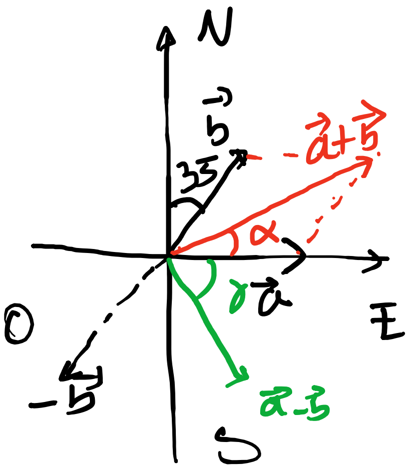
25/03/2020

RHK, CAP. 2 Moto in 1 dimensione

P. 36

$\boxed{3}$ $\vec{a}$: $\|\vec{a}\| = 5.2$ E

$\vec{b}$: $\|\vec{b}\| = 4.3$ 35° E rispetto a N



$\vec{a} = (5.2, 0)$

$\vec{b} = \|\vec{b}\| (\sin 35, \cos 35)$

$\vec{a}+\vec{b} = (5.2 + \|\vec{b}\| \sin 35, \|\vec{b}\| \cos 35)$

$= (7.66, 3.52)$

$\boxed{\text{artg} = tg^{-1} \neq \frac{1}{tg}}$

$\|\vec{a}+\vec{b}\| = \sqrt{7.66^2 + 3{,}52^2} = 8.43$

$tg\,\alpha = \frac{3.52}{7.66} \Rightarrow \alpha = \text{arctg}\, 0.46 \Rightarrow \alpha = 24.7^\circ$

$\vec{a}-\vec{b} = \vec{a} + (-\vec{b})$

$\vec{a}-\vec{b} = (5.2 - \|\vec{b}\| \sin 35, - \|\vec{b}\| \cos 35)$

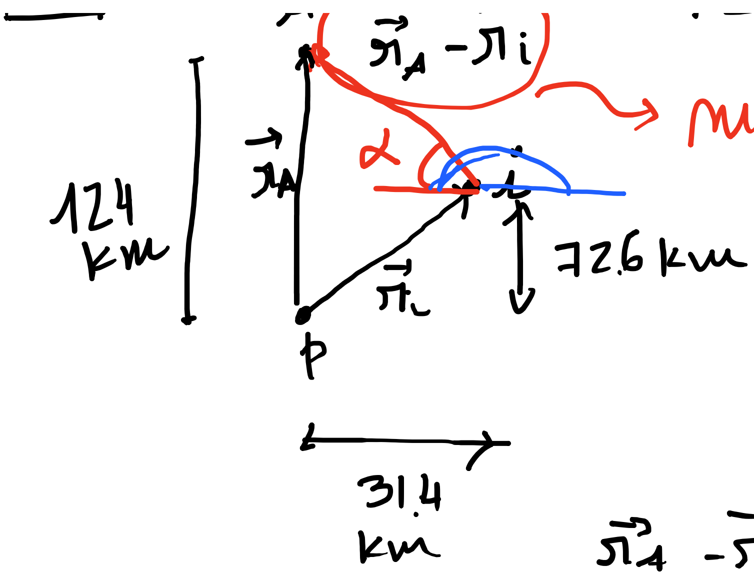
$= (2.73, -3.52)$

$\|\vec{a}-\vec{b}\| = \sqrt{2.73^2 + (3.52)^2} = 4.4$

$tg\,\gamma = -\frac{3.52}{2.73} \Rightarrow \gamma = -52.2^\circ$

$\boxed{6}$

$P = (0\ 0)$



$\vec{r}_A = (0, 124)$

$\vec{r}_i = (31.4, 72.6)$

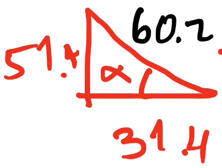
$$\vec{r}_A - \vec{r}_i = (-31.4,\ 124 - 72.6) = (-31.4,\ 51.4)$$

- distanza $\to \|\vec{r}_A - \vec{r}_i\|$

$$\|\vec{r}_A - \vec{r}_i\| = \sqrt{31.4^2 + 51.4^2} = 60.2 \text{ km}$$

- direzione $\quad Tg\,\alpha = \dfrac{51.4}{31.4} \Rightarrow \alpha = \operatorname{arctg} 1.64$

$$\alpha = 58.6^\circ$$



$\to tg\,\alpha = \dfrac{51.4}{31.4}$

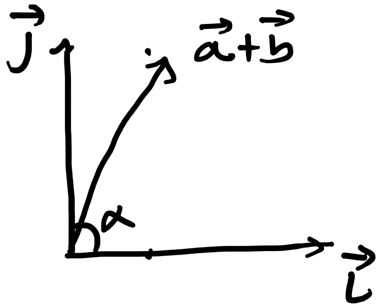
direzione $\to$ 58.6° a Nord di ovest

$\boxed{7}$ $\quad \vec{a} = 5\vec{i} + 3\vec{j} \qquad \vec{b} = -3i + 2j$

modulo, direzione, verso di $\vec{a} + \vec{b}$

$$\vec{a} + \vec{b} = (5\vec{i} + 3\vec{j}) + (-3\vec{i} + 2\vec{j})$$
$$= (5-3)\vec{i} + (3+2)\vec{j}$$
$$= 2\vec{i} + 5\vec{j}$$

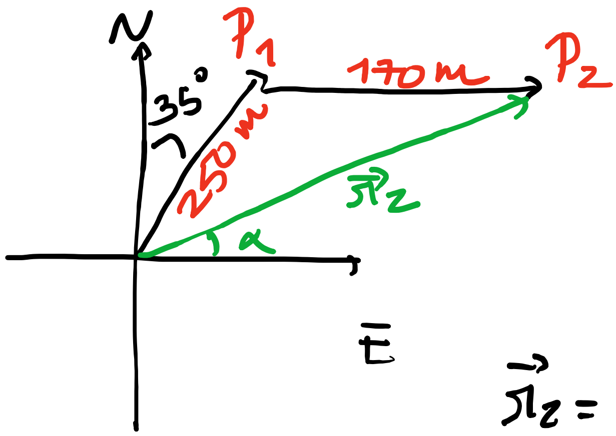
- modulo $\|\vec{a} + \vec{b}\| = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29} \simeq 5.4$



α: $tg\,\alpha = 5/2$

$\Leftrightarrow \alpha = \arctg(5/2) \Leftrightarrow \alpha = \underline{\underline{68.2^\circ}}$

$\boxed{9}$



$\vec{r}_1 = \|\vec{r}_1\| (\sin 35, \cos 35)$

dove $\|\vec{r}_1\| = 250\,m$

$\vec{r}_2 = \vec{r}_1 + (170, 0)$

$\vec{r}_2 = (\underbrace{\|\vec{r}_1\| \sin 35 + 170}_{313.4}, \underbrace{\|\vec{r}_1\| \cos 35}_{204.8}$

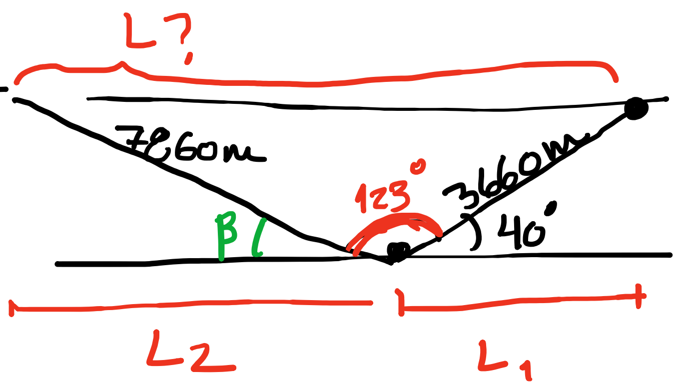
$\vec{r}_2 = (313.4, 204.8)\,m$

$\|\vec{r}_2\| = \sqrt{313.4^2 + 204.8^2} = 374.4\,m$

$tg\,\alpha = \frac{204.8}{313.4} \Leftrightarrow \alpha = \underline{56.8^\circ}$

direzione 56.8° a Nord di Est

$\boxed{15}$



$L = L_1 + L_2$

$\cos 40^\circ = \frac{L_1}{3660} \Leftrightarrow L_1 = 2803.7\,m$

$\beta = 180 - 123 - 40 = 17^\circ$

$\cos\beta = \frac{L_2}{7860} \Rightarrow L_2 = 7516.6\ m$

$L = L_1 + L_2 \Rightarrow L = 2803.7 + 7516.6$

$= \underline{\underline{10320\ m}}$

$\boxed{17}$

$\vec{r} = (2t^3 - 5t)\vec{i} + (6 - 7t^4)\vec{j}$

$\vec{r}(t=2) \xrightarrow{\frac{d}{dt}} \vec{v}(t=2) \xrightarrow{\frac{d}{dt}} \vec{a}(t=2)$

$\vec{v} = \frac{d}{dt}\vec{r}$; $\vec{a} = \frac{d}{dt}\vec{v}$

$\vec{v} = \frac{d}{dt}\left[(2t^3 - 5t)\vec{i} + (6 - 7t^4)\vec{j}\right]$, $\boxed{\frac{d}{dt}t^p = pt^{p-1}}$

$\vec{v}(t) = (6t^2 - 5)\vec{i} + -28t^3 j$

$\vec{v}(t=2) = (6 \times 4 - 5)\vec{i} - 28 \times 8\vec{j} = \underline{19\vec{i} - 224\vec{j}}$

$\vec{a}(t) = \frac{d}{dt}\vec{v}(t) = \frac{d}{dt}\left((6t^2 - 5)\vec{i} - 28t^3\vec{j}\right)$

$\vec{a}(t) = 12t\,\vec{i} - 3 \times 28\,t^2\,\vec{j}$

$\vec{a}(t=2) = 24\vec{i} - 12 \times 28\vec{j} = \underline{24\vec{i} - 336\vec{j}}$

$\boxed{19}$

19

$v = (6t - 4t)\,i + 8j \quad (m/s)$

a) $\vec{a}(t=3)$

$$\vec{a}(t) = \frac{d}{dt}\vec{v} = \boxed{(6-8t)\vec{i} + 0\vec{j}}$$

$$\vec{a}(t=3) = (6-8\times 3)\vec{i} + 0\vec{j}$$

$$= -18\vec{i} + 0\vec{j} \quad (m/s^2)$$

b) $\tilde{t}$ tale che $\vec{a}(\tilde{t}) = 0$

$$6 - 8t = 0 \Rightarrow t = 6/8 \quad (s)$$

c) t tale che $\vec{v} = 0 = 0\vec{i} + 0\vec{j}$

$$\begin{cases} 6t - 4t^2 = 0 \\ 8 = 0 \quad \times \end{cases}$$

d) t tale che $\|\vec{v}\| = 10\ m/s$

$$\|\vec{v}\|^2 = (6t - 4t^2)^2 + 8^2 = 10^2 \quad (m^2/s^2)$$

$$(6t - 4t^2)^2 = 100 - 8^2$$

$$6t - 4t^2 = \pm 6$$

$$4t^2 - 6t \pm 6 = 0 \Rightarrow t = \frac{6 \pm \sqrt{36 - 4\times 4\times \pm 6}}{8}$$

$$t = \frac{1}{4}(3 \pm \sqrt{33}) \qquad 2\sqrt{33} \simeq 5.7$$

$$t = \frac{1}{4}(3 + 5.7) \simeq 2.18\ (s)$$

$\boxed{20}$ $\vec{v} = v_x\vec{i} + v_y\vec{j} \qquad \vec{a} = a_x\vec{i} + a_y\vec{j}$

dimostrare che $\|\vec{v}\| = \text{costante} \iff a_x v_x + a_y v_y = 0$

$$\|\vec{v}\|^2 = v_x^2 + v_y^2 \iff \|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$

$\downarrow$

costante $\iff \boxed{\dfrac{d}{dt}\|\vec{v}\| = 0}$

$$\frac{d}{dt}\|\vec{v}\|^2 = 2\|\vec{v}\|\frac{d}{dt}\|\vec{v}\|$$

regola della catena

$$\frac{d}{dt}\|\vec{v}\|^2 = 0 \iff \frac{d}{dt}\left(v_x^2 + v_y^2\right) = 0$$

$$\cancel{2} v_x \underbrace{\frac{d}{dt} v_x}_{= a_x} + \cancel{2} v_y \underbrace{\frac{d}{dt} v_y}_{a_y} = 0$$

$$v_x a_x + v_y a_y = 0$$

$$\iff \vec{v} \cdot \vec{a} = 0 \Rightarrow \vec{a} \perp \vec{v}$$

$\boxed{23}$

$v_m = 112\,\text{km/h}$ $\quad\quad \Delta t = 1\,\text{s}$

$$v_m = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = v_m \Delta t$$

$$\Delta x = 112\,\frac{\text{km}}{\text{h}} \times 1\,\text{s}$$

Passare $\vec{v}$ da

$$= 112\,\frac{1000\,m}{60 \times 60\,s} \times 1\,s \quad \swarrow \quad \frac{km}{h} \rightarrow m/s$$

$$= 112 \times \frac{1000}{3600} = 31.1\,m$$

$\boxed{26}$

$D = 100\,m \qquad \Delta t = 9.81\,s$

$D = 42.25\,km \qquad \Delta t = 2h\,05m\,42s$

(A) velocità (scalare) media

$$v_1 = \frac{100\,m}{9.81\,s} = 10.19\,m/s$$

$$v_2 = \frac{42250\,m}{2h\,5m\,42s} = \frac{42250\,m}{2 \times 60 \times 60 + 5 \times 60 + 42}$$

$$= 5.60\,m/s$$

(B)

$$v = \frac{D}{\Delta t} \Rightarrow \Delta t = \frac{D}{v}$$

$$\Delta t = \frac{42250\,m}{10.19\,m/s} = 4144.7\,s$$

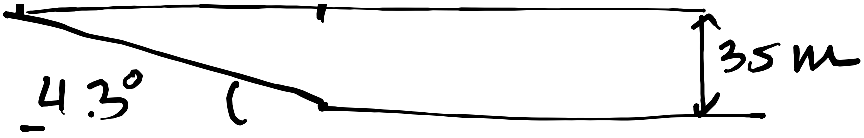
$$\Downarrow$$

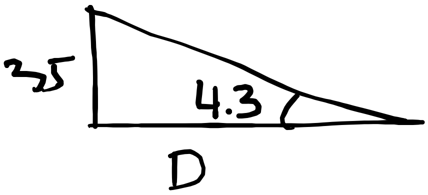
$$1.15h = 1h\,09m$$

$\boxed{28}$

$\overbrace{\qquad\qquad\qquad}^{D?}$

$v = 1300\,km/h$





$$\mathrm{tg}\,4.3^\circ = \frac{35\,m}{D} \Leftrightarrow D = \frac{35}{\mathrm{tg}\,4.3}$$

$$D = 465.49\ m$$

$$u = \frac{D}{\Delta t} \Rightarrow \Delta t = \frac{D}{u} = \frac{465.49\ m}{1300\ \frac{km}{h}}$$

$$\Delta t = \frac{465.49\ m}{1300\ \frac{1000\,m}{3600\,s}} \Leftrightarrow \Delta t = \underline{\underline{1.3}}\ \text{Secondo}$$

$\boxed{31}$

quanta strada?

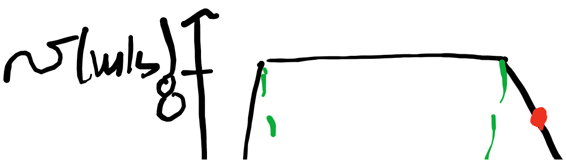
$$\Delta r = \int_{t_i}^{t_f} dt\, v(t) \rightarrow \text{inverso di } \vec{v} = \frac{d}{dt}\vec{r}$$

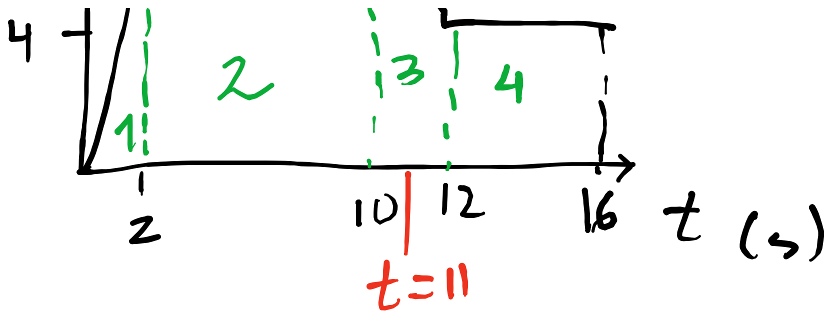
$$\frac{dr}{dt} = v \Leftrightarrow dr = v\,dt \rightarrow \int_{r_i}^{r_f} dr = \int_{t_i}^{t_f} v\,dt$$

$$\Delta r = r_f - r_i = \int_{t_1}^{t_f} v\,dt$$

$\downarrow$

Area soto la curva $v(t)$





$A_1 = \frac{2 \times 8}{2}$, $A_2 = 8 \times 8$, $A_3 = 4 \times 2 + 4 \times \frac{2}{2}$

$A_4 = 4 \times 4$

$$\Delta x = A_1 + A_2 + A_3 + A_4 = \underline{\underline{100\,m}}$$

$\boxed{32}$ $a(t = 11 s)$

tra 10 e 12 secondi

$a = \frac{dv}{dt}$ $a = \frac{\Delta v}{\Delta t} = \frac{4-8}{2} = -2 m/s$

v è funzione lineare del t

$\Downarrow$

a é costante

$$v = c_1 t + c_0 \rightarrow a = \frac{dv}{dt} = c_1$$

$$\boxed{v = at + v_0} \rightarrow \frac{v(t) - v_0}{\Delta t} = a$$