1/04/2020

RHK, cap 2

39

$$\boxed{x(t) = At^2 - Bt^3}$$

$[x] = L$, $[t] = T$

$[A] = \frac{L}{T^2}$; $[B] = \frac{L}{T^3}$

$\downarrow$ SI : $\begin{cases} T = s \\ L = m \end{cases}$

$[A] = m/s^2$; $[B] = m/s^3$

$A = 3\ m/s^2$, $B = 1\ m/s^3$

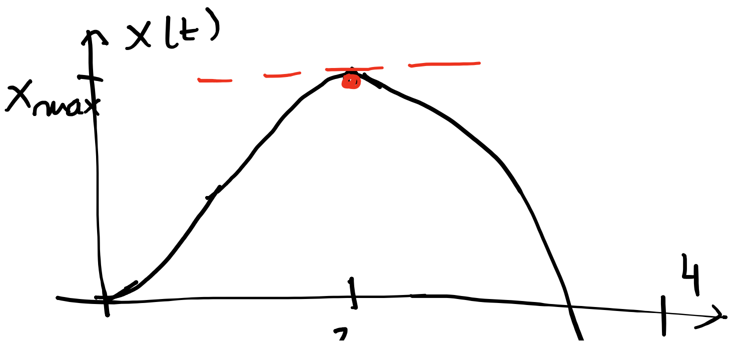
(B) t per il quale $x = x_{max}$

$$\left.\frac{dx}{dt}\right|_{x_{max}} = 0 \iff \frac{d}{dt}(3t^2 - t^3) = 0 \Rightarrow 6t - 3t^2 = 0$$

$t = 0 \vee t = 2\ s$

$$\boxed{x(t) = 3t^2 - t^3}$$

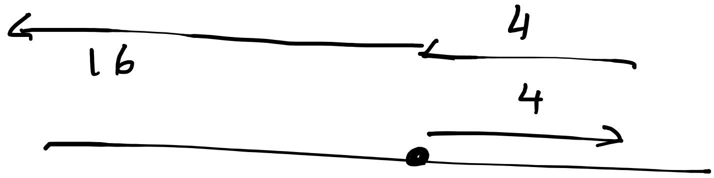
$x_{max} = x(t=2) = 4\ m$



$x_{max} =$

(C)

$$x(t=4) = 3\times 16 - 4^3 = -16\,m$$



Lunghezza totale $\equiv D = 4+4+16 = \underline{\underline{24\,m}}$

(D) spostamento $\equiv \Delta x = x(t=4) - x(t=0)$

$$= -16 - 0 = -16\,m$$

(E)

$$v = \frac{dx}{dt} = \frac{d}{dt}(3t^2 - t^3) = 6t - 3t^2$$

$$\vec{v}(t) = 6t - 3t^2\ \vec{i}$$

$$\begin{cases} \vec{v}(1) = 3\ \vec{i} \\ \vec{v}(2) = 0\ \vec{i} \\ \vec{v}(3) = -9\ \vec{i} \\ \vec{v}(4) = -24\ \vec{i} \end{cases} \quad (m/s) \qquad (\vec{i} \equiv \vec{u}_x)$$

(F)

$$\vec{a} = \frac{d\vec{v}}{dt} = 6 - 6t\ \vec{i}$$

$$\begin{cases} \vec{a}(1) = 0\ \vec{i} \\ \vec{a}(2) = -6\ \vec{i} \\ \vec{a}(3) = -12\ \vec{i} \\ \vec{a}(4) = -18\ \vec{i} \end{cases} \quad (m/s^2)$$

(G) $\vec{v}$ - $\vec{\Delta x}$ $\quad \vec{x}(t=4) - \vec{x}(t=2)$

$$\vec{v}_m = \frac{\Delta x}{\Delta t} = \frac{\ldots}{2}$$

$$\hookrightarrow \vec{v}_m = \frac{3 \times 16 - 64 - (12-8)}{2} = -10\ \vec{i}\ (m/s)$$

$\boxed{47}$

$\|\vec{a}\| = 4.92\ m/s^2$

$\|\vec{v}_0\| = 24.6\ m/s$

(A) t_f tale che $v(t_f) = 0$

freno $\xrightarrow{\vec{v}}$ $\xleftarrow{\vec{a}}$

$$v(t) = v_0 - at$$

$$0 = v_0 - a t_f \Leftrightarrow t_f = v_0 / a$$

$$t_f = \frac{24.6}{4.92} = \underline{\underline{5\ s}}$$

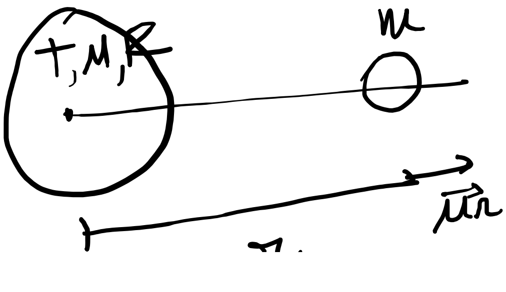
(B)

$$x(t) = x_0 + v_0 t - \frac{a t^2}{2}$$

$$\Delta x \equiv x(t = t_f) - x_0 = v_0 t_f - \frac{a t_f^2}{2}$$

$$\Delta x = 24.6 \times 5 - \frac{4.92}{2} 25 \Leftrightarrow \Delta x = \underline{\underline{61.5\ m}}$$

Caduta Libera



forza gravitazionale

↓

Newton

$$\vec{F}_g = -G\, m\, M_T\, \vec{u}_r$$

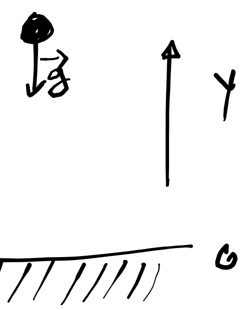
$$\frac{\quad}{r^2}$$

$r \approx R$ $$= -\underbrace{\frac{G M_T}{R^2}}_{g}\, m\, \vec{u_r}$$

$$g = \frac{G M_T}{R^2}$$

g acc. gravità nella superficie della terra

$g = 9.81\ m/s^2$
↓
costante $\Rightarrow$
$$\begin{cases} y(t) = y_0 + v_0 t - \frac{g}{2} t^2 \\ v_y = v_0 - g t \\ a = -g \end{cases}$$

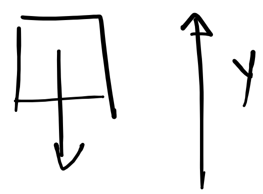


$\boxed{52}$

$$\begin{cases} y(t) = y_0 + v_0 t - \frac{g}{2} t^2 \\ v_y(t) = v_0 - g t \end{cases}$$

$y_0 = 120\ m$
$v_0 = 0$

A, B
↓
$$\begin{cases} y(t) = y_0 - \frac{g}{2} t^2 \\ v_y(t) = -g t \end{cases}$$



tempo di caduta $\Rightarrow$ $0 = y_0 - \frac{g}{2} t_c^2$

$$\boxed{t_c = \sqrt{\frac{2 y_0}{g}}}$$

$$v(t_c) = -g t_c = -g \sqrt{\frac{2 y_0}{g}} = \boxed{-\sqrt{2 y_0 g}}$$

$$t_c = \sqrt{2\,\frac{120}{9.81}} = \underline{\underline{4.95\,s}}, \quad |v(t_c)| = \sqrt{2 \times 120 \times 9.81} = \underline{\underline{48.5\,m/s}}$$

C, D

$v(t_{cut})$, t_{cut} ?

↳ definito come t per il quale

$$y(t_{cut}) = \frac{y_0}{2} = 60\,m$$

$$\begin{cases} y(t) = y_0 - \frac{g}{2}t^2 \\ v(t) = -gt \end{cases}$$

$$y(t) = y_0 - \frac{g}{2}t^2 \rightarrow \frac{y_0}{2} = y_0 - \frac{g}{2}t_{cut}^2$$

$$\Downarrow$$

$$-\frac{y_0}{\cancel{2}} = -\frac{g}{\cancel{2}}\,t_{cut}^2$$

$$\boxed{t_{cut} = \sqrt{y_0/g}}$$

$$[T] = \left[\frac{\cancel{L}}{\cancel{L}/T^2}\right]^{1/2} = [T] \checkmark$$

$$v(t_{cut}) = -g\sqrt{\frac{y_0}{g}}$$

$$\boxed{v(t_{cut}) = -\sqrt{y_0 g}} \rightarrow \sqrt{L\,\frac{L}{T^2}} = \frac{L}{T} \checkmark$$

$$t_{cut} = \sqrt{\frac{120}{9.81}} = 3.5\,s$$

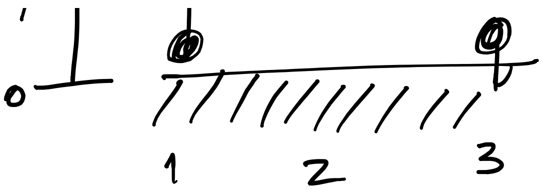
$$v(t_{cut}) = 34.3\,m/s$$

$\boxed{56}$

① ⇒ $y_1 = 0$

$t_1 = 0$

y ↑



$v_0 \equiv v_1 = +|\vec{v}_1| = 14.6 \text{ m/s}$

(2) $\Rightarrow$ y_2
t_2
$v_2 = 0 \text{ m/s}$

(3) $\Rightarrow$ $y_3 = 0$
v_3
$t_3 = 7.72 \text{ s}$

$$\begin{cases} y = y_0 + v_0 t - g\dfrac{t^2}{2} \\ v = v_0 - gt \end{cases}$$

$\hookrightarrow$ t_2 : $0 = v_0 - g t_2 \Rightarrow \boxed{t_2 = v_0/g}$

$\downarrow$

y_2: $y_2 = y_0 + v_0 t_2 - \dfrac{g}{2} t_2^2 \Rightarrow y_2 = y_0 + \dfrac{v_0^2}{g} - \dfrac{g}{2}\dfrac{v_0^2}{g^2}$

$$\boxed{y_2 = y_0 + \frac{1}{2}\frac{v_0^2}{g}}$$

t_3: $y_3 = 0 = y_0 + v_0 t_3 - g\dfrac{t_3^2}{2} \Rightarrow$

$$t_3 = \frac{2v_0}{g} \Rightarrow 7.72 = 2 \times \frac{14.6}{g}$$

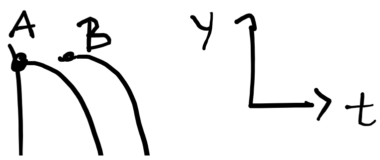
$\downarrow$

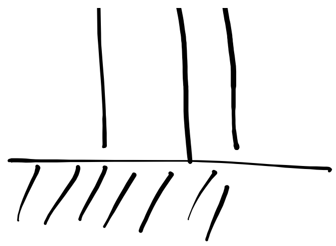
$g = 3.78 \text{ m/s}^2$

$\downarrow$

<u>Mercurio</u>

$\boxed{59}$





$y = y_0 + v_{0y}(t - t_0) - \frac{g}{2}(t - t_0)$

Corpi A e B $\begin{cases} \text{stessa altezza} = y_0 \\ v_{0y} = 0 \\ \text{A parte a } t = 0 \\ \text{B parte a } t = 1\,s \end{cases}$

$$y_A = y_0 - \frac{g}{2}t^2 \quad ; \quad y_B = y_0 - \frac{g}{2}(t-1)^2$$

distanza: $y_B - y_A = -\frac{g}{2}\left[(t-1)^2 - t^2\right]$

$$= -\frac{g}{2}\left[\cancel{t^2} - 2t + 1 - \cancel{t^2}\right]$$

$$\boxed{y_B - y_A = g(t - 1/2)}$$

t per il quale $y_B - y_A = 10\,m$

$$10 = 9.81(t - 1/2) \Leftrightarrow t = \frac{10}{9.81} + \frac{1}{2} \Rightarrow t = 1.52\,s$$

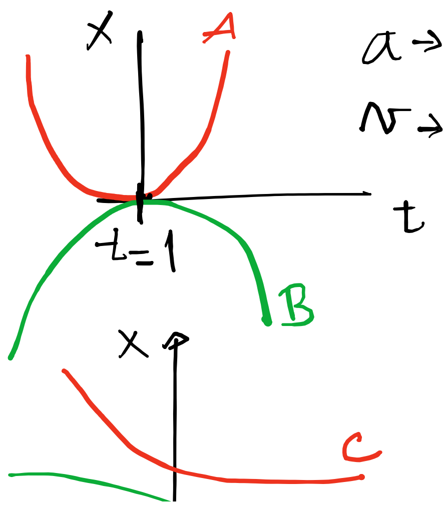
Problemi, Pag 40

$\boxed{10}$

A) $v = 0$, $a > 0$

B) $v = 0$, $a < 0$

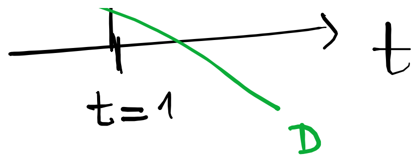
C) $v < 0$, $a > 0$



$a \rightarrow$ curvatura

$v \rightarrow$ inclinazione della tangente

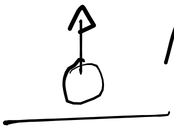
D) $v<0, a<0$



$$x(t) = x_0 + v_0 t + a\frac{t^2}{2}$$

$$v(t) = v_0 + at$$

$\boxed{18}$



$v_i = 25 \pm 6$ (m/s)

$$\begin{cases} i & y=0,\ t=0,\ v_i \\ int & y=y_{max},\ t=t_{int},\ v=0 \\ f & y=0,\ v,\ t=t_{tot} \end{cases}$$

$$v = v_0 - gt \to t_{int}:\ 0 = v_0 - g t_{int}$$

$$t_{int} = v_0/g$$

$$y_{max} = y_0 + v_0 t_{int} - \frac{g}{2} t_{int}^2$$

$$y_{max} = \frac{v_0^2}{g} - \frac{g}{2}\frac{v_0^2}{g^{\cancel{2}}} \quad \boxed{y_{max} = \frac{1}{2}\frac{v_0^2}{g}}$$

t_{tot} corresponde a

$$0 = y_{max} - \frac{g}{2}(t - t_{int})^2$$

$$\boxed{t_{tot} = \frac{2|v_0|}{g}}$$

t τ

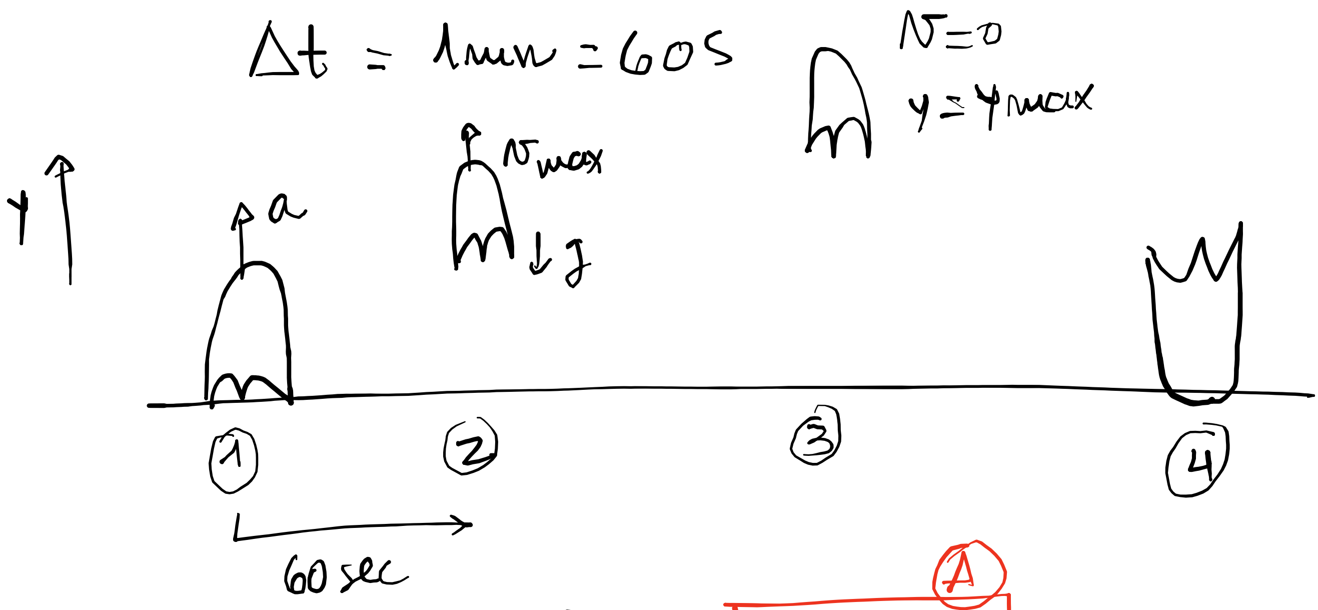
$$t_{tot} = \frac{2(v_0 \pm \epsilon)}{g} \Rightarrow t_{tot} = \frac{2v_0}{g} \pm \frac{2\epsilon}{g}$$

$$\begin{cases} t \approx 5 \, m/s \\ \tau = \frac{2\epsilon}{g} \end{cases}$$

$\boxed{21}$

$a = 20 \, m/s^2$

$\Delta t = 1 min = 60 s$



$y_1 = 0$	$y_2 = ?$	$\boxed{y_3 = y_{max}}$ A	$y_4 = 0$
$v_1 = 0$	$v_2 = v_{max}$	$v_3 = 0$	$v_4 = ?$
$\underbrace{a = 20 \, m/s^2}_{\equiv \tilde{a}}$	$a = -g$	$a = -g$	$a = -g$
$t_1 = 0$	$t_2 = 60 sec$	$t_3 = ?$	$\boxed{t_4 = ?}$ B

$$\begin{cases} y = y_0 + v_0 t + \frac{a}{2} t^2 \\ v = v_0 + at \end{cases}$$

y_3 e' funzione di y_2

$$y_2 = \cancel{y_1} + \cancel{v_1 t} + \underline{\tilde{a}} t^2 \quad \text{con } t = 60 \text{ sec}$$

$=0 \quad =0 \quad 2 \qquad \tilde{a} = 20\, m/s^2$

$$y_2 = \frac{\tilde{a}}{2} t^2 = \frac{20}{2}(60)^2 = 3.6 \times 10^4\, m$$

y_3 bisogna conoscere $t_3 \rightarrow$ tempo quando $v = 0$

$$v = v_i - gt$$

$$v_2 = v_{max} = v(t=60) = \cancel{v_0}^{=0} + \tilde{a} t = \tilde{a} t = 20 \times 60 = 1200\, m/s$$

$$\begin{cases} y_3 = y_2 + v_{max}(t_3 - t_2) - \frac{g}{2}(t_3 - t_2)^2 \\ v_3 = v_{max} - g(t_3 - t_2) \end{cases} \qquad \boxed{y_3 = y_2 + \frac{v_2^2}{2g}} = y_{max}$$

$$\Rightarrow \boxed{t_3 - t_2 = \frac{v_2}{g}}$$

$$\begin{cases} y_4 = y_3 - \frac{g}{2}(t_4 - t_3)^2 \\ v_4 = \underbrace{\cancel{v_3}}_{=0} - g(t_4 - t_3) \end{cases} \qquad \boxed{t_4 - t_3 = \sqrt{\frac{2y_3}{g}}}$$

$$\downarrow$$

$$t_4 = t_3 + \sqrt{\frac{2y_3}{g}}$$

$$= t_2 + \frac{v_2}{g} + \sqrt{\frac{2}{g}\left(y_2 + \frac{v_2^2}{2g}\right)}$$

$$\boxed{t_4 = t_2 + \frac{\tilde{a}}{g} t_2 + t_2 \sqrt{\frac{\tilde{a}}{g} + \left(\frac{\tilde{a}}{g}\right)^2}}$$

$t_2 = 60s$, $\tilde{a} = 2g \Rightarrow t_4 \approx 327$ sec B

A $\quad y_3 = y_2 + v_2^2 \Rightarrow \boxed{y_3 = y_{max} = \frac{\tilde{a} t_2^2}{2}(1 + \tilde{a}/g)}$

$$\frac{\ }{2g} \quad \frac{\ \ \ \ \ max \qquad 2}{}$$

$$y_{max} \simeq \underline{\underline{108\ km}}$$