15/04/2020

RHK, CAP. 4

Moto in 2 & 3 dimensioni

$$\vec{F} = m\vec{a} \Leftrightarrow \vec{a} = \frac{\vec{F}}{m} \rightarrow 3D \text{ cartesiano } \begin{cases} a_x = F_x/m \\ a_y = F_y/m \\ a_z = F_z/m \end{cases}$$

$$\vec{a} = \frac{d\vec{v}}{dt} \quad ; \quad \vec{v} = \frac{d\vec{r}}{dt}$$

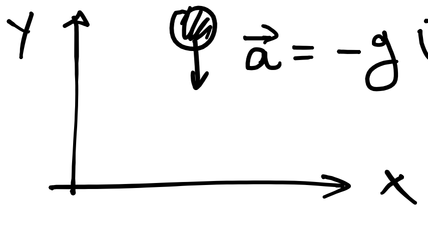
Integrando troviamo

$$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \qquad v_x = v_{0x} + a_x t$$

$$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \qquad v_y = v_{0y} + a_y t$$

$$z(t) \quad '' \qquad\qquad ''$$

→ **Moto dei proiettili** (2D)



$$\vec{a} = -g\,\vec{u}_y$$

$$a_y = -g$$

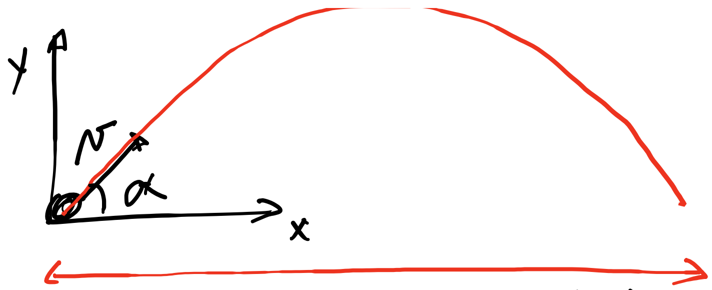
$$a_x = 0$$

$$\downarrow$$

$$x = x_0 + v_{0x}t \qquad v_x = v_{0x} \rightarrow \text{velocità costante}$$

$$\rightarrow \left[\; y = y_0 + v_{0y}t - \frac{g}{2}t^2 \qquad v_y = v_{0y} - gt \right.$$

moto unif. acc.



R = gittata

Δx per il quale $\Delta y = 0$

$$\begin{cases} \Delta x = x - x_0 = v_{0x} t \\ \Delta y = v_{0y} t - \frac{g}{2} t^2 \end{cases}$$

$$\downarrow$$

$$\begin{cases} R = v_{0x} t \\ 0 = v_{0y} t - \frac{g}{2} t^2 \Leftrightarrow t = 0 \vee t = \frac{2 v_{0y}}{g} \end{cases}$$

$v_{0x} = v \cos\alpha$

$v_{0y} = v \sin\alpha$

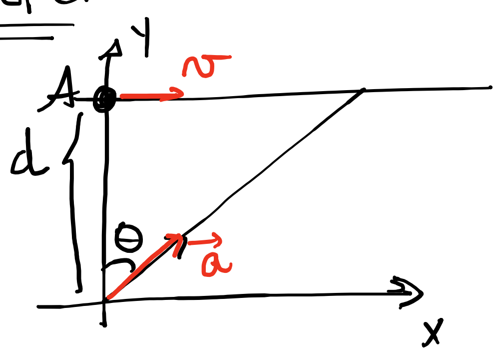
$$\boxed{R = v_{0x} \frac{2 v_{0y}}{g}}$$

$$\Downarrow$$

$$R = \frac{2 v_0^2}{g} \sin\alpha \cos\alpha$$

Problemi

$\boxed{1}$



(A)

$$\begin{cases} d = 30 m \\ v_{iA} = 3 m/s \\ \vec{r}_{0A} = (0, d) \\ a_A = 0 \end{cases}$$

(B)

$$\begin{cases} \vec{v}_{iB} = \vec{0} \\ |\vec{a}| = 0.4 m/s^2 \\ \vec{r}_{0B} = (0, 0) \end{cases}$$

$\hookrightarrow$ $a_{xB} = |a| \sin\theta$

$a_{yB} = |a| \cos\theta$

? θ per il quale c'è collisione?

$$\begin{cases} \vec{r}_A(t) \\ \vec{r}_B(t) \end{cases} \rightarrow \bar{t} \text{ tale che } \vec{r}_A(\bar{t}) = \vec{r}_B(\bar{t})$$

$\downarrow$

dipende da θ

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{\vec{a}}{2} t^2$$

$$\begin{cases} x(t) = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \end{cases}$$

$$\hookrightarrow \begin{cases} x(t) = x_0 + v_{0x} t + \frac{a_x}{2} t^2 \\ y(t) = y_0 + v_{0y} t + \frac{a_y}{2} t^2 \end{cases}$$

$$\begin{cases} x_A(\bar{t}) = x_B(\bar{t}) \\ y_A(\bar{t}) = y_B(\bar{t}) \end{cases} \Leftrightarrow \begin{cases} \cancel{x_{0A}} + v_{0xA}\bar{t} + \cancel{\frac{a_{xA}}{2}}\bar{t}^2 = \cancel{x_{0B}} + \cancel{v_{0xB}}\bar{t} + \frac{a_{xB}}{2}\bar{t}^2 \\ \underbrace{y_{0A}}_{d} + \cancel{v_{0yA}}\bar{t} + \cancel{\frac{a_{yA}}{2}}\bar{t}^2 = \cancel{y_{0B}} + \cancel{v_{0yB}}\bar{t} + \frac{a_{yB}}{2}\bar{t}^2 \end{cases}$$

$$\begin{cases} v_{0xA}\bar{t} = \frac{a_{xB}}{2}\bar{t}^2 \Rightarrow \bar{t} = \frac{2v_{0xA}}{a_{xB}} \\ d = \frac{a_{yB}}{2}\bar{t}^2 \Rightarrow d = \frac{1}{2} a_{yB} \, 4 \frac{v_{0xA}^2}{a_{xB}^2} \end{cases}$$

$$\boxed{d = 2\frac{a_{yB}}{a_{xB}^2} v_{0xA}^2}$$

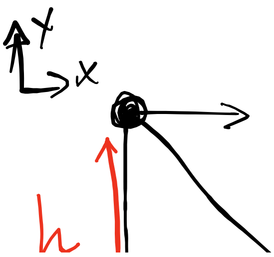
$$d = 2\frac{\cancel{|a|}\sin\theta}{|a|^{\cancel{2}}\cos^2\theta} v_{0xA}^2$$

$$\boxed{d = 2\frac{\sin\theta}{|a|\cos^2\theta} v_{0xA}^2}$$

$$30 = \frac{2\sin\theta}{0.4\cos^2\theta} \times 3^2 \qquad \cos^2\theta + \sin^2\theta = 1$$

$$30 = \frac{2\sin\theta}{0.4(1-\sin^2\theta)} 9 \rightarrow \cos\theta = \frac{1}{2} \Rightarrow \boxed{\theta \approx 60^\circ}$$

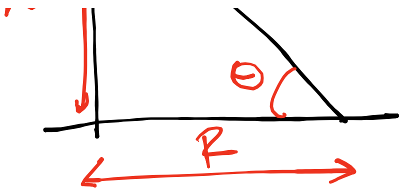
2



$\tilde{a} = 1.2 m/s^2$

dimostrare

$$\boxed{y(x) = \alpha x + \beta \quad \alpha, \beta \in \mathbb{R}}$$



$$\begin{cases} x = x_0 + v_{0x} t + \frac{1}{2}\tilde{a} t^2 \\ y = y_0 + v_{0y} t - \frac{1}{2} g t^2 \end{cases}$$

$\vec{a} = \tilde{a}\,\vec{u}_x - g\,\vec{u}_y$

cond. iniziali $= \begin{cases} \vec{r} = (0, h) \\ \vec{v}_0 = (0, 0) \end{cases}$ dove $h = 39\,m$

$$\begin{cases} x(t) = 0 + 0 + \frac{1}{2}\tilde{a} t^2 \Leftrightarrow \boxed{\frac{2x(t)}{\tilde{a}} = t^2} \\ y(t) = h + 0 - \frac{g}{2} t^2 \end{cases}$$

tempo di caduta

$$y = h - \frac{g}{\cancel{2}} \frac{\cancel{2}x}{\tilde{a}}$$

$$\boxed{y = -\frac{g}{\tilde{a}} x + h}$$

→ trovare R e θ

$\operatorname{tg}\theta = h/R$

R è x tale che $y = 0$

$$0 = -\frac{g}{\tilde{a}} R + h$$

$$\Rightarrow \boxed{R = h\tilde{a}/g}$$

$$R = 39 \times \frac{1.2}{9.8} = \underline{\underline{4.78\,m}}$$

$$\theta = \operatorname{arctg}(h/R) = \operatorname{arctg} \frac{39}{4.78} = \underline{\underline{83^\circ}}$$

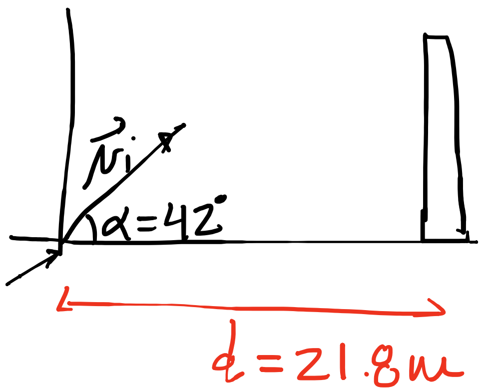
$$t=\sqrt{\frac{2R}{\tilde{a}}} = \sqrt{\frac{2\times 4.78}{1.2}} = 2{,}82 \text{ s}$$

→ con che velocità colpisce il terreno

$$\begin{cases} v_x = \cancel{v_{0x}}^{\,0} + \tilde{a}\, t \\ v_y = \cancel{v_{0y}}^{\,0} - g\, t \end{cases} \rightarrow \text{calcolare per } t = 2.82 \text{ s}$$

$$\boxed{\vec{v} = 3.4\, \vec{u}_x - 27.7\, \vec{u}_y} \quad (\text{m/s})$$

5



$|\vec{v}_i| = 25.3$ m/s

$$\boxed{\begin{cases} v_{0x} = |\vec{v}_i| \cos\alpha \\ v_{0y} = |\vec{v}_i| \sin\alpha \end{cases}}$$

$X_0 = Y_0 = 0$

A) Per quanto tempo rimane in aria?

$$\begin{cases} x = \cancel{x_0} + v_{0x}\, t \\ y = \cancel{y_0} + v_{0y}\, t - \frac{g}{2} t^2 \end{cases}$$

t tale che $x - x_0 = \Delta x = d$

$$x = |\vec{v}_i| \cos\alpha \; t_v$$

$t_v \rightarrow$ tempo di volo

$$\boxed{t_v = \frac{d}{|\vec{v}_i| \cos\alpha}} \rightarrow t_v = \frac{21.8}{25.3 \cos 42} = 1.16 \text{ s}$$

B) $y(t_v)$?

$$y(t_v) = v_{oy} t_v - \frac{g}{2} t_v^2$$

$$= d \,tg\,\alpha - \frac{g}{2} \frac{d^2}{|v_i|^2 \cos^2\alpha} \longrightarrow \underline{\underline{13\,m}}$$

e) $\vec{v}(t_v)$

$$v_x = v_{ox}$$

$$v_y = v_{oy} - \frac{g}{2} t$$

$$\vec{v}(t_v) = \underset{\substack{\| \\ |\vec{v}_i| \cos\alpha}}{v_{ox}} \vec{u}_x + \Big(\underset{\substack{\| \\ |\vec{v}_i| \sin\alpha}}{v_{oy}} - \frac{g}{2} t_v\Big) \vec{u}_y$$

$$\vec{v}(t_v) = |\vec{v}_i| \cos\alpha \, \vec{u}_x + \left(|\vec{v}_i| \sin\alpha - \frac{g\,d}{v_{ox}}\right) \vec{u}_y$$

$$\boxed{\vec{v}(t_v) = 18.8\,\vec{u}_x + 5.5\,\vec{u}_y} \quad (m/s)$$

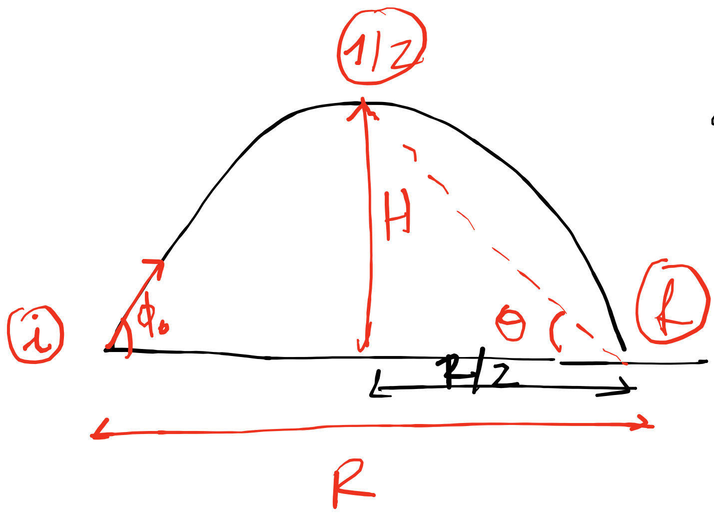
D $v_y(t_v) > 0 \Rightarrow$ sta ancora salendo

$\downarrow$

Non ha superato il vertice

$$\begin{cases} \text{Prima del vertice} & v_y > 0 \\ \text{nel vertice} & v_y = 0 \\ \text{dopo il vertice} & v_y < 0 \end{cases}$$

6



dimostrare che

$$tg\,\theta = \frac{1}{2}\, tg\,\phi_0$$

$$tg\,\theta = \underbrace{H/R/2}_{\text{scrivere come funzione di } \phi_0} \quad (1)$$

i $\Rightarrow$ $x_0 = y_0 = 0$

$v_{0x} = |\vec{v}_0| \cos\phi_0$

$v_{0y} = |v_0| \sin\phi_0$

1/2 $\Rightarrow$ $v_y(t_{1/2}) = 0$

$x(t_{1/2}) = R/2$

$y(t_{1/2}) = H$

f $\Rightarrow$ $x(t_f) = R$

$y(t_f) = 0$

Eq. del moto
$$\begin{cases} x = x_0 + v_{0x} t \\ y = y_0 + v_{0y} t - \frac{1}{2} g t^2 \end{cases} \Rightarrow \begin{cases} v_x = v_{0x} \\ \boxed{v_y = v_{0y} - g t} \end{cases}$$

$t_{1/2}$ definito da $v_y(t_{1/2}) = 0$

$$\Leftrightarrow \boxed{t_{1/2} = v_{0y}/g}$$

$$y(t_{1/2}) = H = \frac{v_{0y}^2}{g} - \frac{1}{2} g \frac{v_{0y}^2}{g^2}$$

$$\Leftrightarrow \boxed{H = \frac{v_{oy}^2}{2g}} \quad (2)$$

$$x(t_{1/2}) = R/2$$

$$\boxed{v_{ox}\,\frac{v_{oy}}{g} = R/2} \quad (3)$$

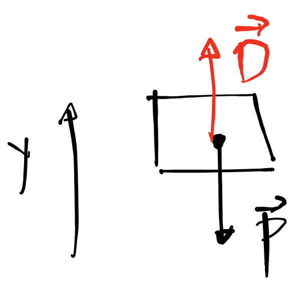
(2),(3) → (1)

$$tg\,\theta = 2\,\frac{H}{R} = \cancel{2}\,\frac{v_{oy}^{\cancel{2}}}{2g}\;\frac{g}{\cancel{2}\,v_{ox}\,\cancel{v_{oy}}}$$

$$tg\,\theta = \frac{1}{2}\,\frac{v_{oy}}{v_{ox}} = \frac{1}{2}\,\frac{|\cancel{\vec{v}_0}|\sin\phi_0}{|\cancel{\vec{v}_0}|\cos\phi_0}$$

$$\boxed{tg\,\theta = \frac{1}{2}\,tg\,\phi_0}$$

$\boxed{15}$



$$\boxed{|\vec{D}| = b\,v^{\alpha}}$$

$$\downarrow$$

$$b\,v^2$$

$$\vec{F}_{tot} = \vec{P} + \vec{D} = (b v^2 - mg)\,\vec{u}_y$$

$$\downarrow$$

$$m\vec{a} \rightarrow \boxed{\vec{a} = \left(-g + \frac{b}{m}\,v^2\right)\vec{u}_y}$$

$$\vec{a} = \frac{d\vec{v}}{dt} \Leftrightarrow \vec{v} = \int dt\,\vec{a}$$

- $\vec{a}$ iniziale $\qquad \vec{a}(t_i) = -g\,\vec{u_y}$

- Velocità limite $v_L \to$ definita da $\vec{a} = 0$

anche velocità terminale

$$\Updownarrow$$

$$|\vec{P}| = |\vec{D}|$$

$$mg = b v_L^2 \Leftrightarrow \boxed{v_L = \sqrt{\frac{mg}{b}}}$$

- $\vec{a}$ quando $v = v_L/2$

$$\vec{a} = \left(\frac{b}{m} v^2 - g\right)\vec{u_y}$$

$$= \left(\frac{b}{m}(v_L/2)^2 - g\right)\vec{u_y}$$

$$= \left(\frac{\cancel{b}}{4\cancel{m}}\frac{\cancel{m}g}{\cancel{b}} - g\right)\vec{u_y}$$

$$\boxed{\vec{a}\left(\frac{v_L}{2}\right) = -\frac{3}{4} g\,\vec{u_y}}$$

$\boxed{11}$ dimostrare che

$$\frac{d^2}{d^2t}(v^2) = 2g^2$$

$$\vec{v} = v_{0x}\vec{u_x} + (v_{0y} - gt)\vec{u_y}$$

$$v^2 = \|\vec{v}\|^2 = v_x^2 + v_y^2$$

$$v_{0x}^2 + (v_{0y} - gt)^2$$

$= v_{0x} + (v_{0y} - gt)$

$$v^2 = v_{0x}^2 + v_{0y}^2 + g^2t^2 - 2gtv_{0y}$$

$$\frac{d}{dx}x^p = px^{p-1}$$

$$\frac{d}{dt}v^2 = g^2 2t - 2gv_{0y}$$

$$\frac{d^2}{dt^2}v^2 = \frac{d}{dt}\left(\frac{d}{dt}v\right) = 2g^2$$

$$\boxed{\frac{d^2}{dt^2}v^2 = 2g^2}$$