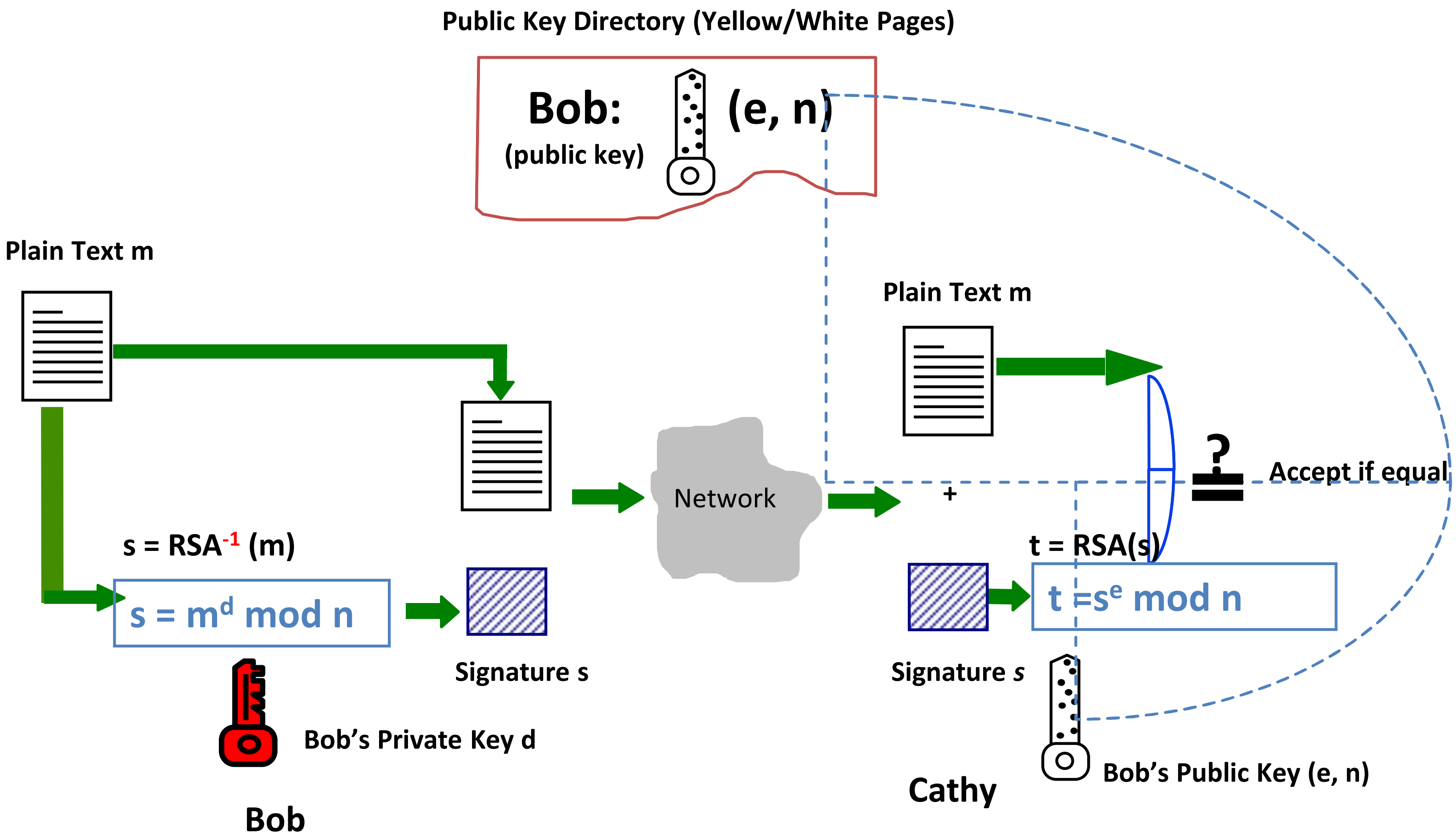
- Make sure random number generator is properly seeded when generating keys

# Digital Signatures

# Digital Signature



# Digital Signature (based on RSA)



# RSA Signature - small example

- Bob (**keys generation**):
  - chooses 2 primes: **$p=5$, $q=11$**
  - multiplies p and q: **$n = p \times q = 55$**
  - chooses a number **$e=3$** s.t. **$\gcd(e, 40) = 1$**
  - compute **$d=27$** that satisfy **$(3 \times d) \bmod 40 = 1$**

  - **Bob's public key: (3, 55)**
  - **Bob's private key: 27**

# RSA Signature - small example

- Bob:
  - has a document **m=19** to sign:
  - uses **his private key** **d=27** to **calculate the digital signature** of **m=19**:

$$
\begin{aligned}
s &= m^d \bmod n \\
&= 19^{27} \bmod 55 \\
&= 24
\end{aligned}
$$

  - **appends 24 to 19**.

    Now **(m, s) = (19, 24**) indicates that the **doc is 19**, and **Bob's signature on the doc is 24**.

# RSA Signature - small example

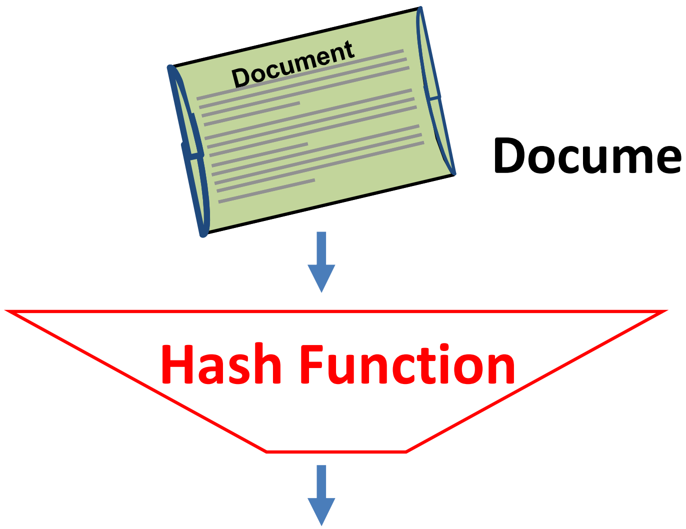
- Cathy, a verifier:
  - **receives** a pair **(m,s)=(19, 24)**
  - looks up the phone book and **finds out Bob's public key** (e, n)=(3, 55)
  - **calculates** $\mathbf{t} = s^e \bmod n$
    $= 24^3 \bmod 55$
    $= 19$
  - **checks whether t=m**
  - **confirms that (19,24) is a genuinely signed document of Bob if t=m.**

# How about Long Documents ?

- In the previous example, a document has to be an integer in [0,...,n)
- To sign a very long document, we need a so called one-way **hash algorithm**
- **Instead of signing directly on a doc**,
  - we **hash the doc first**,
  - and **sign the hashed data** which is normally short.

# Hash Functions

- **Hash functions:**
  - **Input:** arbitrary length
  - **Output:** fixed length (generally much shortern than the input)

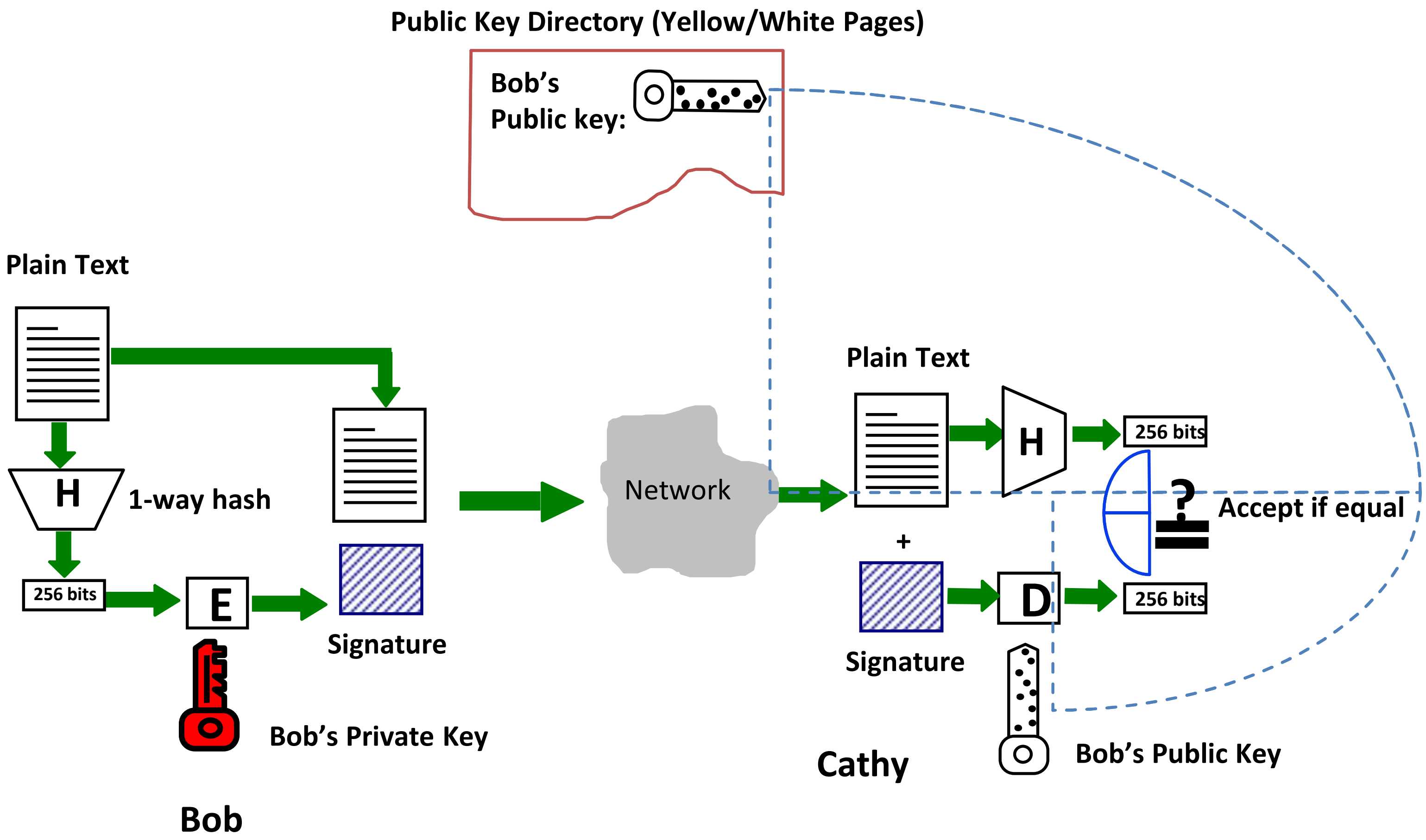



**Document with arbitrary length**

**Hash value** for the document
(fixed length, e.g. 256 bit)

Rather than signing the original document, we **sign its hash value**

# Digital Signature (for long docs)



# Why Digital Signature ?

- Unforgeable
  - takes 1 billion years to forge !
- Un-deniable by the signatory
- Universally verifiable
- Differs from doc to doc

# Digital Signature - summary

- Three (3) steps are involved in digital signature
  - Setting up public and private keys
  - Signing a document
  - Verifying a signature

# Setting up Public & Private Keys

- Bob does the following
  - prepares a pair of public and private keys
  - Publishes his public key in the public key file (such as an on-line phone book)
  - Keeps the private key to himself
- Note:
  - Setting up needs only to be done **once** !

# Signing a Document

- Once setting up is completed,  Bob can sign a document (such as a contract, a cheque, a certificate, ...) using the private key
- The pair of document & signature is a proof that Bob has signed the document.

# Verifying a Signature

- Any party, say Cathy, can verify the pair of document and signature, by using Bob’s public key in the public key file.
- Important !
  - Cathy does NOT have to have public or private key !

# (Other) Asymmetric Cryptosystems

# ElGamal Cryptosystem

Encryption schemes built from the Diffie-Hellman protocol

- **Key Generation** (for Bob)
  - chooses a prime **p** and a number **g** *primitive root modulo p*
    - i.e., for every integer **a** coprime to **p**, there is an integer **k** such that $\mathbf{g^k = a \bmod p}$
      - Two integers are coprime if their gcd is 1
  - chooses a random exponent **a** in [0, p-2]
  - computes $\mathbf{A = g^a \bmod p}$
  - **public key** (published in the phone book): **(p,g,A)**
  - **private key**: **a**

# ElGamal Cryptosystem

- **Encryption:** Alice has a message **m** (0<=m<n) to be sent to Bob:

  – finds out **Bob's public key (p,g,A)**.
  – chooses a random exponent **b** in [0,p-2]
  – computes **$B = g^b \bmod p$**
  – computes **$c = A^b m \bmod p$**.
  – The complete ciphertex is **(B,c)**
  – sends the ciphertext **(B,c)** to Bob.

# ElGamal Cryptosystem

- **Decryption**: Bob
  - receives the ciphertext **(B,c)** from Alice.
  - uses his matching private decryption key a to calculate **m** as follows.
    - Compute $x = p-1-a$
    - Compute $m = B^x c \bmod p$

# ElGamal Cryptosystem

- Randomized cryptosystem
- Based on the Diffie–Hellman key exchange
- Efficiency
  - The ciphertext is twice as long as the plaintext. This is called message expansion and is a disadvantage of this cryptosystem.
- Security
  - Its security depends upon the difficulty of a certain problem related to computing discrete logarithms.

# Rabin Cryptosystem

**Key Generation** (for Bob)

- generates 2 large random and distinct primes **p, q** s.t.

  p (mod 4) = q (mod 4) = 3 (other options are possible, this makes decryption more efficient)

- multiplies p and q: **n = p × q**
- **public key** (published in the phone book): **n**
- **private key**: **(p, q)**

# Rabin Cryptosystem

- **Encryption:** Alice has a message **m** (0<=m<n) to be sent to Bob:
  - finds out **Bob's public key n**.
  - calculates the ciphertext **$c = m^2 \bmod n$**.
  - sends the ciphertext **c** to Bob.

# Rabin Cryptosystem

- **Decryption:** Bob
  - receives the ciphertext **c** from Alice.
  - uses his matching private decryption key **(p,q)** to calculate **m** as follows.
    - Compute $\mathbf{m_p = c^{(p+1)/4} \bmod p}$
    - Compute $\mathbf{m_q = c^{(q+1)/4} \bmod q}$
    - Find $\mathbf{y_p}$ and $\mathbf{y_q}$ such that $\mathbf{y_p\, p + y_q\, q = 1}$ (Euclidean algorithm)
    - Compute $\mathbf{r = (y_p\, p\, m_q + y_q\, q\, m_p) \bmod n}$
    - Compute $\mathbf{s = (y_p\, p\, m_q - y_q\, q\, m_p) \bmod n}$
    - One of **r, -r, s, -s** must be the original message **m**

# Rabin Cryptosystem

- Efficiency
  - Encryption more efficient than RSA encryption

- Security
  - The Rabin cryptosystem has the advantage that the problem on which it relies has been proved to be as hard as integer factorization
    - Recovering the plaintext ***m*** from the ciphertext ***c*** and the public key ***n*** is computationally equivalent to factoring
    - Not currently known to be true for the RSA problem.