6/05/2020

Quantità di moto

quantità di moto = momento lineare

$$\vec{p} = m\vec{v} \qquad \left[kg \frac{m}{s} \right]$$

ricordiamo che $[N] = \left[kg \frac{m}{s^2} \right] \rightarrow \left[\frac{\Delta p}{\Delta t} \right] = [N]$

2ª legge di Newton

$$\frac{d\vec{p}}{dt} = \vec{F}_{ris} = \sum_i \vec{F}_i \qquad \boxed{1}$$

$$\frac{d}{dt}(m\vec{v}) = \underbrace{\frac{dm}{dt}}_{0} \vec{v} + m\frac{d\vec{v}}{dt}$$

0 per sistemi di massa costante

$$\boxed{1} \Rightarrow m\frac{d\vec{v}}{dt} = \boxed{m\vec{a} = \sum_i \vec{F}_i}$$

integrando nel tempo si trova la dfn. di Impulso

$$d\vec{p} = \sum \vec{F}_i \Rightarrow \int d\vec{p} = \int dt \sum_i \vec{F}_i$$

dt

$$\boxed{\Delta\vec{p} = \int_{t_i}^{t_f} \underbrace{\sum_i \vec{F}_i \, dt}_{\text{Impulso} = \vec{J}}}$$

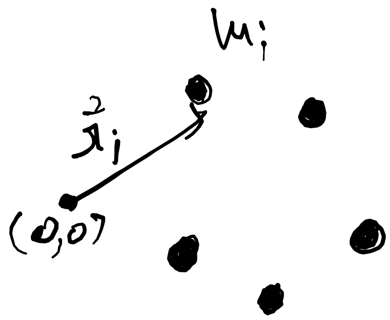
se $\sum_i \vec{F}_i = 0 \Rightarrow \frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p}$ è costante

↓

Legge di conservazione della quantità di moto

Urti unidimensionali & centro di massa

$$\boxed{\vec{r}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i}}$$



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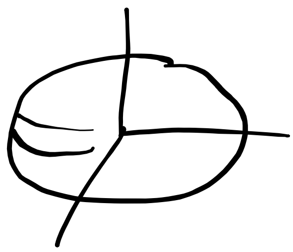
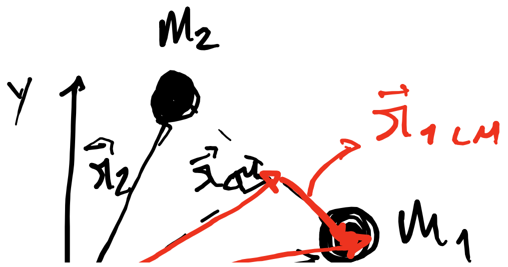
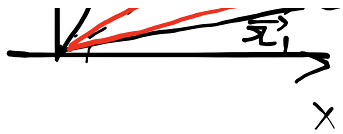
in un sistema continuo

$$x_{cm} = \frac{1}{M} \int x \, dm$$



$$\frac{d}{dt}\vec{r}_{CM} = \vec{v}_{CM} = \frac{\sum_{i=1}^{N} m_i \frac{d}{dt}\vec{r}_i}{\sum m_i} = \frac{\sum_i m_i \vec{v}_i}{\sum_i m_i}$$





$$\begin{cases} \vec{r}_1 = \vec{r}_{CM} + \vec{r}_{1CM} \\ \vec{r}_2 = \vec{r}_{CM} + \vec{r}_{2CM} \end{cases}$$

rif. del lab. — rif. del centro di massa

$$\frac{d}{dt}$$

$$\begin{cases} \vec{v}_1 = \vec{v}_{CM} + \vec{v}_{1CM} \\ \vec{v}_2 = \vec{v}_{CM} + \vec{v}_{2CM} \end{cases}$$

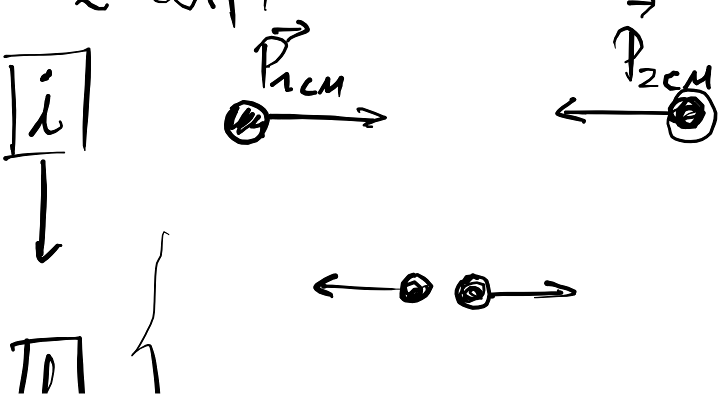
$$\vec{P} = m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1(\vec{v}_{CM} + \vec{v}_{1CM}) + m_2(\vec{v}_{CM} + \vec{v}_{2CM})$$

$$= (m_1 + m_2)\vec{v}_{CM} + m_1 \vec{v}_{1CM} + m_2 \vec{v}_{2CM}$$

$$\vec{P} = \vec{P}_{CM} + \underbrace{m_1 \vec{v}_{1CM} + m_2 \vec{v}_{2CM}}_{0}$$

La quantità del moto del sistema è nulla nel riferimento del centro di massa

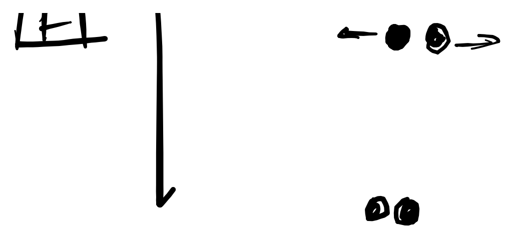
2 corpi



$$|P_{1CM}| = |P_{2CM}|$$

finale — iniziale

urto elastico $|P_{1CMf}| = |P_{1CMI}|$



urto anelastico $|P_{1CMf}| < |P_{1CMI}|$

$|P_{2CMf}| < |P_{1CMI}|$

completamente anelastico $P_{1CMF} = P_{2CMF} = 0$

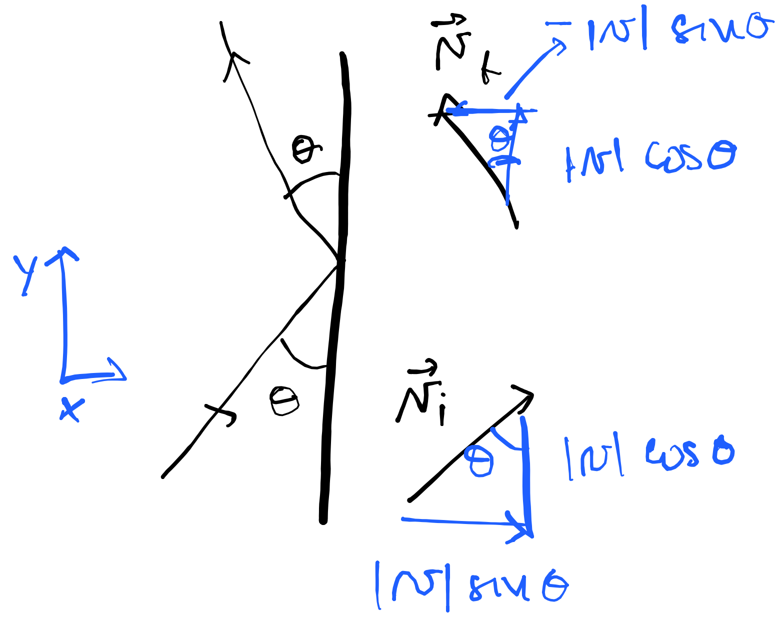
$\boxed{3}$ Pg 139

$m = 0.325\ kg$

$v = 6.22\ m/s$

$\theta = 33^\circ$

$\Delta t = 10.4\ ms$



(A) Impulso

$$\frac{d\vec{P}}{dt} = \vec{F} \rightarrow \boxed{\Delta\vec{P} = \underbrace{\int_{t_i}^{t_i+\Delta t} dt\, \vec{F}}_{\text{Impulso}}}$$

$$\boxed{\Delta\vec{P} = m(\vec{v}_f - \vec{v}_i)}$$

$$\vec{v}_i = |v|(\sin\theta\, \vec{u}_x + \cos\theta\, \vec{u}_y)$$

$$\vec{v}_f = |v|(-\sin\theta\, \vec{u}_x + \cos\theta\, \vec{u}_y)$$

$$\hookrightarrow \vec{v}_f - \vec{v}_i = -2|v| \sin\theta\, \vec{u}_x$$

$$\Delta P = J = -2m\,|\vec{v}|\,\sin\theta\; u_x$$
$$= -2 \times 0.325 \times 6.22 \times \sin 33^\circ\; \vec{u}_x$$
$$= -2.2\,\vec{u}_x \quad (kg\, m/s)$$

(B) Forza media dalla palla sulla parete

$$\vec{\Delta p} = \int_{t_i}^{t_i+\Delta t} dt\, \vec{F}$$ Assumendo $\vec{F}$ costante

$$= \vec{F}_{med} \int_{t_i} dt = \vec{F}_{med}\, \Delta t$$

$\Delta t \to 10.4\, ms$

$-2.2\,\vec{u}_x$

$$\vec{F}_{med} = \frac{\vec{\Delta p}}{\Delta t} = \frac{-2.2}{10.4 \times 10^{-3}}\, \vec{u}_x$$
$$= -211.73\, \vec{u}_x \quad (N)$$

3ª Legge di Newton

$$\vec{F}_{Pal,Par} = -\vec{F}_{Par,Pal} \longrightarrow \vec{F}_{Pal,Par} = 211.73\, \vec{u}_x \quad (N)$$

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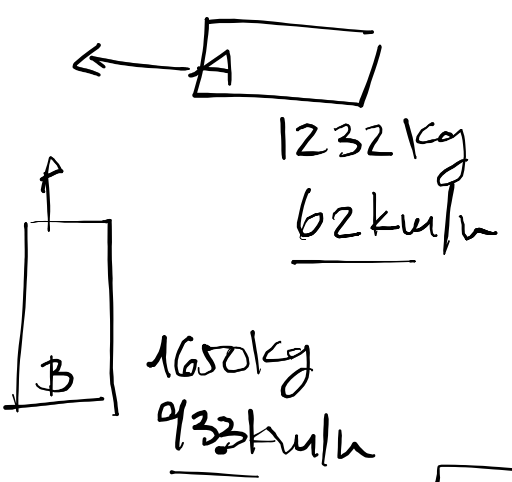


? Velocità (comune) subito dopo l'urto?

$$\boxed{\vec{P}_i = \vec{P}_f}$$

$$\vec{P}_i = m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi}$$

$\to P_f = (m_A + m_B)\, v_f$

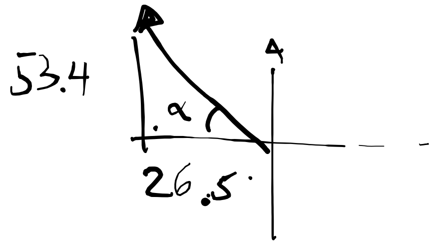
$$\vec v_f = \frac{1}{m_A+m_B}\left(m_A \vec v_{Ai} + m_B \vec v_{Bi}\right)$$

$\vec v_{Ai} = -62\, \vec u_x$

$\vec v_{Bi} = 93.3\, \vec u_y$

$$\vec v_f = \frac{1}{m_A+m_B}\left[m_A(-62\, \vec u_x) + m_B(93.3\, \vec u_y)\right]$$

$$\Leftrightarrow \vec v_f = -26.5\, \vec u_x + 53.4\, \vec u_y$$



$\|\vec v_f\| = 59.63$ km/h

$\alpha = 63.6°$

$tg\, \alpha = \frac{53.4}{26.2}$

$\alpha = arctg\left(\frac{53.4}{26.2}\right) = 63.6°$

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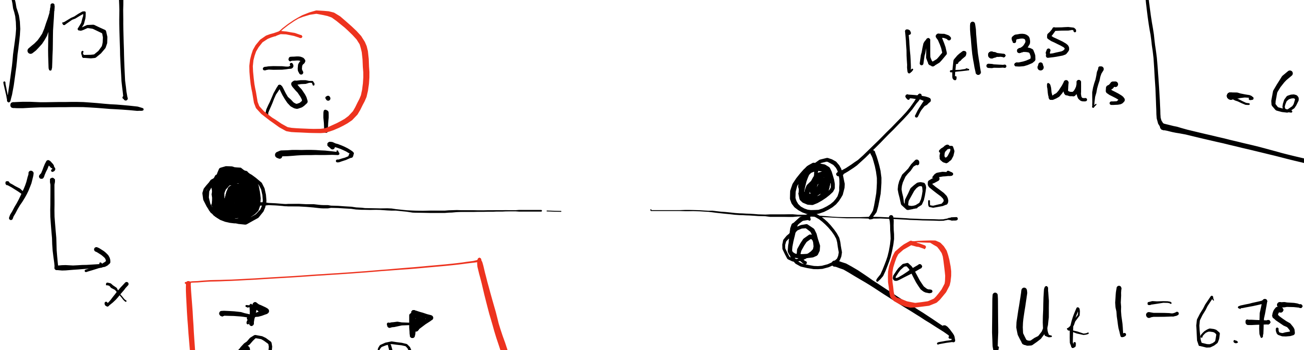

$\boxed{\vec P_i = \vec P_f}$

2 q.

(A) $\vec u_y$: $P_{iy} = 0 = P_{fy} = m|\vec v_f| \sin 65 - m|\vec u_f| \sin\alpha$

$$\sin\alpha = \frac{|\vec v_f| \sin 65}{|\vec u_f|}$$

$$\alpha = arcsin\left(\frac{|\vec v_f| \sin 65}{|\vec u_f|}\right)$$

$\alpha = 28.03°$

(B) $\vec v_i$?

$\underline{\underline{U_x}}$ $\quad P_{x\,i} = P_{x\,f}$

$$m|\vec v_i| = m|\vec v_f|\cos 65^\circ + m|\vec u_f|\cos\alpha$$

$$|\vec v_i| = |\vec v_f|\cos 65 + |\vec u_f|\cos\alpha$$

$$\hookrightarrow |\vec v_i| = \underline{\underline{7.44\ m/s}}$$

$\boxed{15}$

$M = 0.22\,kg \qquad m = 0.046\,kg$

$v = 45\,m/s \qquad u_i = 0$

Assumendo urto elastico $\Rightarrow E_c = \frac{1}{2}\sum_i m_i v_i^2$ è costante

$$\vec P_f = \vec P_i \quad Mv = m u_f + M v_f \qquad \boxed{1}$$

$$E_{cf} = E_{ci} \quad \frac{1}{2}Mv^2 = \frac{1}{2}Mv_f^2 + \frac{1}{2}m u_f^2 \Leftrightarrow v_f^2 = v^2 - \frac{m}{M}u_f^2 \qquad \boxed{2}$$

$$\boxed{1}^2 \quad M^2v^2 = m^2u_f^2 + M^2v_f^2 + 2mMu_fv_f$$

$$\boxed{2} \to \boxed{1}^2 \Leftrightarrow \boxed{u_f = \frac{2M}{M+m}v} \qquad \boxed{v_f = \frac{M-m}{m+M}v}$$

$$\frac{2\times 0.22}{0.22+0.046}\cdot 45 = 74.44\ m/s$$

$M \to \tilde{M} \qquad u_f = \frac{2M}{\tilde{M}+m} v$

$\tilde{M} = 2M \to u_f = \frac{4M}{2M+m} v = 81.48$ m/s

$\tilde{M} = 3M \to u_f = 84.14$ m/s

$\tilde{M} \gg m : \quad u_f \to 2v = 90$ m/s

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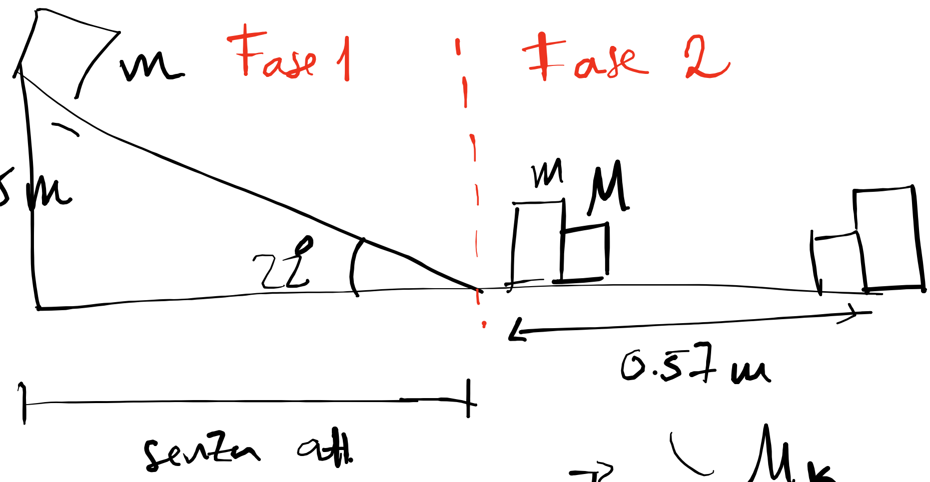
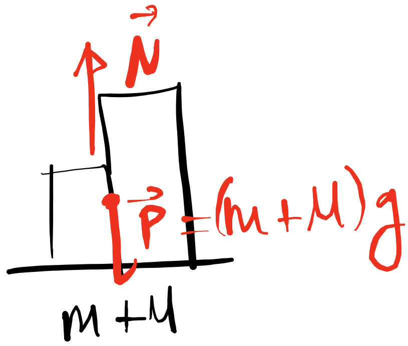


Fase 2

$\vec{F}_a = \frac{d\vec{p}}{dt}$ ($F_a = \mu_k N$)

$\mu_k N = (M+m)\, a \Leftrightarrow \boxed{a = \mu_k g}$

$N = (M+m) g$



Fase 1

v di m alla fine del piano inclinato

Energia è conservata

$E_i = E_f \Rightarrow \frac{1}{2} m 0^2 + mgh = \frac{1}{2} m v_1^2 + mg0$

$v_i = 0$ $\qquad h_f = 0$

$$\boxed{v_1 = \sqrt{2gh}}$$

vel di m in fondo al piano inclinato

Nell'urto $\vec{p}_i = \vec{P}_f \Leftrightarrow m v_1 = (M+m) v_2$

$$\boxed{v_2 = \frac{m v_1}{M+m} = \frac{m}{M+m}\sqrt{2gh}}$$

v di M e m subito dopo l'urto.

Fase 2 → moto uniformemente acc.

$$\begin{cases} x = x_0 + v_0 t - \frac{1}{2} a t^2 \Leftrightarrow \boxed{x_f - x_0 = \frac{v_0^2}{2a}} \\ v = v_0 - a t \Rightarrow \boxed{t_f = v_0 / a} \end{cases}$$

dove $v_0 = v_2 = \frac{m}{M+m}\sqrt{2gh}$

$$a = \frac{v_0^2}{2(x_f - x_0)} \Leftrightarrow \mu_k g = \left(\frac{m}{M+m}\right)^2 \frac{2g\ldots}{2(x_f\ldots}$$

$$\boxed{\mu_k = \left(\frac{m}{m+M}\right)^2 \frac{h}{\Delta x}}$$