13/05/2020

Quantità di moto

CAP 6 RHK, Problemi

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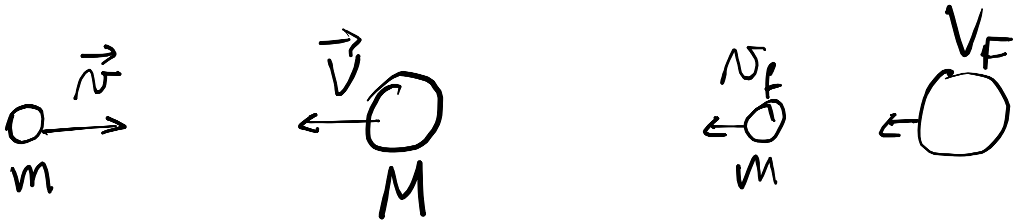


Urto elastico

$$\begin{cases} mv - MV = -m v_f - M V_F \rightarrow \vec{P}_i = \vec{P}_f \\ v' = -v_F' \\ V' = -V_F' \end{cases}$$

$v' = -v_F'$, $V' = -V_F'$ } urto elastico

$v', v_F' \ldots \rightarrow$ nel rif del centro di massa

$$v_f = v_{CM} + v_f' = v_{CM} - v' = 2 v_{CM} - v \quad [1]$$

$\uparrow$ $v = v_{CM} + v'$

nello stesso modo

$$V_F = 2 v_{CM} - V \quad [2]$$

$$v_{CM} = \frac{mv + MV}{m+M}$$

$\hookrightarrow$

[1] $\left\{ v_f = 2 \frac{mv + MV}{\ldots} - v \iff \boxed{v_f = \frac{m-M}{\ldots} v + \frac{2M}{\ldots} V} \right.$

$m + M$ $\qquad$ [3] $M+m \qquad m+M$

[2] $$V_F = 2\,\frac{mv + MV}{m+M} - V \iff V_F = V\,\frac{M-m}{M+m} + \frac{2mv}{m+M}$$ [4]

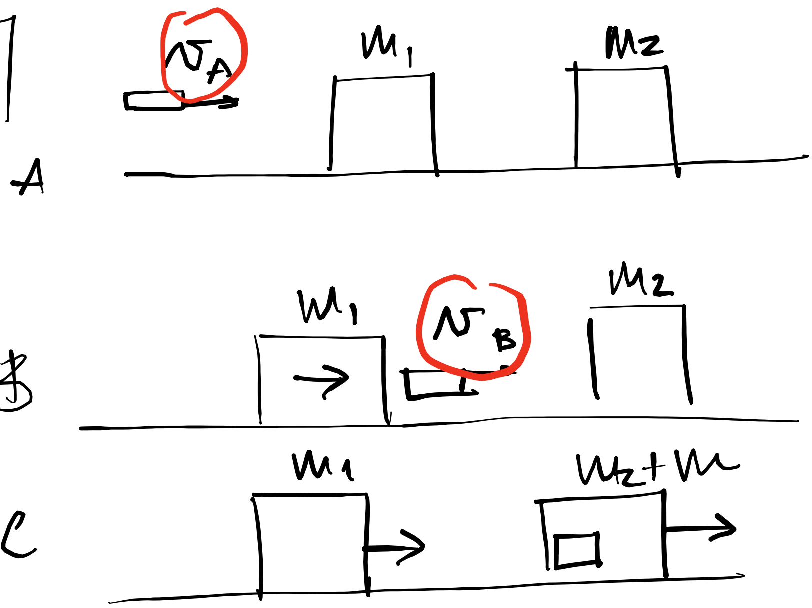
$m \ll M$

$$\begin{cases} \dfrac{m-M}{M-m} = -1 + O(m/M) \\ \dfrac{2M}{m+M} = 2 + O(m/M) \\ \dfrac{2m}{m+M} = 0 + O(m/M) \end{cases}$$

done $\frac{m}{M} \ll 1$

[3] & [4] $\Rightarrow$ $$\begin{cases} v_F = -v + 2V + O(m/M) \\ V_F = V + O(m/M) \end{cases}$$

[19]



$m = 3.54\,g$

$m_1 = 1.22\,kg$

$v_{1C} = 0.63\,m/s$

$\hookrightarrow = v_{1B}$

$m_2 = 1.78\,kg$

$v_{2C} = 1.48\,m/s$

$$\vec{P}_A = \vec{P}_B = \vec{P}_C$$

$\vec{P}_A = \vec{P}_B \qquad m v_A = m_1 v_{1B} + m v_B$

$\hookrightarrow \boxed{v_B} = v_A - v_{1B} \dfrac{m_1}{m}$

$\vec{P}_A = \vec{P}_C \qquad m v_A = m_1 v_{1C} + (m_2 + m) v_{2C}$

$\hookrightarrow \boxed{v_A} = \dfrac{m_1}{m} v_{1C} + \left(\dfrac{m_2}{m} + 1\right) v_{2C}$

$v_A = \dfrac{1.22}{3.54 \times 10^{-3}} \times 0.63 + \left(\dfrac{1.78}{3.54 \times 10^{-3}} + 1\right) 1.48$

$= \underline{\underline{962.8 \text{ m/s}}}$

$v_B = \underline{\underline{745.6 \text{ m/s}}}$

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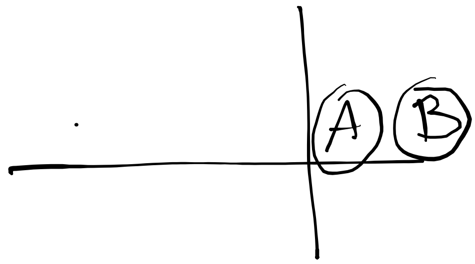
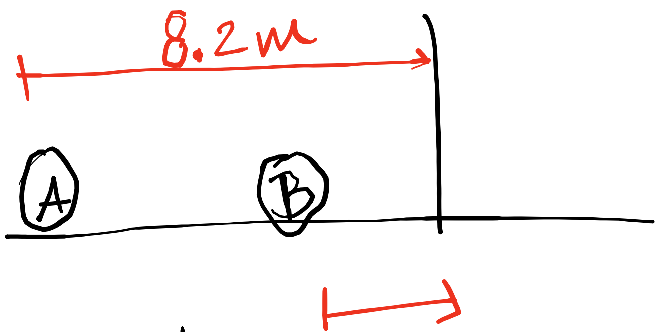
$m_A = 1100 \text{ kg}$

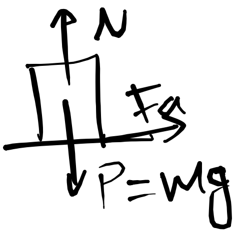
$m_B = 1400 \text{ kg}$

$\mu_k = 0.13$





Dopo l'urto → moto unif. accelerato 6dm



con $\vec{F}_{ris} = \vec{F}_a = \mu_k \vec{N} = m \mu_k g$

$\vec{F}_{ris} = m a$

$N = P = mg$

↳ $a = \mu_k g$ [1]

$$\begin{cases} x = x_0 + v_{0x} t - \frac{a}{2} t^2 \\ v = v_{0x} - a t \end{cases} \Rightarrow t = \frac{v_{0x}}{a} \Rightarrow \Delta x = \frac{v_{0x}^2}{2a} \quad [2]$$

[2] $v_{0x} = \sqrt{2 \Delta x \, a} \Rightarrow v_{0x} = \sqrt{2 \mu_k g \Delta x}$

↑ [1]

$v_{0A} = 4.4 \, m/s$
$v_{0B} = 3.82 \, m/s$
} velocità delle auto dopo l'urto

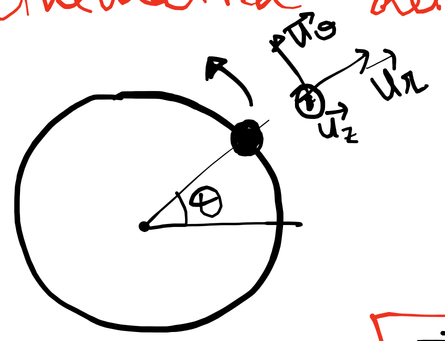
(b) velocità di tamponamento
↳ cons. di p immediatamente prima e dop l'urto

$$m_B v_B = m_A v_{0A} + m_B v_{0B}$$

$$v_B = v_{0B} + \frac{m_A}{m_B} v_{0A}$$

$\hookrightarrow$ $v_B = 7.27 \text{ m/s}$

Cinematica dei moti rotatori



In coord polari il moto è descritto da $r(t)$ $\theta(t)$

$$\vec{v} = \dot{r}\,\vec{u}_r + r\dot{\theta}\,\vec{u}_\theta \quad [1]$$

$\dot{r}$ → vel. radiale; $r\dot{\theta}$ → vel. tang.

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{u}_\theta \quad [2]$$

acc. radiale; acc. tang.

velocità angolare $\frac{d\theta}{dt} = \dot{\theta} \equiv \omega$

acc. angolare $\frac{d\omega}{dt} = \frac{d^2\theta}{dt} = \ddot{\theta} \equiv \alpha$

Se α è costante possiamo integrare (2x) per trovare $\omega(t)$, $\theta(t)$

$$\frac{d\omega}{dt} = \alpha \Rightarrow \omega = \omega_0 + \int \alpha\, dt$$

$$\omega = \omega_0 + \alpha t$$

$$\omega = \frac{d\theta}{dt} \Rightarrow \theta(t) = \theta_0 + \int dt\, \omega(t)$$

$$\theta(t) = \theta_0 + \omega_0 t + \alpha \frac{t^2}{2}$$

[1] & [2] per moto circolare uniforme
↳ $\dot{r} = \ddot{r} = 0$

$$\vec{v} = r\omega \vec{u}_\theta = \vec{\omega} \times \vec{r}$$

↑ Prod. Vettoriale

$$\vec{a} = \underbrace{\vec{\omega} \times \vec{v}}_{\text{radiale}} + \underbrace{\vec{\alpha} \times \vec{r}}_{\text{tang}}$$

Cap 8

Problemi

[1] $\phi(t) = 4t - 3t^2 + t^3$ (rad)

a) $\omega(t=2)$ & $\omega(t=4)$

$$\omega = \frac{d\phi}{dt} = 4 - 6t + 3t^2 \quad (\text{rad/s})$$

$$\omega(t=2) = 4 - 6 \times 2 + 3 \times 4 = 4 \; ? \; - \; ..$$

$$\omega(t=4) = 4 - 6 \times 4 + 3 \times 16 = 28 \text{ rad/s}$$

b) accelerazione angolare media tra $t=2$ e $t=4$

$$\alpha = \frac{d\omega}{dt} \rightarrow \bar{\alpha} = \frac{\Delta\omega}{\Delta t} =$$

$$\bar{\alpha} = \frac{\omega(t=4) - \omega(t=2)}{4-2} = 12 \text{ rad/s}^2$$

c) acc. angolare istantanea a $t=2$ & $t=4$

$$\alpha = \frac{d\omega}{dt} = -6 + 6t$$

$$\alpha(t=4) = 18 \text{ rad/s}^2$$

$$\alpha(t=2) = 6 \text{ rad/s}^2$$

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FASE 1

$\alpha_1 = 3 \text{ rad/s}^2$

$\Delta t = 4 \text{ s}$

$$\phi = \phi_{01} + \omega_0 t + \alpha_1 \frac{t^2}{2}$$

FASE 2

$\alpha_2 =$ costante

arresto entro $\Delta t = 0.1 \text{ s}$

$$\phi(t) = \phi_{02} + \omega_{02} t + \frac{1}{2}\alpha_2 t^2$$

dove

$\omega = \omega_0 + \alpha_1 t$

$\phi_{01} = 0$, $\omega_0 = 0$

↳ $\begin{cases} \phi(t) = \alpha_1 t^2/2 \\ \omega(t) = \alpha_1 t \end{cases}$

$\phi(t=4) = 8\alpha_1$

$\omega(t=4) = 4\alpha_1$

$\begin{cases} \omega_{02} = 4\alpha_1 \\ \phi_{02} = 8\alpha_1 \end{cases}$

$\begin{cases} \phi(t) = 8\alpha_1 + 4\alpha_1 t + \frac{1}{2}\alpha_2 t^2 \\ \omega(t) = 4\alpha_1 + \alpha_2 t \end{cases}$ ⊠*

arresto in $\Delta t = \frac{1}{10}$ s

$\omega(0.1) = 0 \iff \alpha_2 = -\frac{4\alpha_1}{1/10}$

$\boxed{\alpha_2 = -40\alpha_1}$

⊠*

$\phi(t=0.1) = 8\alpha_1 + 4\alpha_1 \frac{1}{10} - 20\alpha_1 \left(\frac{1}{10}\right)^2$

$= 24.6$ rad

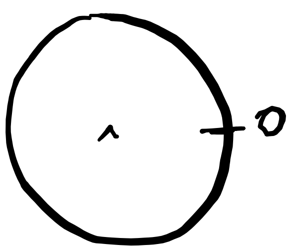
↳ 3.92 giri

↓

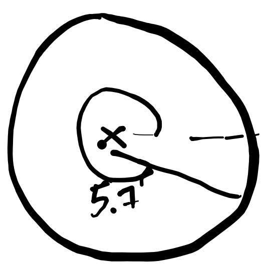
0.92 giri

⟺ 5.7 rad

inizio



fine



(B) velocità angolare media

$\bar{\omega} = \frac{\Delta\phi}{\dots} = \frac{24.6}{\dots} = 6$ rad/s

6

centrifuga di raggio 10.4 m = r
che ruota con

$$\phi(t) = 0.326 t^2 \text{ (rad)}$$

(A) $w = \frac{d\phi}{dt} = 2 \times 0.326\, t$

$= 0.652\, t$

(B) velocita tangenziale

$v_T = w r = 6.52 t$

(C) acc. tang.

$a_T = r \frac{dw}{dt} = r \times 0.652$

$= 6.78\, m/s$

(D) acc. radiale dopo 5.6s :

$a_r = r w^2 = 10.1\, (0.652 \times 5.6)^2$

$= 133.3 m/s^2$