18/03/2020

Capitolo 1 RHK

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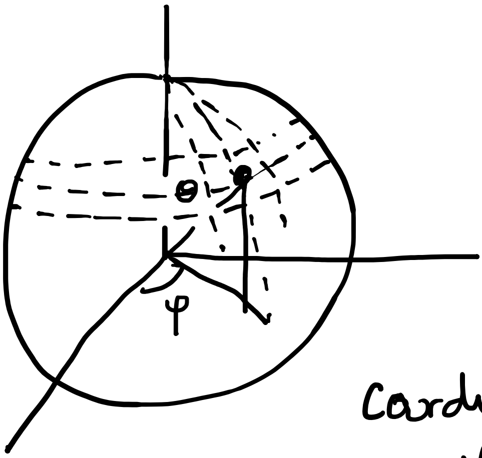
$\boxed{3}$



$\theta = 43° \; 36' \; 25.3''$ N

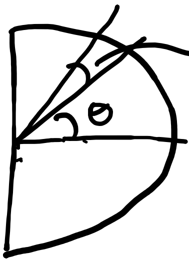
$\phi = 77° \; 31' \; 48.2''$ W

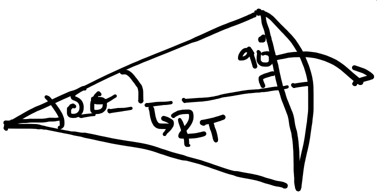
Errore $\frac{\Delta\theta}{2} = \frac{\Delta\phi}{2} = \pm 0.5''$

Coordinate sferiche

$\varphi \in [0, 2\pi] \quad \theta = [0, \pi]$

$\boxed{N\text{-}S}$





$$tg \frac{\delta\theta}{2} = \frac{\Delta_{NS}}{2} \frac{1}{R_T}$$

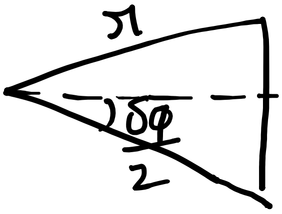
$$\Delta_{NS} = 2 R_T \, tg \, \delta\theta/2$$

$$\Delta_{NS} = 2 \underbrace{(6.4 \times 10^6 \, m)}_{R_T} \, tg \left(\frac{1/2}{60 \times 60} \right)$$

$$\Delta_{NS} = \underline{\underline{31 \, m}}$$

$\boxed{EW}$

$r \neq R_T$



Δ_{EW}

$$tg \frac{\delta\phi}{2} = \frac{\Delta_{EW}/2}{r}$$

$$\Delta_{EW} = 2 r \, tg \, \delta\phi / 2$$

$r = R_T \sin\theta \longrightarrow \Delta_{EW} = 2 R_T \sin\theta \, tg(\delta\phi/2)$

$$\Delta_{EW} = 2 \times (6.4 \times 10^6 \, m) \sin\left(43 + \frac{36}{60} + \dots\right) tg\left(\frac{1}{2} \frac{1}{60 \times 60}\right)$$

$$\Delta_{EW} = \underline{\underline{21.4 \, m}}$$

$\boxed{15}$

$V_{O_2} = 0.3 \, l$ $\qquad V_{CO_2} = 0.3 \, l$

$\rho_{O_2} = 1.43 \, g/l$ $\qquad \rho_{CO_2} = 1.96 \, g/l$

Assumendo un periodo del ciclo respiratorio di circa 10 secondi

Per ogni ciclo perdiamo

$$\left(1.43 \frac{g}{l} \cdot 0.3 \, l\right) - \left(1.96 \frac{g}{l} \cdot 0.3 \, l\right) = -0.159 \, g$$

in 8 ore perdiamo

$$\frac{8h}{10 \sec} \times 0.159 \, g = \frac{8 \times 60 \times 60 \, \cancel{sec}}{10 \, \cancel{sec}} \times 0.159$$

$$= 458 g$$

$\boxed{16}$ $V = 5700 \, m^3$

$\Delta t = 12 h$

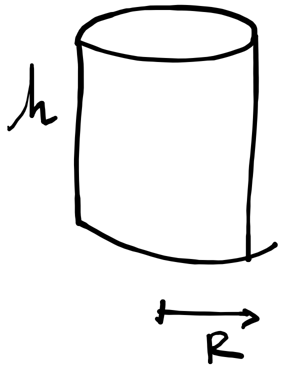
? Portata massica media? $\rightarrow$ (Kg/s)

$\rho_{H_2O} = 1000\, kg/m^3 \quad (1\, kg/l)$

$$\frac{V}{\Delta t} = \frac{5700\, m^3}{12\, h} = \frac{5700}{12 \times 60 \times 60} = 0.132\, \frac{m^3}{s}$$

$$\rho_{H_2O} \frac{V}{\Delta t} = 1000\, \frac{kg}{\cancel{m^3}} \times 0.132\, \frac{\cancel{m^3}}{s} = 132\, \frac{kg}{s}$$

$\boxed{8}$



$V = \pi R^2 h$

$A = 2\pi R h + 2\pi R^2 = 2\pi R (R + h)$

minimizzare A, mantenendo V fisso

A come funzione di V

$$V = \pi R^2 h \Leftrightarrow \boxed{h = \frac{V}{\pi R^2}}$$

$$A = 2\pi R \left(R + \frac{V}{\pi R^2} \right) \Leftrightarrow \boxed{A = 2\pi R^2 + 2\frac{V}{R}}$$

$$\frac{\partial A}{\partial R} = 0 \Leftrightarrow 2\pi \frac{\partial}{\partial R} R^2 + 2V \frac{\partial}{\partial R} (1/R) = 0$$

$\frac{\partial}{\partial R} R^q = q R^{q-1}$

$$2\pi\, 2R + 2V \frac{(-1)}{R^2} = 0$$

$$\Leftrightarrow R = \left(\frac{V}{2\pi} \right)^{1/3}$$

visto che $h = \frac{V}{\pi R^2} = \frac{\pi R^2 h}{\pi \left(\frac{V}{2\pi}\right)^{2/3}} = \frac{R^2 h}{\left(\frac{R^2}{2} h\right)^{2/3}}$

$$h^{2/3} = 2^{2/3} R^{2/3} \Rightarrow h = 2R$$

⟹ Vettori Posizione, velocità & accelerazione

- **Posizione**:

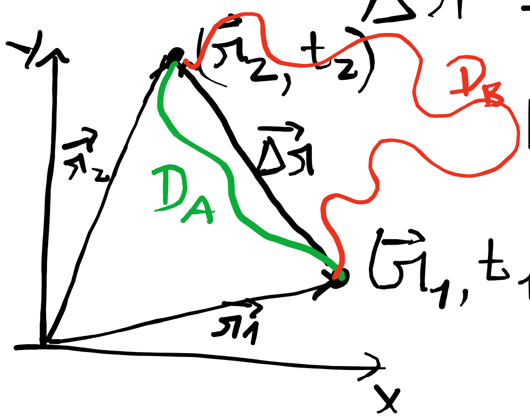
$$\vec{r} = (x, y, z)$$
$$= x \vec{u}_x + y \vec{u}_y + z \vec{u}_z$$
$$= x \vec{i} + y \vec{j} + z \vec{k}$$

↓

In generale sono funzioni del tempo t

Vet. spostamento: se a $t = t_1$ la particella si trova a $\vec{r}_1$ e a $t = t_2$ si trova a $\vec{r}_2$ il vettore spostamento $\vec{\Delta r}$ è definito da

$$\vec{\Delta r} = \vec{r}_2 - \vec{r}_1$$



$\|\vec{\Delta r}\| \neq$ distanza percorsa D

$\|\vec{\Delta r}\| < D_A < D_B$

- Velocità

• velocità media (vettoriale)

$\vec{v}$ $\vec{v}_{med}$

$$\vec{v}_{med} \equiv \frac{\vec{\Delta r}}{\Delta t}$$ → moltiplicazione di un vettore $\vec{\Delta r}$ per uno scalare $1/\Delta t$

$$\vec{v}_{med} \parallel \vec{\Delta r}$$

• velocità (istantanea) vettoriale

$$\vec{v} = \underbrace{\lim_{\Delta t \to 0} \frac{\vec{\Delta r}}{\Delta t}}_{\equiv \frac{d\vec{r}}{dt}} = \lim_{\Delta t \to 0} \vec{v}_{med}$$

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt}}$$

$$\vec{v} = \frac{d}{dt} x \, \vec{u}_x + \frac{d}{dt} y \, \vec{u}_y + \frac{d}{dt} z \, \vec{u}_z$$

$$\parallel$$

$$\vec{v} = v_x \vec{u}_x + v_y \vec{u}_y + v_z \vec{u}_z$$

$$v_x = \frac{dx}{dt} ; \quad v_y = \frac{dy}{dt} ; \quad v_z = \frac{dz}{dt}$$

$$\vec{v} = \left(\frac{d}{dt} x , \frac{d}{dt} y , \frac{d}{dt} z \right) = \left(v_x , v_y , v_z \right)$$

nel SI $[v] = m/s$

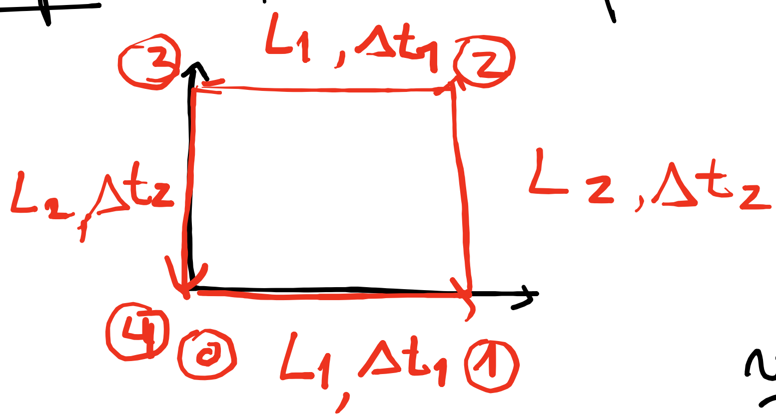
$[r] = m$

$[a] = m/s^2$

velocità scalare media

$$u = \frac{D}{\Delta t}$$

Esempio: Particella che parte da ⓪ e arriva a ④



velocità media vettoriale

$$\vec{v}_{med} = \frac{\vec{\Delta r}}{\Delta t} = 0$$

vel. scalare media

$$u = \frac{2L_1 + 2L_2}{2\Delta t_1 + 2\Delta t_2} = \frac{L_1 + L_2}{\Delta t_1 + \Delta t_2} \neq 0$$

quindi $\|\vec{v}_m\| \neq u$

- ACCELERAZIONE

↳ misura la variazione nel tempo della velocità

- acc. media: $\vec{a}$ $\quad \vec{a}_{med}$

$$\vec{a}_{med} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_{finale} - \vec{v}_{ini}}{\Delta t}$$

$$\vec{a}_{media} \parallel \Delta \vec{v}$$

- <u>acc. istantanea</u> $\vec{a}$

$$\vec{a} = \lim_{\Delta t \to 0} \vec{a}_m = \underbrace{\lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}}_{= \frac{d\vec{v}}{dt}}$$

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt}}$$

$$\vec{a} = a_x \vec{u}_x + a_y \vec{u}_y + a_z \vec{u}_z$$

$$= \frac{dv_x}{dt} \vec{u}_x + \frac{dv_y}{dt} \vec{u}_y + \frac{dv_z}{dt} \vec{u}_z$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt}$$

$$a_z = \frac{dv_z}{dt} = \frac{d^2z}{\dots}$$

dt

⇒ Cinematica Unidimensionale

considoriamo un punto materiale descritto da posizione $x(t)$

velocità $v(t)$

accelerazione $a(t)$

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} = \frac{d^2}{dt^2} x$$

Se conosciamo possiamo trovare $X(t)$, integrando queste equazioni

<u>Step 1</u>

$$a = \frac{dv}{dt} \Leftrightarrow dv = a\,dt$$

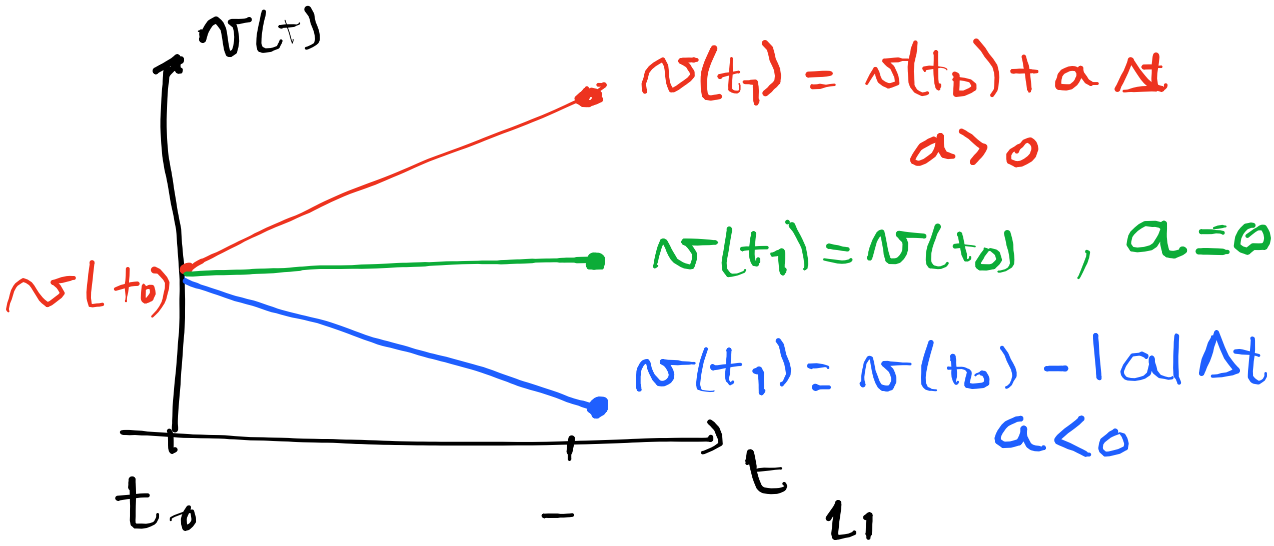
↓ integrare tra t_0 e t_1

$$\int_{v_0}^{v(t_1)} dv = \int_{t_0}^{t_1} a\,dt \Leftrightarrow \boxed{v(t_1) - v(t_0) = \int_{t_0}^{t_1} a\,dt}$$

assumendo a costante (indep. di t)

$$v(t_1) - v(t_0) = a \int_{t_0}^{t_1} dt \Rightarrow \boxed{v(t_1) = v(t_0) + a\,(t_1, t_0)} \quad (*)$$

Per a costante v è funzione lineare di t



<u>Step 2</u>

$$v = \frac{dx}{dt} \Leftrightarrow dx = v\,dt$$

$$\int_{x_0}^{x(t_1)} dx = \int_{t_0}^{t_1} v\,dt \Leftrightarrow x(t_1) - x(t_0) = \int_{t_0}^{t_1} \left(v(t_0) + a(t - t_0)\right) dt$$

Assumendo a costante (*)

$$\int dt\, t = \frac{t^2}{2}$$

$$\boxed{x(t_1) = x(t_0) + v(t_0)(t_1 - t_0) + \frac{a}{2}(t_1 - t_0)^2}$$

velocita iniziale

pos. iniziale

$t_0 \rightarrow$ tempo iniziale

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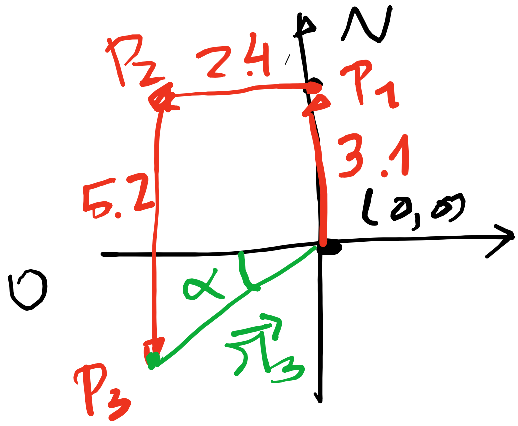
$\boxed{2}$

3.1 km N ; 2.4 km O ; 5.2 km S



$P_1 = (0, 3.1)$ km

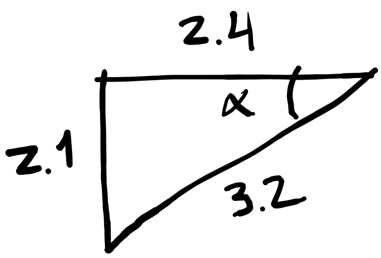
$P_2 = (-2.4, 3.1)$ km

e $P_3 = (-2.4, 3.1-5.2)$

$\underbrace{(-2.4, -2.1)}_{=\vec{r}_3}$ km

distanza $\|\vec{r}_3\| = \sqrt{2.4^2 + 2.1^2}$

$= 3.2$ km



$tg\,\alpha = 2.1/2.4 \Leftrightarrow \alpha = \mathrm{arctg}\,\frac{2.1}{2.4}$

$= 41.2°$