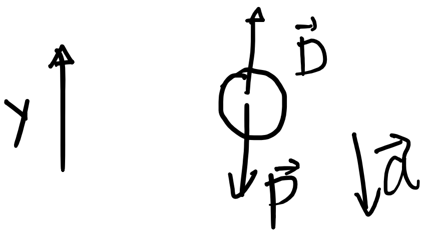
22/04/2020

RHK, CAP4 Problemi

$\boxed{17}$ & $\boxed{18}$

Resistenza Aerodinamica



$\vec{D} = c\, v^{p}$ c, p sono parametri/costanti

$$\boxed{D = b\, v}$$

→ parametro costante che viene misurato

$$\sum_i \vec{F}_i = m\vec{a}$$

$$\vec{D} + \vec{P} = m\vec{a}$$

$$b v - m g = -m a \Rightarrow \boxed{a = g - \frac{b}{m} v}$$

$$\boxed{\vec{a} = -a\, \vec{u}_y} \qquad \boxed{1}$$

tempi piccoli

$$a(t \ll b/m) = g$$

tempi grandi : $t \gg b/m$ $a \to 0$ perché $\vec{D} = -\vec{P}$

velocità limite/terminale

v_L definita da $a = 0$ in Eq $\boxed{1}$

$$\boxed{v_L = g m / b}$$

→ trovare $y(t)$

$$\vec{a} = \frac{d\vec{v}}{dt}\,; \quad \vec{v} = \frac{d\vec{y}}{dt} \Rightarrow y(t) = \int dt\, v(t)$$

Eq [1] $\quad \boxed{\frac{dv}{dt} = -\frac{b\,v}{m} + g}$ → Eq. dif. di primo grado

$d \quad \vec{v} = -v\,\vec{u}_y$

se $g \to 0$ l'eq. è bungee

$$\frac{dv}{dt} = -\frac{b}{m} v$$

sol: $\boxed{v(t) = v_L \left(1 - e^{-b/m\, t}\right)}$

dim. $\quad v(t) = v_h(t) + v_p$ → costante dove

v_h è soluzione di $\frac{dv_h}{dt} = -\frac{b\,v_h}{m}$

$$\frac{dv_h}{v_h} = -\frac{b}{m} dt \rightarrow \int \frac{dv_h}{v_h} = \frac{b}{m} \int dt$$

$$\log v_h = -\frac{b}{m} t \Leftrightarrow \boxed{v_h(t) = e^{-b/m\, t}} + \tilde{\alpha}$$

$$\frac{d}{dt}(v_h + v_p) = -\frac{b}{m}(v_h + v_p) + g \Leftrightarrow -\frac{b\,v_p}{m} + g = 0$$

costante $\Leftrightarrow \frac{d}{dt} v_p = 0$

$$\boxed{v_p = \frac{g\,m}{b}}$$

$$v(t) = \frac{g\,m}{b} + e^{-b/m\, t} + \tilde{\alpha}$$

→ costante di integrazione

fissata da $v(t=0) = 0$

$$0 = \frac{gm}{b} + 1 + \tilde{\alpha} \Rightarrow \tilde{\alpha} = -\frac{gm}{b} - 1$$

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$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \rightarrow e^x \underset{x \ll 1}{\approx} 1 + x + \frac{x^2}{2} + \dots$$

$$\boxed{v(t) = v_L \left(1 - e^{-b/m\, t}\right)}$$

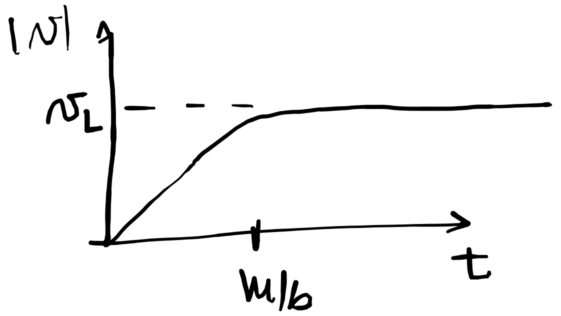
- $\frac{b}{m} t \ll 1 \rightarrow v \approx v_L\left(\cancel{1} - \cancel{1} - \frac{b}{m}t + \dots\right)$

$$v = v_L \frac{b}{m} t$$

$$v_L = g\frac{m}{b}$$

$$v = g t$$

- $\frac{b}{m} t \gg 1 \rightarrow v \approx v_L$



vogliamo trovare $y(t)$

$$\vec{v} = \frac{dy}{dt} \Leftrightarrow dy = -v\,dt \Rightarrow dy = -v_L\left(1 - e^{-\frac{b}{m}t}\right) dt$$

$$\int dy = -v_L \int dt \left(1 - e^{-b/m\, t}\right)$$

$$y(t) = -v_L \Big[\underbrace{\int dt}_{t} - \underbrace{\int dt\, e^{-b/m\, t}}_{\frac{e^{-b/m\, t}}{-b/m}} \Big]$$

$$\int e^{\alpha x} dx = \frac{e^{\alpha x}}{\alpha}$$

$$y(t) = -v_L\left(t + \frac{m}{b} e^{-b/m\, t}\right) + \tilde{y}$$

$\tilde{y}$ ↳ costante

$$y(t=0) = y_0 \Rightarrow y_0 = v_L\left(0 + \frac{m}{b}\right) + \tilde{y}$$

~

$$y = y_0 - v_L \frac{m}{b}$$

$$\boxed{y(t) = y_0 - v_L\left(t + \frac{m}{b} e^{-b/m\, t} - \frac{m}{b}\right)}$$

- $\frac{bt}{m} \ll 1 \Rightarrow y(t) = y_0 - g t^2/2 \Rightarrow$ moto unif. acc.
- $\frac{bt}{m} \gg 1 \rightarrow y(t) = y_0 - v_L t \rightarrow$ moto a $\vec{v}$ costante

$\boxed{18}$ dimostrare $y_{95} = \frac{v_L^2}{g} \log\left(20 - \frac{19}{20}\right)$

t_{95} quando $v = v_L \frac{95}{100} \Leftrightarrow v(t_{95}) = \frac{95}{100} v_L$

$$\frac{95}{100} \cancel{v_L} = \cancel{v_L}\left(1 - e^{-b/m\, t_{95}}\right)$$

$$\frac{95}{100} - 1 = -e^{-b/m\, t_{95}} \Leftrightarrow \log\left(1 - \frac{95}{100}\right) = -\frac{b}{m} t_{95}$$

$$t_{95} = -\frac{m}{b} \log(5/100)$$

$$\boxed{t_{95} = -\frac{m}{b} \log(1/20)} \quad \boxed{A}$$

tempo per arrivare a $v = \frac{95}{100} v_L$

La distanza percorsa in t_{95} è di $y_{95} = y(t_{95})$

$$y_{95} = v_L\left(t_{95} + m\left(e^{-b/m\, t_{95}} - 1\right)\right) + y_0$$

$\boxed{A} \downarrow$ in $\boxed{B}$

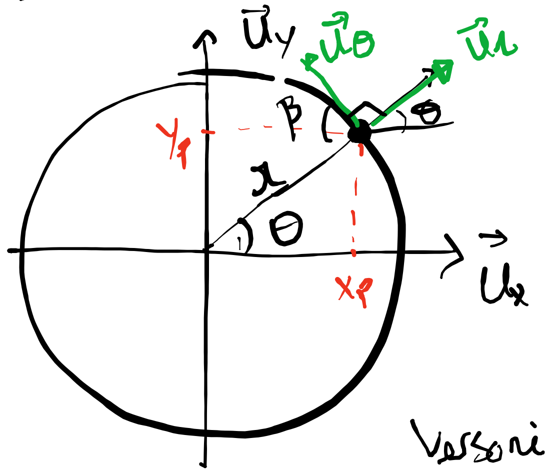
$$y_{95} = \frac{v_L^2}{g}\left(\lg 20 - 19/20\right)$$

B) per la palla di baseball $v_L = 42\,m/s$

$$y_{95} = \frac{(42)^2}{9.8}\left(\lg 20 - \frac{19}{20}\right) \simeq 367.9\,m$$

Moto Circolare Uniforme

Coord cartesiane $\longrightarrow$ Coord. polari



vett. posizione $\vec{R} = \begin{cases} x\vec{u}_x + y\vec{u}_y \\ r\vec{u}_r + \theta\vec{u}_\theta \end{cases}$

$$\begin{matrix} x = r\cos\theta \\ y = r\sin\theta \end{matrix} \iff \begin{cases} r = \sqrt{x^2+y^2} \\ \theta = \operatorname{arctg}(y/x) \end{cases}$$

Versori

$$\begin{cases} \vec{u}_r = \cos\theta\,\vec{u}_x + \sin\theta\,\vec{u}_y \\ \vec{u}_\theta = -\cos\beta\,\vec{u}_x + \sin\beta\,\vec{u}_y = -\sin\theta\,\vec{u}_x + \cos\theta\,\vec{u}_y \end{cases}$$

$$\theta + \beta + \frac{\pi}{2} = \pi$$

vettore posizione $\vec{R} = x\vec{u}_x + y\vec{u}_y = r(\cos\theta, \sin\theta) = r\,\vec{u}_r$

vettore velocità $\vec{v} = \frac{d\vec{R}}{dt} = \dot{r}\,\vec{u}_r + r\,\dot{\vec{u}}_r$ $\quad \left(\dot{x} \equiv \frac{dx}{dt}\right)$

$$\ddot{x} = \frac{d^2x}{dt^2}$$

$$\vec{v} = \dot{r}\,\vec{u}_r + r\dot{\theta}\,\vec{u}_\theta$$

$\dot{r}\,\vec{u}_r$: componente radiale

$r\dot{\theta}\,\vec{u}_\theta$: comp. tangenziale

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\dot{r}\,\vec{u}_r + r\dot{\theta}\,\vec{u}_\theta\right) = \ddot{r}\,\vec{u}_r + \dot{r}\,\dot{\vec{u}}_r + \dot{r}\dot{\theta}\,\vec{u}_\theta + r\ddot{\theta}\,\vec{u}_\theta + r\dot{\theta}\,\dot{\vec{u}}_\theta$$

derivata dei versori

$$\boxed{\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\vec{u}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\,\vec{u}_\theta}$$

2ª Legge di Newton in coord. Polari

$$\boxed{\vec{F}_{res} = m\vec{a} = m\left[(\ddot{r} - r\dot{\theta}^2)\,\vec{u}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\,\vec{u}_\theta\right]}$$

Moto circolare & Uniforme

- circolare: r è costante $\Leftrightarrow$ $\ddot{r} = \dot{r} = 0$
- uniforme: $\dot{\theta}$ è costante → $\ddot{\theta} = 0$ → acc. angolare

$$\vec{R} = r\,\vec{u}_r \qquad \vec{v} = r\dot{\theta}\,\vec{u}_\theta \,, \qquad \vec{a} = -r\dot{\theta}^2\,\vec{u}_r$$

$$\vec{F} = -mr\dot{\theta}^2\,\vec{u}_r$$

$$a = r\left(\frac{v}{r}\right)^2 = \frac{v^2}{r}$$

Velocità angolare $\quad \omega = \dot{\theta} = \frac{d\theta}{dt}$

$[\omega] = rad/s$ in SI

acc. angolare $\quad \gamma = \ddot{\theta} \quad$ ($\gamma = 0$ M.C.U.)

$[\gamma] = rad/s^2$ in SI

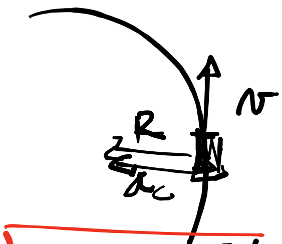
$|v| = \omega r \quad ; \quad |\vec{a}| = r\dfrac{v^2}{r^2} = \dfrac{v^2}{r} = a_c$

$\downarrow$ acc. centripeta

$|F| = m\dfrac{v^2}{r} \rightarrow$ Forza centripeta $\rightarrow$ orientate verso il centro della traiettoria

$\boxed{19}$ TGV $\quad |\vec{v}| = 310\,km/h$

$|\vec{a}_{max}| = 0.050\,g$



$\boxed{a_c = v^2/R}$

A) R_{min} tale che $a = a_{max}$

$a = v^2/R \Rightarrow R = v^2/a$

$R_{min} = v^2/a_{max}$

$\downarrow \dfrac{(310 \times 10^3/3600)^2}{0.05 \times 9.81} = \underline{\underline{15117\,m}}$

B) $a = v^2/R \Rightarrow v = \sqrt{aR}$

$R = 940\,m$

$\downarrow$

$v = (0.05 \times 9.8 \times 940)^{1/2}$

$= 21.47\,m/s = 77.3\,km/h$

$R = 1.4\,m$

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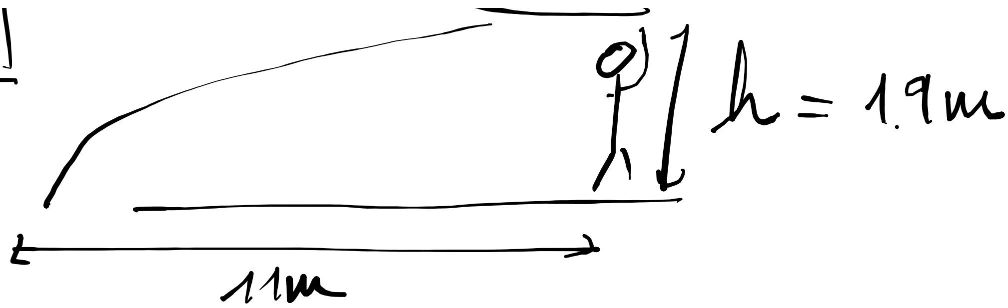

? acc. centripeta? $a_c = v^2/R$

moto partendo da $(x_0, y_0) = (0, 1.9)$ m e fino a

$(x_f, y_f) = (11, 0)$ m

moto di un proiettile quindi

$$\begin{cases} x = x_0 + v_{0x} t \\ y = y_0 + \cancel{v_{0y} t} - \frac{1}{2} g t^2 \end{cases}$$

$\to v_{0x} = \frac{x(t_f) - x_0}{t_f} \Rightarrow \boxed{v_{0x} = \sqrt{\frac{g}{2\Delta y}}\,\Delta x}$

$\Rightarrow \sqrt{(y_0 - y)\frac{2}{g}} = t_f$

0

lancio orizzontale

$$|a_c| = v^2/R = \frac{v_{0x}^2}{R} = \frac{g}{2\Delta y}\frac{\Delta x^2}{R} \to 223.12 \text{ m/s}$$

$$[a_c] = \frac{m/s^2}{\cancel{m}}\,\frac{\cancel{m^2}}{\cancel{m}} = m/s^2 \;//$$