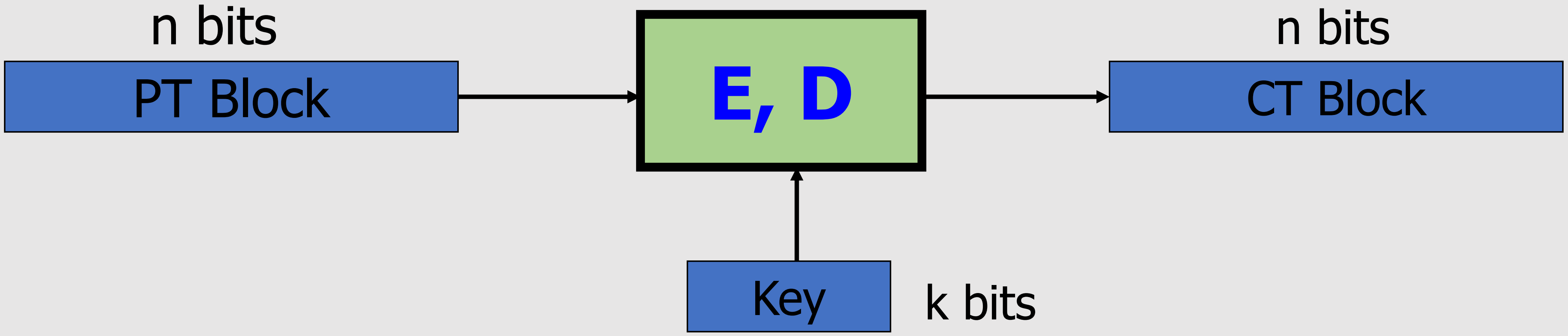
Modes of Operation
(using block ciphers)

Outline

- One-Time Key
  - Semantic Security
  - Electronic Code Book (ECB)
  - Deterministic Counter Mode (DETCTR)
- Many-Time Key
  - Semantic Security for Many-Time Key:
    Semantic Security under Chosen-Plaintext Attack (CPA)
  - Cipher Block Chaining (CBC)
    - Randomized
    - Nonce-based

Review: PRPs and PRFs

Block Ciphers



Canonical examples:

- **DES**: n= 64 bits, k = 56 bits
- **3DES**: n= 64 bits, k = 168 bits
- **AES**: n=128 bits, k = 128, 192, 256 bits

Abstractly: PRPs and PRFs

- **Pseudo Random Function** (**PRF**) defined over (K,X,Y):

$$F: K \times X \to Y$$

such that there exists "efficient" algorithm to evaluate F(k,x)

- **Pseudo Random Permutation** (**PRP**) defined over (K,X):

$$E: K \times X \to X$$

such that:

1. There exists "efficient" deterministic algorithm to evaluate E(k,x)
2. The function $E(k, \cdot)$ is one-to-one, for every k
3. There exists "efficient" inversion algorithm D(k,y)

Using block ciphers

- Don’t think about the **inner-workings** of AES and 3DES.
- We assume both are **secure PRPs** and will see how to use them

Modes of Operation

How to use a **block cipher** on **messages consisting of more than one block**

- **One-Time Key**
  - Electronic Code Book
  - Deterministic Counter Mode
- **Many-Time Key**
  - Cipher Block Chaining
  - Counter Mode

Modes of Operation
One-Time Key

(example: encrypted email, new key for every message)

Using PRPs and PRFs

Goal: build "secure" encryption from a secure PRP (e.g., AES).

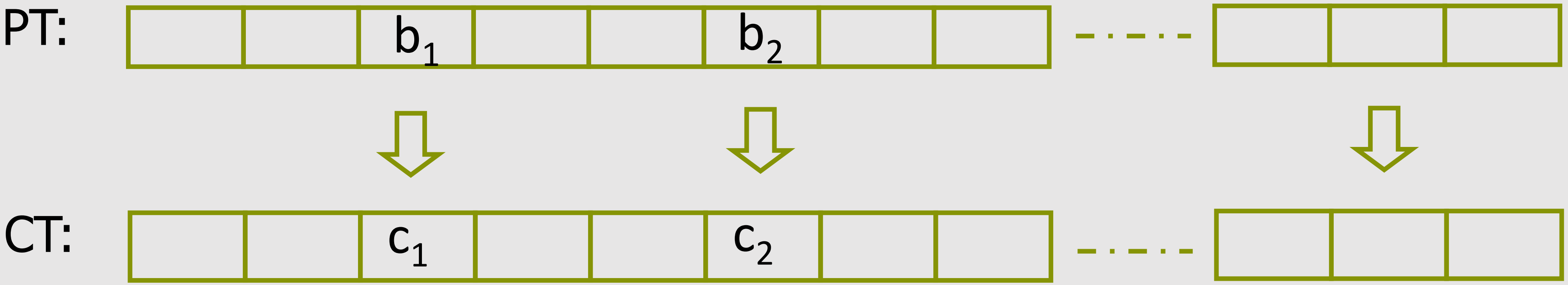
This segment: **one-time key**

1. **Adversary's power:** Adversary sees only one ciphertext (one-time key)
2. **Adversary's goal:** Learn info about PT from CT (semantic security)

Next segment: many-time keys (a.k.a. *chosen-plaintext security*)

Incorrect use of a PRP

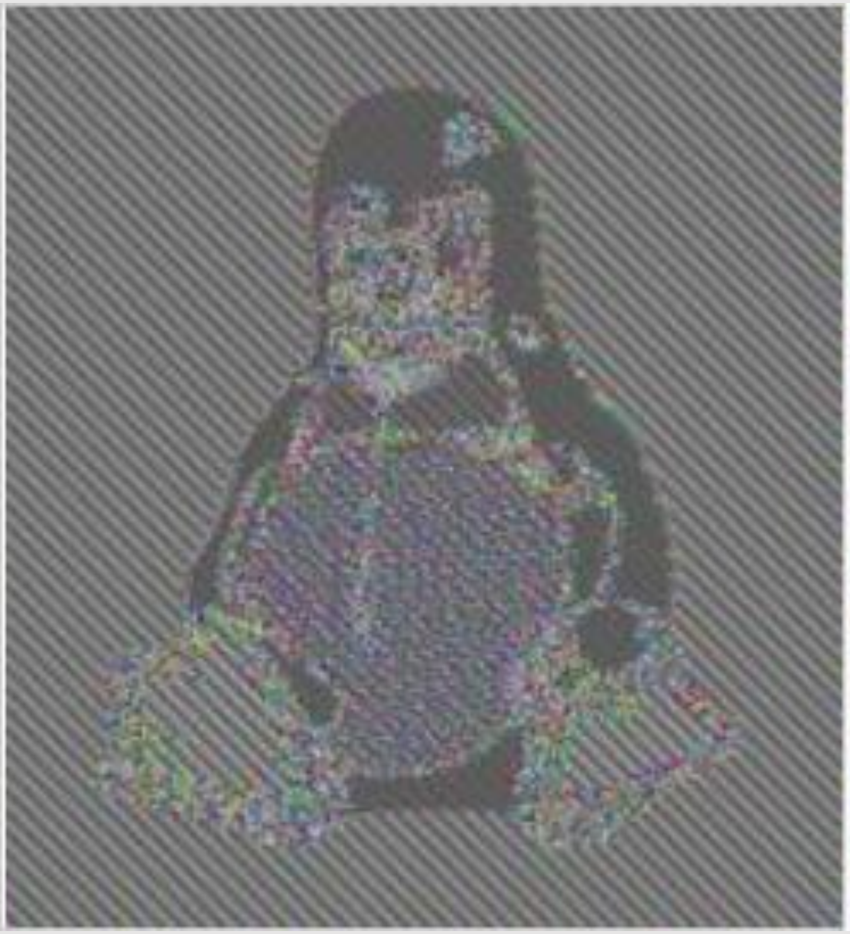
Electronic Code Book (ECB):



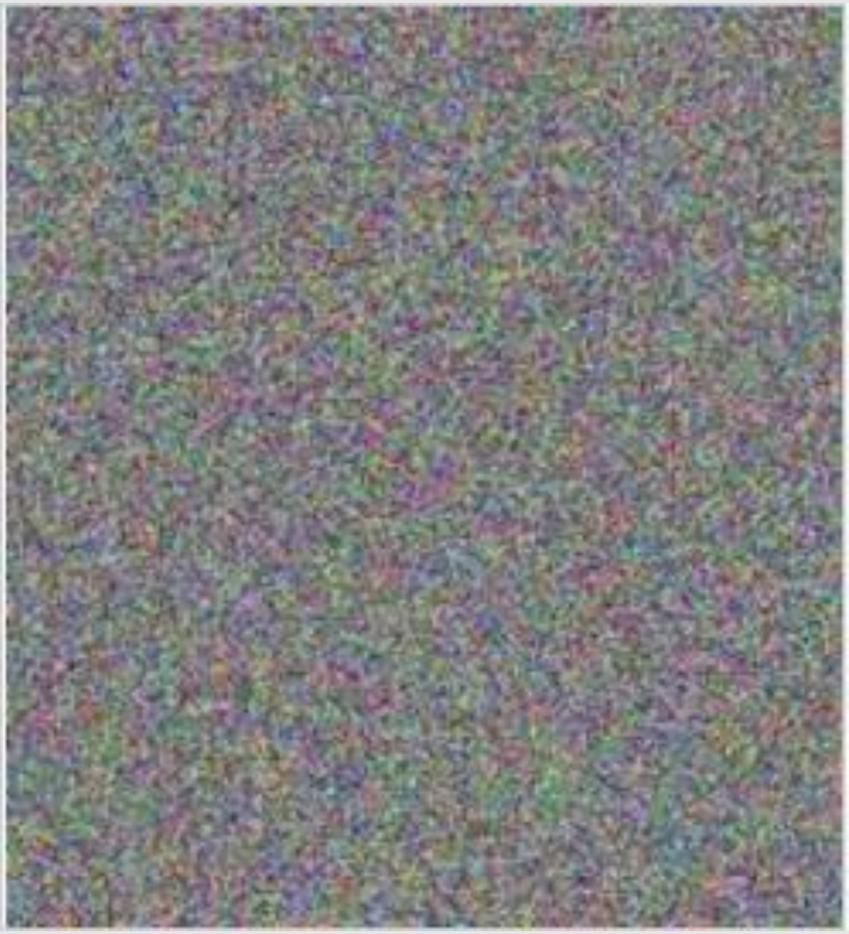
Problem: if $b_1 = b_2$ then $c_1 = c_2$

In pictures

Plain text



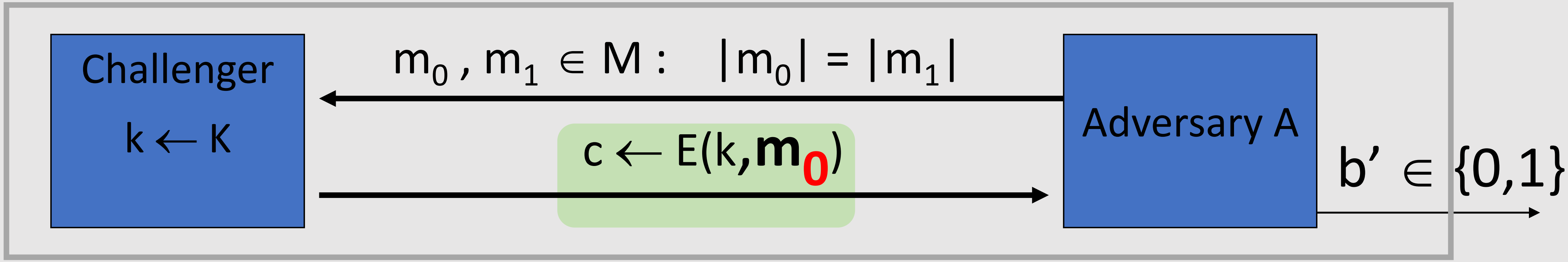
Cipher text with **ECB**



Cipher text with
other modes of operation

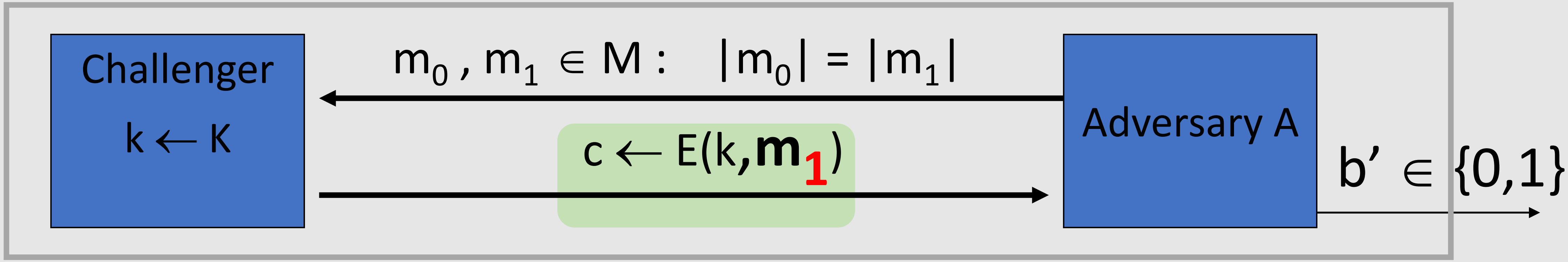
Semantic Security (one-time key)

EXP(0):



one time key ⇒ adversary sees only one ciphertext

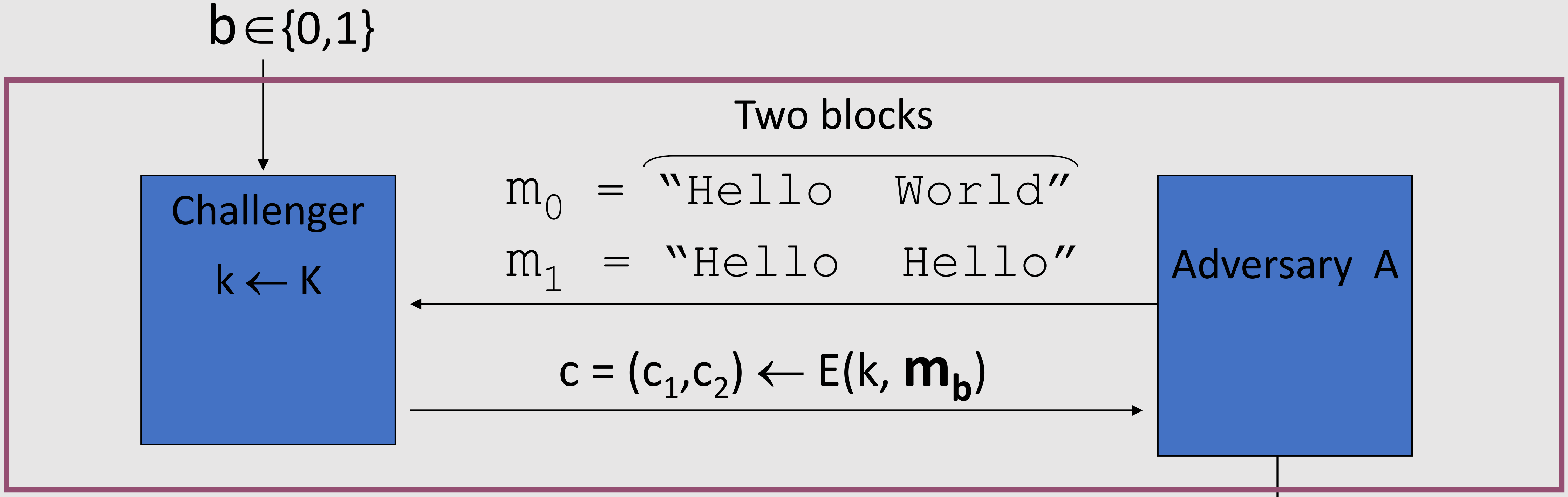
EXP(1):



$\text{Adv}_{SS}[A,\text{Cipher}] = \Big| \Pr[\,\textbf{EXP(0)}=1\,] - \Pr[\,\textbf{EXP(1)}=1\,] \Big|$ should be “negligible” for all “efficient” A

ECB is not Semantically Secure

ECB **is not semantically secure** for messages that contain **more than one block.** (known-plaintext attack)



If $c_1 = c_2$ output 1, else output 0

Then Adv_{SS} [A, ECB] =

Deterministic Counter Mode (Secure Construction)

- **PRF** $F : K \times \{0,1\}^n \to \{0,1\}^n$ (e.g., n=128 with AES)
- **E_{DETCTR} (k, m) =** (Encryption)

⇒ Stream cipher built from a PRF (e.g., AES, 3DES)

Deterministic Counter Mode (Secure Construction)

- **PRF** $F : K \times \{0,1\}^n \rightarrow \{0,1\}^n$ (e.g., n=128 with AES)
- **D_{DETCTR} (k, c) =** (Decryption)

	c[0]	c[1]	…	c[L]
$\oplus$	**F(k,0)**	**F(k,1)**	…	**F(k,L)**
	m[0]	m[1]	…	m[L]

No need to **invert** F when decrypting

Deterministic Counter Mode Security

Theorem: For any L>0,

If **F** is a **secure PRF** over (K,X,X) then

DETCTR is **semantically secure** over (K,X^L,X^L).

In particular, for every efficient adversary **A attacking DETCTR**

there exists an efficient adversary **B attacking F** s.t.:

$$\text{Adv}_{SS}[A, \text{DETCTR}] = 2 \cdot \text{Adv}_{PRF}[B, F]$$

$\text{Adv}_{PRF}[B, F]$ is negligible (since F is a secure PRF)

Hence, $\text{Adv}_{SS}[A, \text{DETCTR}]$ must be negligible.

Modes of Operation
Many-Time Key

Examples:

- File systems: Same AES key used to encrypt many files.
- IPsec: Same AES key used to encrypt many packets.

Semantic Security for Many-Time Key

Key used **more than once** $\Rightarrow$ adversary sees many CTs with same key

(i.e., used for **multiple messages**)

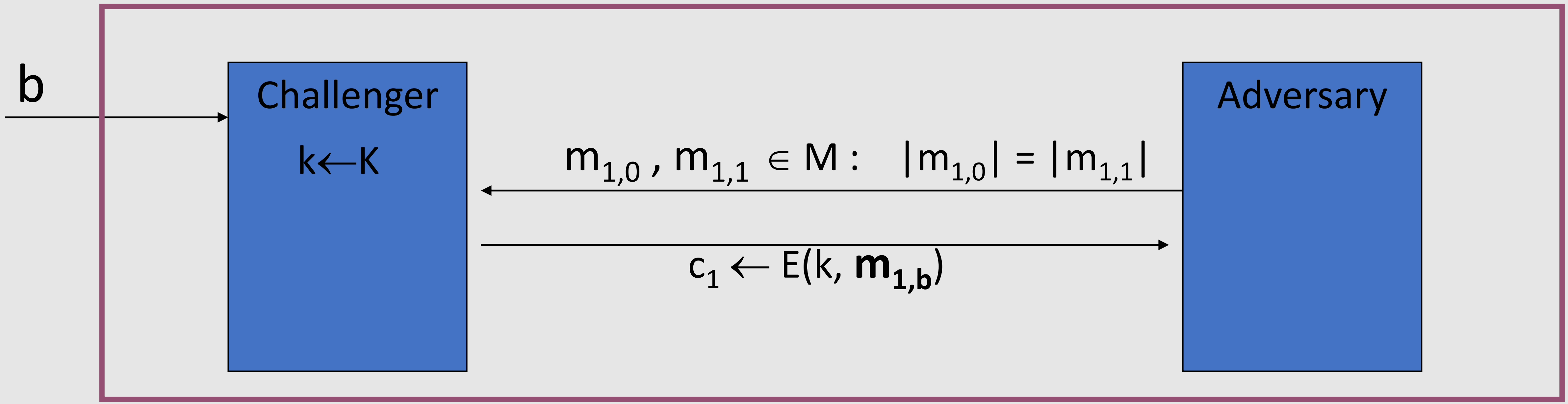
Adversary's power: **Chosen-Plaintext Attack (CPA)**

- Adversary can obtain the encryption of arbitrary messages of his choice (conservative modeling of real life)

Adversary's goal: Break semantic security

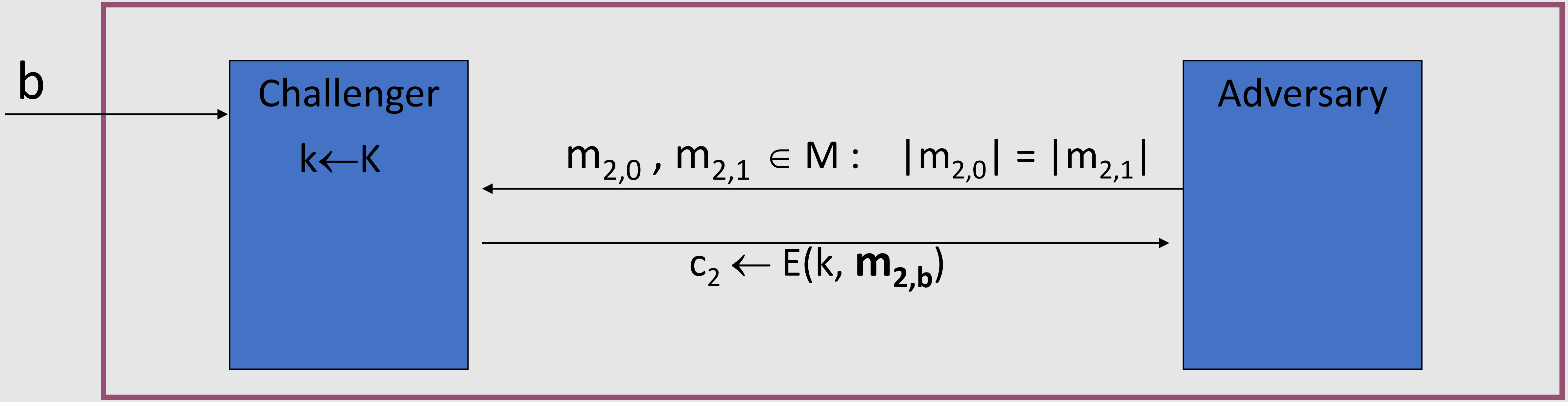
Semantic Security for Many-Time Key (CPA Security)

Q = (E,D) a cipher defined over (K,M,C). For b=0,1 define EXP(b) as:



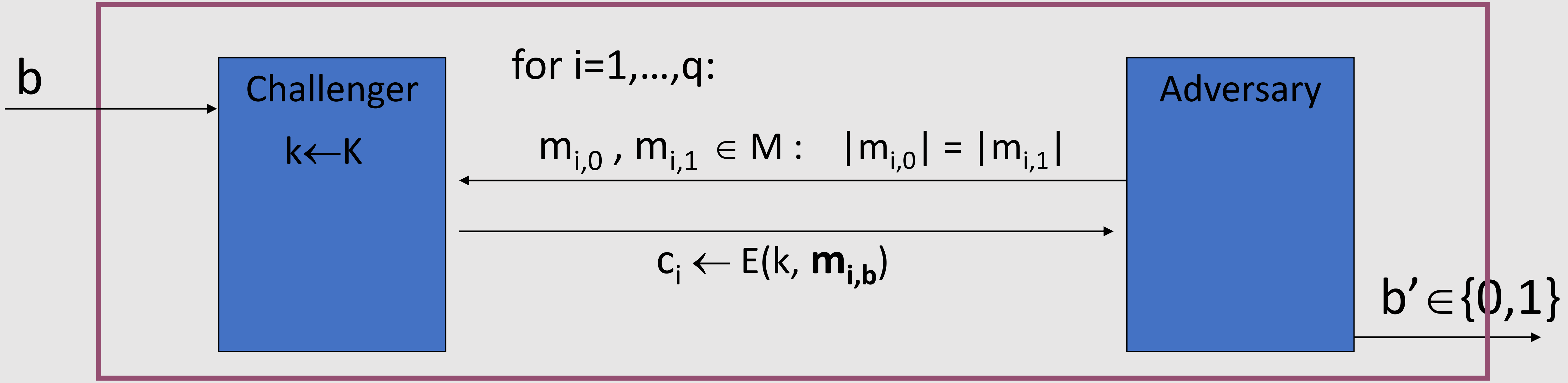
Semantic Security for Many-Time Key (CPA Security)

Q = (E,D) a cipher defined over (K,M,C). For b=0,1 define EXP(b) as:



Semantic Security for Many-Time Key (CPA Security)

Q = (E,D) a cipher defined over (K,M,C). For b=0,1 define EXP(b) as:



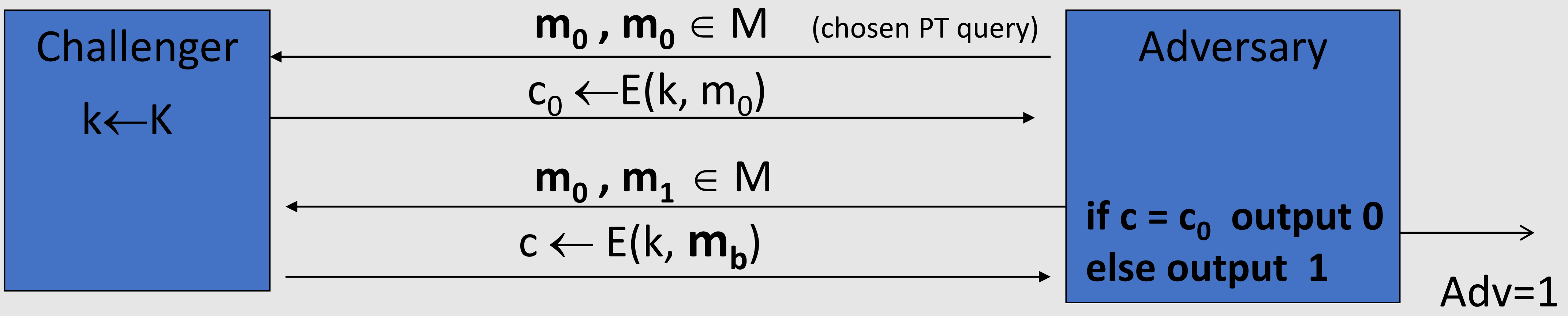
CPA ⇒ if adversary wants c = E(k, m) it queries with $m_{j,0} = m_{j,1} = m$

Definition: Q is **semantically secure under CPA** if for all "efficient" adversary A:

$\mathbf{Adv_{CPA}\,[A,Q] = \left|\, Pr[EXP(0)=1] - Pr[EXP(1)=1] \,\right|}$ is "negligible".

Ciphers Insecure under CPA

Suppose E(k,m) **always outputs same ciphertext for msg m and key k**. Then:

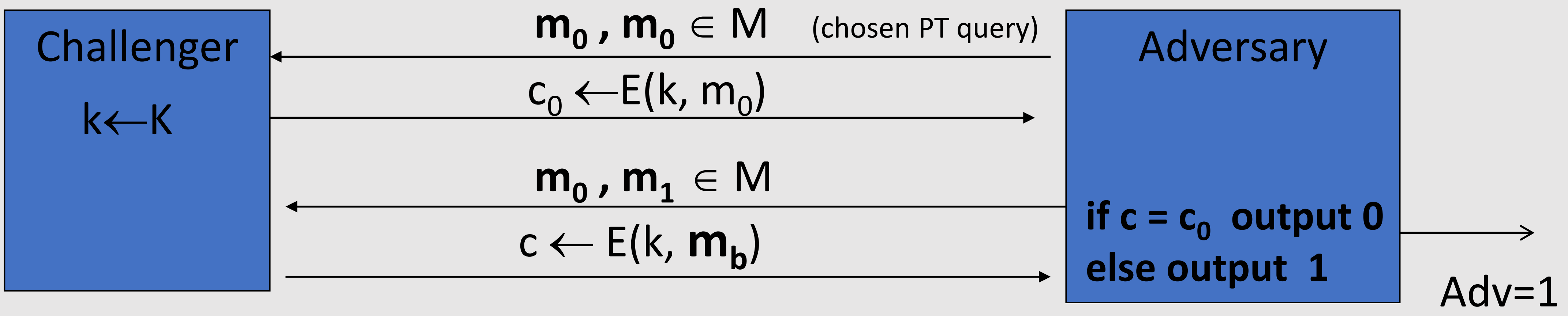


So what? an attacker can learn that two encrypted files are the same, two encrypted packets are the same, etc.

- Leads to significant attacks when the message space M is small

Ciphers Insecure under CPA

Suppose E(k,m) **always outputs same ciphertext for msg m and key k**. Then:

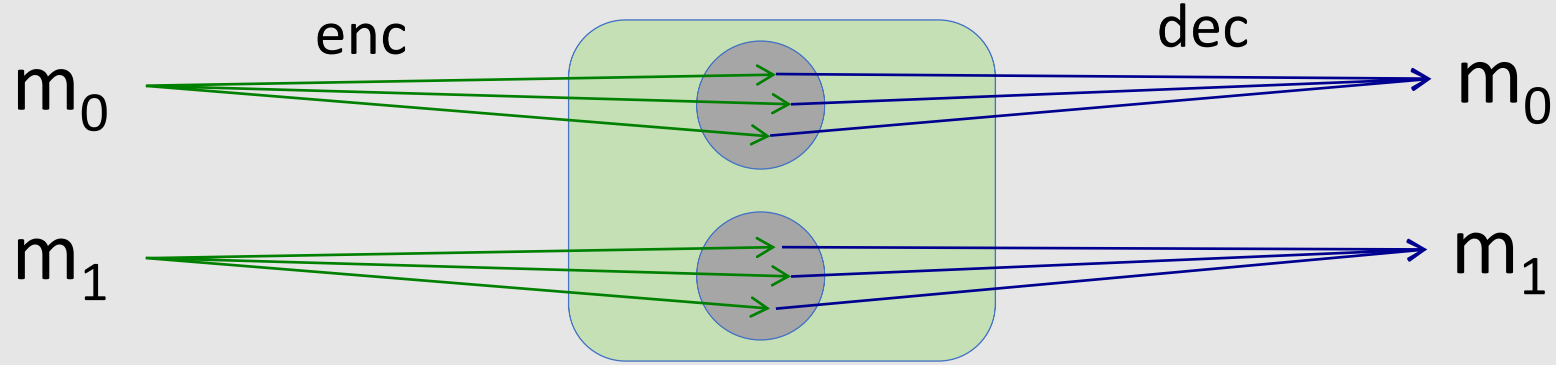


If secret key is to be used multiple times $\Rightarrow$

given **the same plaintext message twice**,
encryption must produce different outputs.

Solution 1:   Randomized Encryption

- E(k,m) is a randomized algorithm:

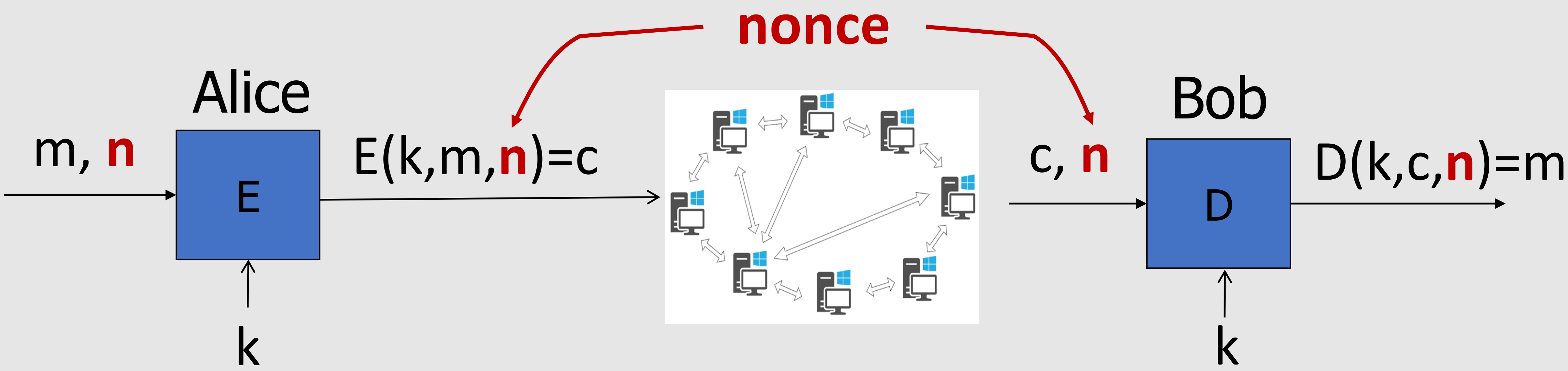


⇒ encrypting same msg twice gives different ciphertexts   (w.h.p.)

⇒ ciphertext must be longer than plaintext

Roughly speaking: CT-size =   PT-size + “# random bits”

Solution 2: Nonce-based Encryption



Nonce n:

- a value that changes from msg to msg
- (k,n) pair **<u>never</u> used more than once**
- n does **not** need to be **secret** and does **not** need to be **random**

Solution 2: Nonce-based Encryption

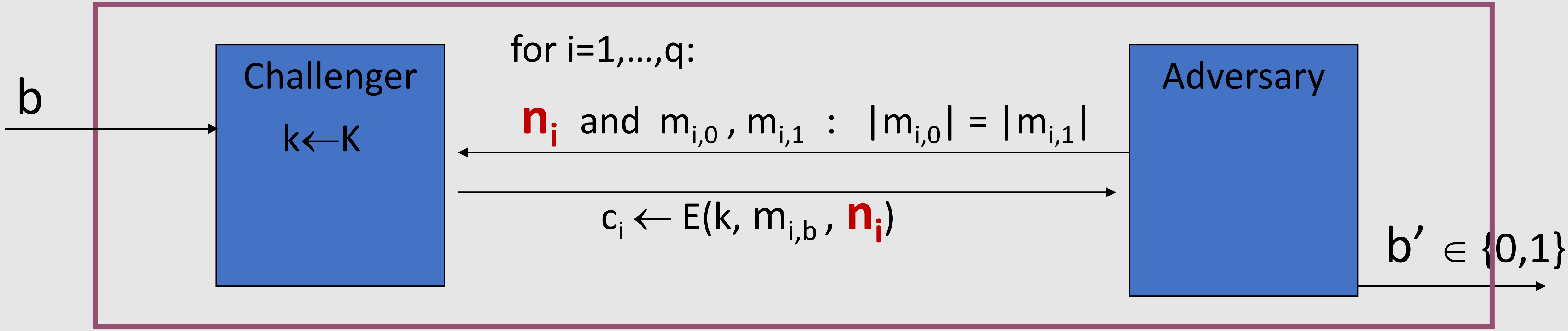
Nonce

- **Method 1:** nonce is a **counter** (e.g., packet counter)
  - used when encryptor keeps state from msg to msg
  - if decryptor has same state, need not send nonce with CT

- **Method 2:** encryptor chooses a **random nonce**, $n \leftarrow \mathcal{N}$
  (It's like randomized encryption)
  (ex. Multiple devices encrypting with the same key)
  - $\mathcal{N}$ must be large enough to ensure that the same nonce is not chosen twice with high probability

CPA Security for Nonce-based Encryption

System should be secure when **nonces are chosen adversarially.**



All nonces $\{n_1, …, n_q\}$ must be distinct.

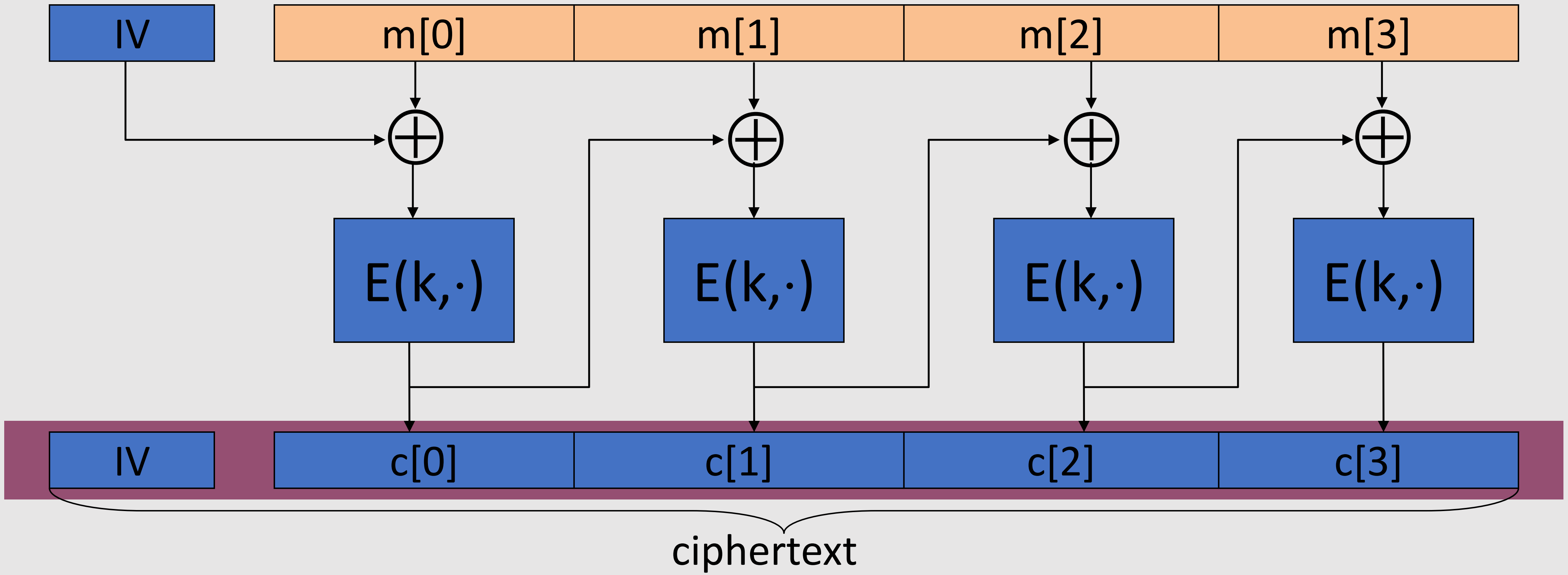
Definition. Nonce-based **Q** is **semantically secure under CPA** if for all "efficient" adversary A:

$$Adv_{nCPA}[A,Q] = |Pr[EXP(0)=1] - Pr[EXP(1)=1]|$$ is "negligible".

Many-time Key Mode of Operation:
Cipher Block Chaining (CBC)

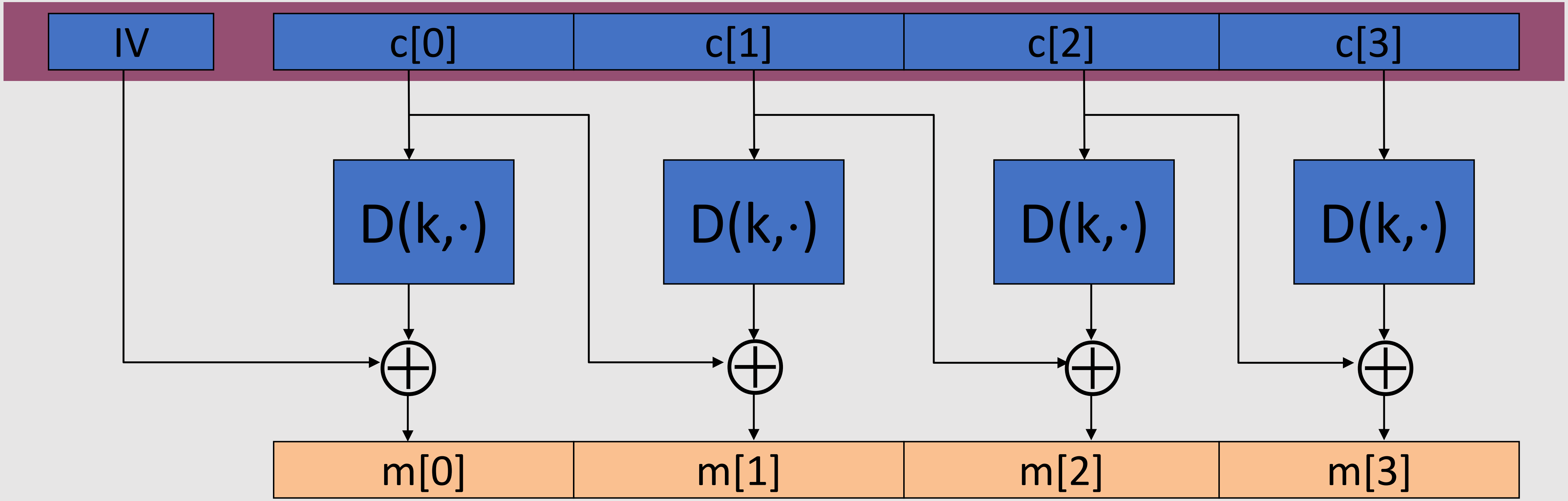
Construction 1: CBC with random IV

- **PRP** $E : K \times \{0,1\}^n \rightarrow \{0,1\}^n$
- (Encryption) $\mathbf{E_{CBC}(k,m)}$**:** choose **random** $IV \in \{0,1\}^n$ and do:



Construction 1: CBC with random IV

- $D : K \times \{0,1\}^n \to \{0,1\}^n$ **inversion algorithm** of E
- (Decryption) $\mathbf{D_{CBC}(k,c)}$:



(Randomized) CBC Security

Theorem: For any L>0 (length of the message we are encrypting),

If **E** is a **secure PRP** over (K,X) then

CBC is **semantically secure under CPA** over (K, X^L, X^{L+1}).

In particular, for every efficient q-query adversary **A** **attacking CBC** there exists an efficient PRP adversary **B** **attacking E** s.t.

$$\mathrm{Adv}_{CPA}[A, CBC] \leq 2 \cdot \mathrm{Adv}_{PRP}[B, E] + \mathbf{2\, q^2 L^2 / |X|}$$

Note: CBC is only secure as long as $q^2L^2 \ll |X|$

(the error term should be negligible)

An example

$$\mathrm{Adv_{CPA}}[A, \mathrm{CBC}] \leq 2 \cdot \mathrm{Adv_{PRP}}[B, E] + \mathbf{2\, q^2 L^2 / |X|}$$

q = # messages encrypted with k , L = length of max message

Suppose we want $\mathrm{Adv_{CPA}}[A, \mathrm{CBC}] \leq 1/2^{32}$ $\Leftarrow$ $q^2 L^2 / |X| < 1/2^{32}$

- AES: $|X| = 2^{128}$ $\Rightarrow$ $q\,L < 2^{48}$

  So, after 2^{48} AES blocks, must change key

- 3DES: $|X| = 2^{64}$ $\Rightarrow$ $q\,L < 2^{16}$

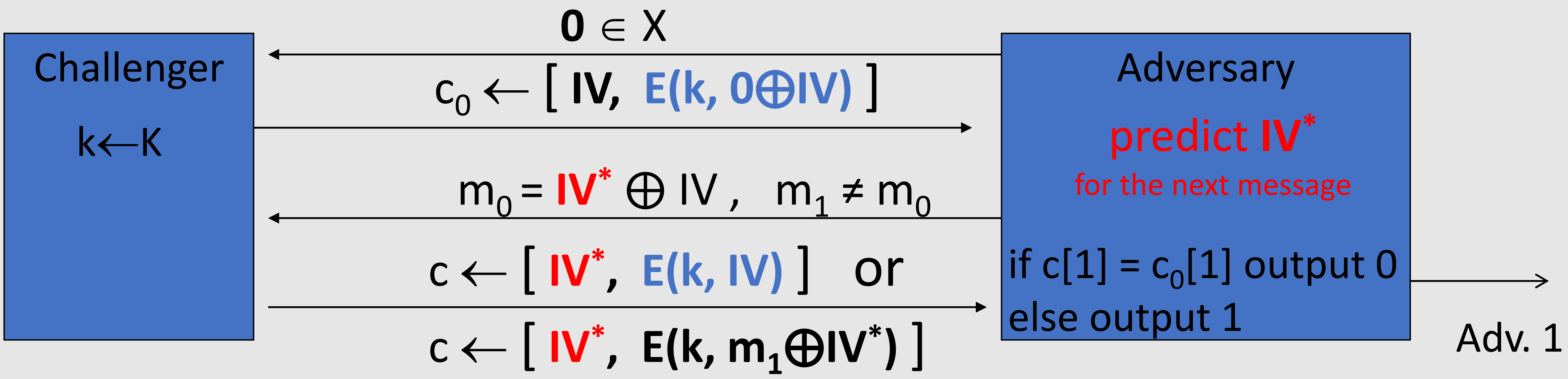
  So, after 2^{16} DES blocks, must change key

  $\Rightarrow$ after 2^{16} blocks (each of 8 bytes) need to change key $\Rightarrow$ $2^{16} \times 8 = ½$ MB !!!

Warning: an attack on CBC with rand. IV

CBC where adversary can **predict** the IV is not CPA-secure !!

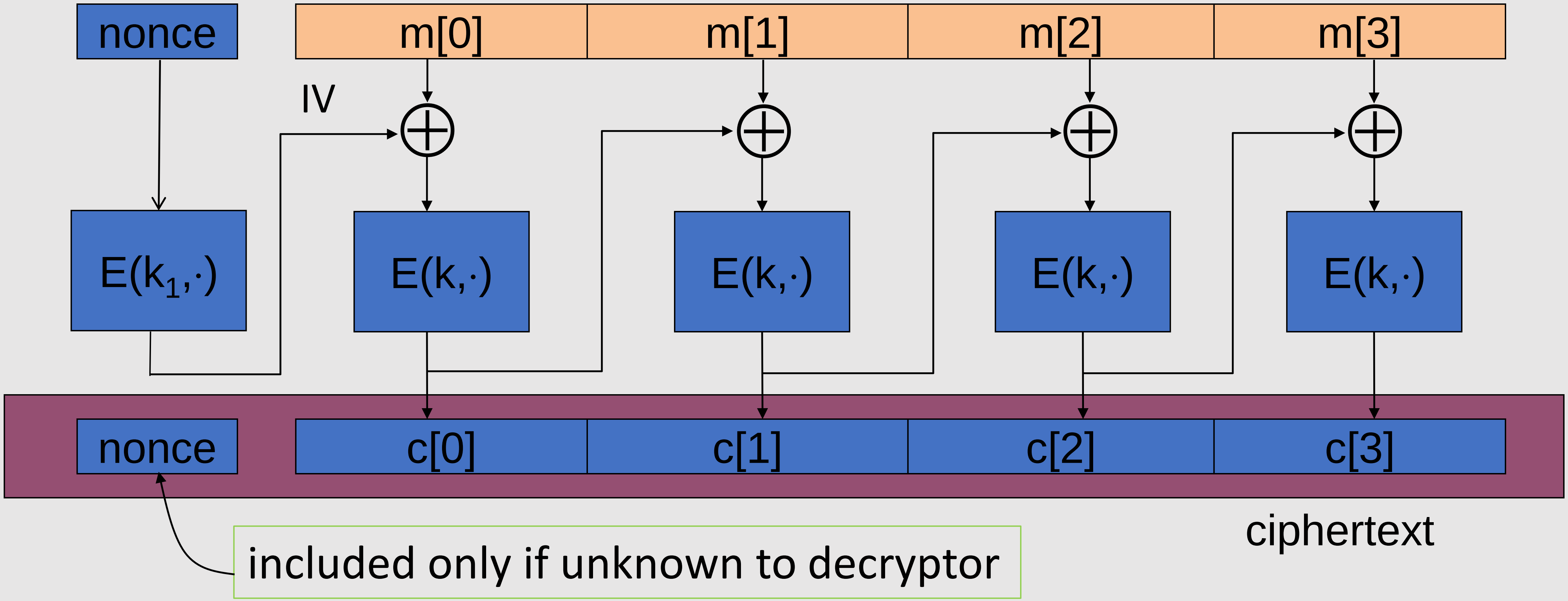
Suppose given $c \longleftarrow E_{CBC}(k,m)$ adversary can predict IV for next message



Bug in SSL/TLS 1.0: IV for record #i is last CT block of record #(i-1)

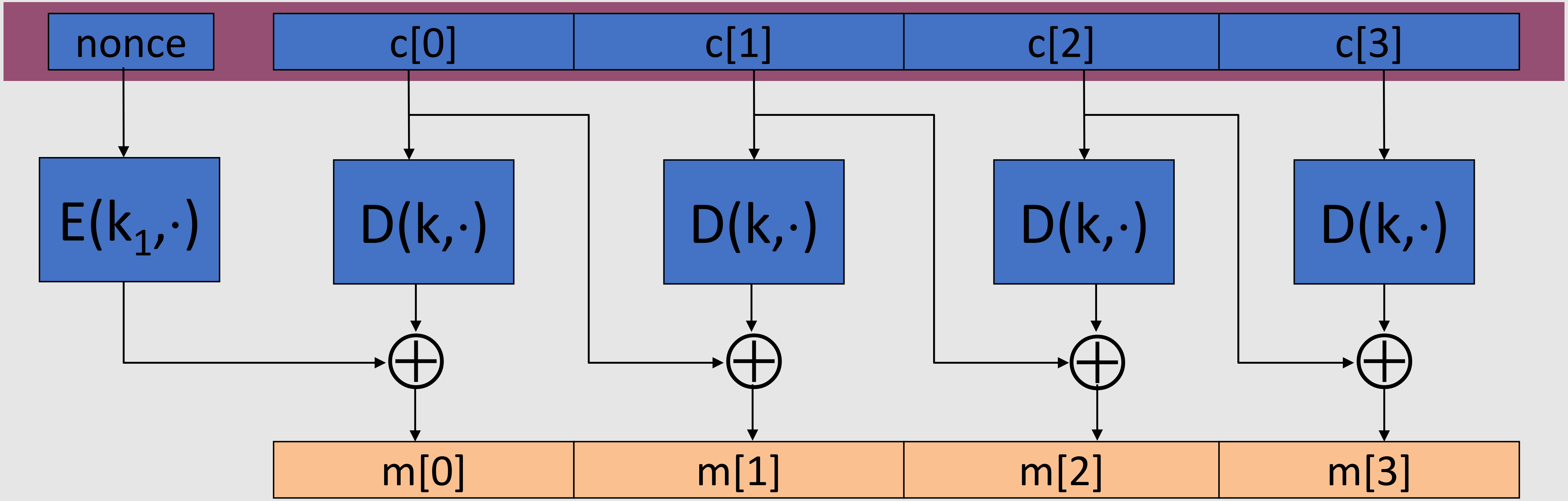
Construction 2: Nonce-based CBC

- key = (**k, k_1**)
- (key, nonce) pair is used for only one message
- **Encryption:**



Construction 2: Nonce-based CBC

- **Decryption:**

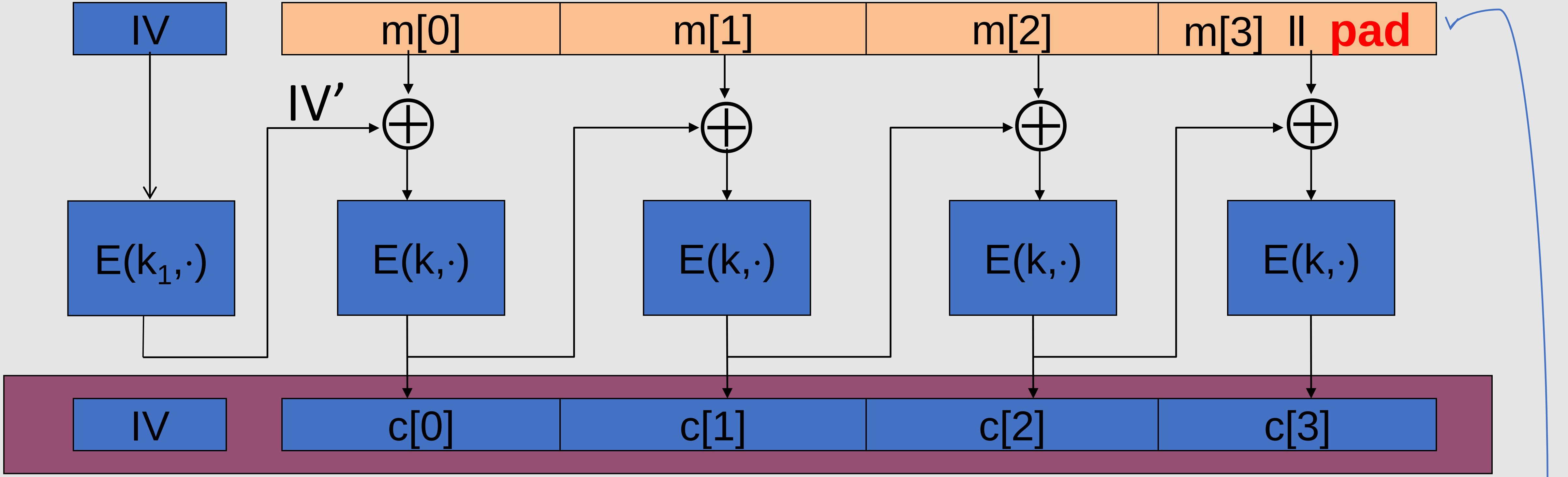


An example Crypto API (OpenSSL)

```
void AES_cbc_encrypt(
        const unsigned char *in,
        unsigned char *out,
        size_t length,
        const AES_KEY *key,
        unsigned char *ivec,            ⟵  user supplies IV
        AES_ENCRYPT or AES_DECRYPT);
```

When it is non-random need to encrypt it before use
(Otherwise, no CPA security!!)

A CBC technicality: padding



removed during decryption

TLS: for n>0, n byte pad is | n | n | n | ••• | n |

if no pad needed, add a dummy block | 16 | 16 | 16 | ••• | 16 |