20/05/2020

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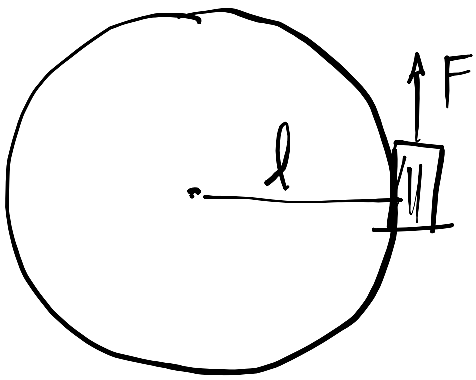


$l = 5m$

$F = 1N$

$T_{max} = 3N$

$m = 15kg$

- t_1 a cui la fune si spezza
- $|\vec{a}|$ a $t = \frac{t_1}{2}$
- Δx tra t_1 e $2t_1$

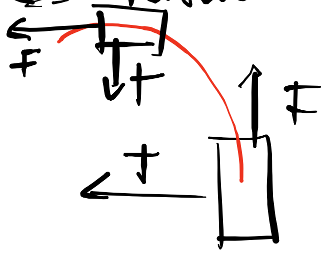
Moto circolare Uniforme ($r = l = 5m$)

$$\underbrace{\theta}_{\text{Pos. angolare}} = \theta_0 + \underbrace{\omega_0}_{\text{velocità angolare}} t + \underbrace{\gamma}_{\text{acc. angolare}} \frac{t^2}{2} \qquad \boxed{4}$$

$$\omega(t) = \frac{d}{dt}\theta = \omega_0 + \gamma t \qquad \boxed{3}$$

inoltre
$$\begin{cases} \vec{v} = \vec{\omega} \times \vec{r} & \boxed{1} \\ \vec{a} = \underbrace{\vec{\omega} \times \vec{v}}_{\text{radiale}} + \underbrace{\vec{\gamma} \times \vec{r}}_{\text{tangenziale}} & \boxed{2} \end{cases}$$

$t_1 \Leftrightarrow$ trovare quando $T = T_{max} = 3N$



forza centripeta

$$T = F_c = m v^2/R$$

$$\frac{v_{max}^2}{R} m = T_{max} \Leftrightarrow \boxed{v_{max} = \sqrt{\frac{R\,T_{max}}{m}}}$$

1

$$\boxed{\omega_{max} = \sqrt{\frac{T_{max}}{Rm}}} \quad 5$$

$\vec{F} \parallel \vec{u}_T$

$$\hookrightarrow F = m a_T = m \gamma R \Rightarrow \boxed{\gamma = \frac{F}{Rm}} \quad 7$$

$$\omega(t) = \underbrace{\cancel{\omega_0}}_{=0} + \gamma t$$

$$\boxed{\omega(t) = \frac{F}{Rm} t} \quad 6$$

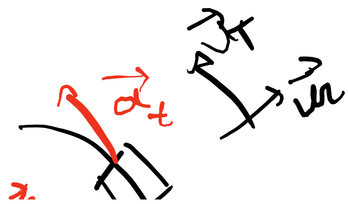
da 5 & 6

$$\omega(t_1) = \omega_{max} \Leftrightarrow \sqrt{\frac{T_{max}}{Rm}} = \frac{F}{Rm} t_1$$

$$\boxed{t_1 = \sqrt{\frac{T_{max} R \cdot m}{F^2}}} \rightarrow \sqrt{3 \cdot 5 \cdot 15}$$

$= 15$ sec

2) $|\vec{a}|$ @ $t = t_1/2$



$$\dot{a} = \vec{\omega}\times\vec{v} + \gamma \times \vec{r}$$

$$= -\omega^2 R\, \vec{u}_r + \gamma R\, \vec{u}_T$$

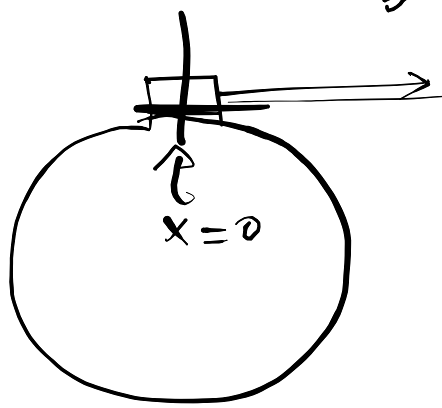
$\vec{a}_r = a_t$

$\boxed{6}\boxed{7} \rightarrow \vec{a} = -\left(\frac{F}{Rm}t\right)^2 R\, \vec{u}_r + \frac{F}{\cancel{R}m} \cancel{R}\, \vec{u}_T$

$$|\vec{a}(t_1/2)| = \sqrt{\frac{F^4 (t_1/2)^4}{R^2 m^4} + \left(\frac{F}{m}\right)^2}$$

$$= \sqrt{\frac{(15/2)^4}{5^2\ 15^4} + \frac{1}{15^2}}$$

3



moto unif. accelerato

$a = F/m$

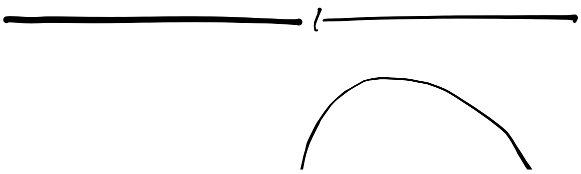
$$X(t) = x_0 + v_{0x} t + a\frac{t^2}{2}$$

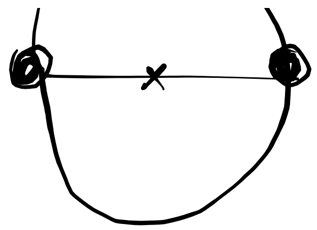
$$\underbrace{X(2t_1) - X(t_1)}_{\Delta x} = v(t_1)(2t_1) + a\frac{(2t_1)^2}{2}$$

$$v_{max} = \sqrt{\frac{R F_{max}}{m}} = 1\ \frac{m}{s}$$

$$\Delta x = 2t_1\left(1 + \frac{F t_1}{m}\right) = 30(1+1) = 60\ \underline{m}$$

9/06/2005





$m = 0.1\,kg$

$|v| = 8\,m/s$

$R = 0.5\,m$

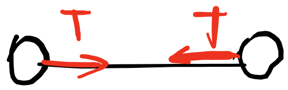
- velocità angolare $v = \omega R \Rightarrow \omega = v/R$
  $= \frac{8}{0.5} = 16\,rad/s$

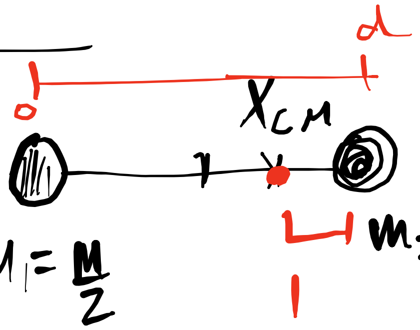
- Periodo $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = 0.39\,s$

- acc. centripeta $a_c = v^2/R = \omega^2 R = \frac{(16)^2}{2} = 128\,m/s^2$

- tensione del filo

$T = F_c = m a_c = 0.1 \times 128$
$= \underline{\underline{12.8\ N}}$





$m_2 = m$

$m_1 = \frac{M}{2}$

$v_1 = v = 8\,m/s$

→ Posizione del baricentro

$$X_{CM} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

$$= \frac{x_1 \cancel{M}/2 + \cancel{M} x_2}{}$$

$\frac{\cancel{m}}{2} + \cancel{m}$

$$= \frac{x_1 + 2x_2}{1+2} = \boxed{\frac{x_1 + 2x_2}{3}}$$

Assumendo $x_1 = 0,\ x_2 = d = 1m$

$$\boxed{x_{cm} = \frac{2}{3} d} \rightarrow R_2 = d/3 = 1/3\ m$$

$$R_1 = \frac{2}{3} d = 2/3\ m$$

→ velocità v_2

$$\begin{cases} v_1 = \omega R_1 \\ v_2 = \omega R_2 \end{cases} \Rightarrow \omega = v_1 / R_1$$

↳ stesso ω

$$v_2 = v_1 \frac{R_2}{R_1} \Rightarrow v_2 = \frac{v}{2}$$

$$\underline{\underline{v_2 = 4\ m/s}}$$

→ Periodo T

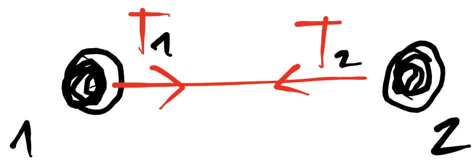
$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{v_1} R_1$$

$$T = 0.5\ sec$$

→ Acc. centripeta

$$a_c = \omega^2 R \begin{cases} a_{c_1} = \omega^2 R_1 = v^2 \frac{3}{2d} = 96\ m/s^2 \\ a_{c_2} = \omega^2 R_2 = v^2 \frac{3/4}{d} = 48\ m/s^2 \end{cases}$$

→ tensione del filo



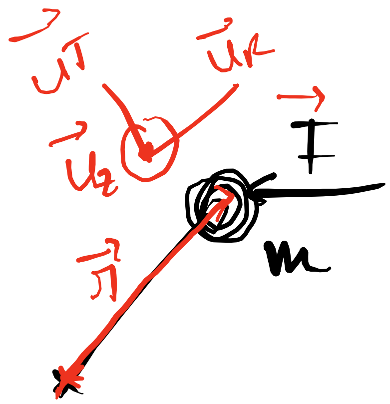
$$\begin{cases} T_1 = F_{c1} = m_1 v_1^2 / R_1 \\ T_2 = F_{c2} = m_2 v_2^2 / R_2 \end{cases}$$

$$\rightarrow \begin{cases} T_1 = \dfrac{3m/2}{2d} v^2 \\ T_2 = \dfrac{3 m v^2}{4d} \end{cases} \Big\} \; T_1 = T_2 = T$$

$$T = \frac{3}{4}\frac{m v^2}{d} = \underline{\underline{4.8\ N}}$$

→ Dinamica dei moti rotatori & Momento angolare

Momento torcente



$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{F} = m\vec{a}$$

$$\vec{\tau} = m\,\vec{r} \times \vec{a}$$

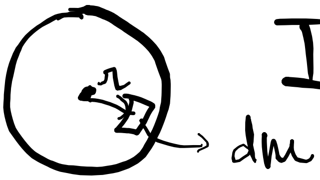
visto che $\vec{a} = a_r \vec{u}_r + a_T \vec{u}_T \rightarrow \vec{\tau} = m\, \underbrace{\vec{r} \times (a_r \vec{u}_r}_{=0\ \vec{r} \parallel \vec{u}_r} + \overbrace{a_T}^{r\alpha} \vec{u}_T)$

$$\boxed{\vec{\tau} = m r^2 \alpha\, \vec{u}_z}$$

$\alpha \rightarrow$ acc. angolare

$I =$ momento d'inerzia (particella puntiforme)

Per corpi rigidi (non-puntiformi)



$$I = \int dI = \int dm\, r^2$$

distanza dall'asse di rotazione

2ª legge di Newton per moti rotatori

$$\boxed{\sum_i \vec{\tau}_i = I\vec{\alpha}}$$

→ Momento Angolare (quantità di moto di rotazione)

$$\vec{l} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{l}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

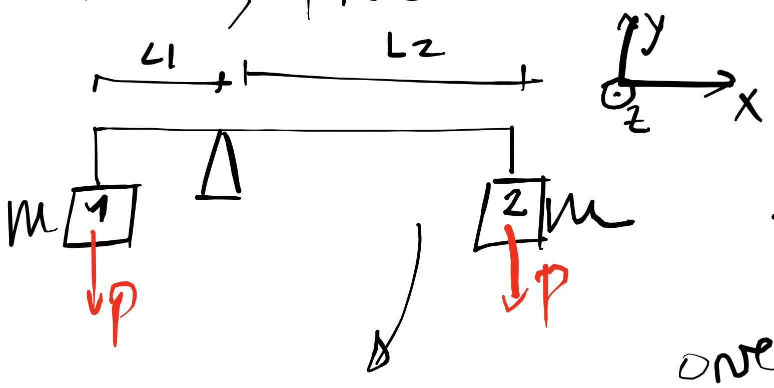
$$= \underbrace{\left(\frac{d\vec{r}}{dt}\right) \times \vec{p}}_{\underbrace{\vec{v} \times (m\vec{v})}_{0}} + \vec{r} \times \underbrace{\left(\frac{d}{dt}\vec{p}\right)}_{\sum_i \vec{F}_i}$$

$$\frac{d\vec{l}}{dt} = \vec{r} \times \sum_i \vec{F}_i = \sum_i \underbrace{\vec{r} \times \vec{F}_i}_{\vec{\tau}_i}$$

$$\boxed{\frac{d\vec{l}}{dt} = \sum_i \vec{\tau}_i}$$

	tras	rot
quantità	$\vec{p} = m\vec{v}$	$\vec{l} = \vec{r} \times \vec{p} = I\vec{\omega}$
	$\sum \vec{F}_i = \frac{d\vec{p}}{dt} = m\vec{a}$ m costante	$\sum_i \vec{\tau}_i = \frac{d\vec{l}}{dt} = I\vec{\alpha}$ I costante
	$\sum_i \vec{F}_i = 0$ $\downarrow$ $\vec{p}$ = costante	$\sum_i \vec{\tau}_i = 0$ $\downarrow$ $\vec{l}$ = costante

cap 9, Problema 17



$$\vec{\tau} = \vec{r}_1 \times \vec{P} + \vec{r}_2 \times \vec{P} \quad \boxed{1}$$

dove $\begin{cases} \vec{r}_1 = -L_1 \vec{u}_x \\ \vec{r}_2 = L_2 \vec{u}_x \end{cases}$

$\vec{P} = -mg\,\vec{u}_y$

$\boxed{1}$ $\vec{\tau} = mg(-L_2\vec{u}_z + L_1\vec{u}_z)$ $\Big\}$ $\vec{\alpha} = g\,\underline{L_1 - L_2}\ \vec{u}_z$

$$\vec{\tau} = I\vec{\alpha} = \underbrace{(mL_1^2 + mL_2^2)}_{I}\,\alpha \qquad \underline{\qquad L_1^2 + L_2^2}$$

acc. lineare $\quad a = \alpha r$

$$\begin{cases} a_1 = \alpha L_1 \iff a_1 = g\,\dfrac{L_1(L_1 - L_2)}{L_1^2 + L_2^2} \rightarrow 1.73 \text{ m/s}^2 \\ a_2 = \alpha L_2 \end{cases}$$

$$a_2 = g L_2 \frac{L_1 - L_2}{L_1^2 + L_2^2} \rightarrow 6.93 \text{ m/s}^2$$