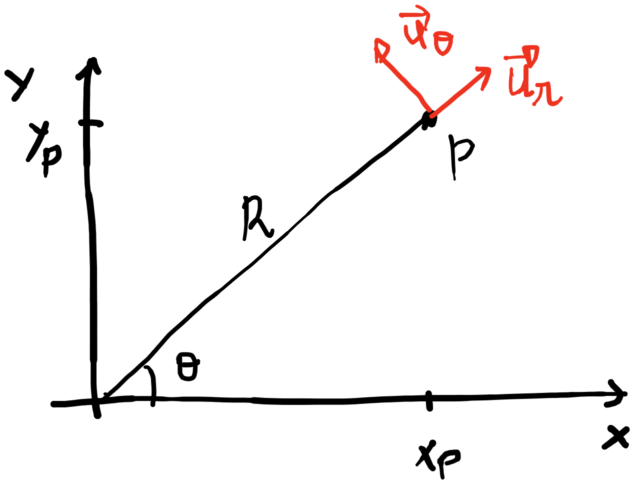
11/03/2020

Coordinate Polari



$\vec{r}_p = x_p \vec{u}_x + y_p \vec{u}_y$

versori

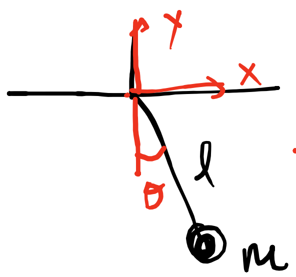
$\|\vec{u}_x\| = \|\vec{u}_y\|$

$\downarrow$

$\vec{r}_p = R\,\vec{u}_r + \theta\,\vec{u}_\theta$

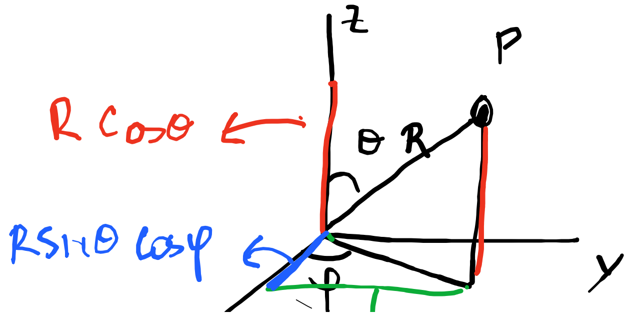
$$\begin{cases} x_p = R\cos\theta \\ y_p = R\sin\theta \end{cases} \qquad x_p^2 + y_p^2 = R^2$$

$R, \theta \rightarrow$ coordinate Polari



→ Più facile in coord. Polari

Coordinate sferiche (3D)



$\vec{r}_p = x_p \vec{u}_x + y_p \vec{u}_y + z_p \vec{u}_z$

$\downarrow$

$\vec{r}_p = (x_p, y_p, z_p)$

$\varphi \in [0, 2\pi)$ coord

$y = R \sin\theta \sin\varphi$

$\theta \in [0, \pi]$
$R \in [0, +\infty[$ } Sferiche

$$\begin{cases} x_p = R \sin\theta \cos\varphi \\ y_p = R \sin\theta \sin\varphi \\ z_p = R \cos\theta \end{cases} \rightarrow R^2 = x_p^2 + y_p^2 + z_p^2$$

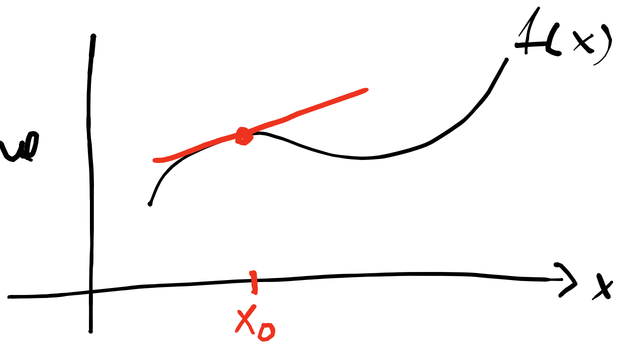
DERIVATE

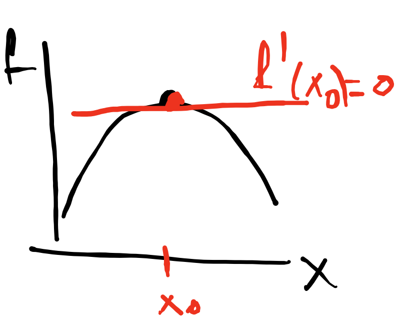
derivata di una funzione reale $f(x)$
dove $x \in \mathbb{R}$:

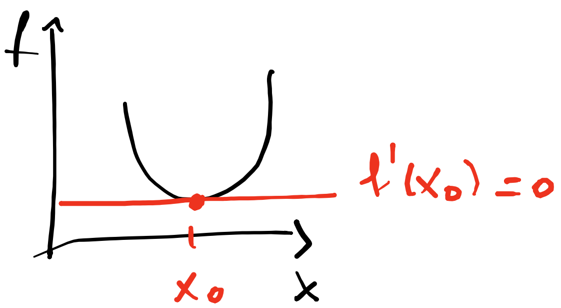
$$f'(x) = \frac{df}{dx}$$

$$f'(x_0) \equiv \lim_{\Delta \to 0} \frac{f(x_0+\Delta) - f(x_0)}{\Delta}$$

↓
misura variazione di f con x
↓
geometricamente
derivata è inclinazione
della retta tangente
a f nel punto x_0

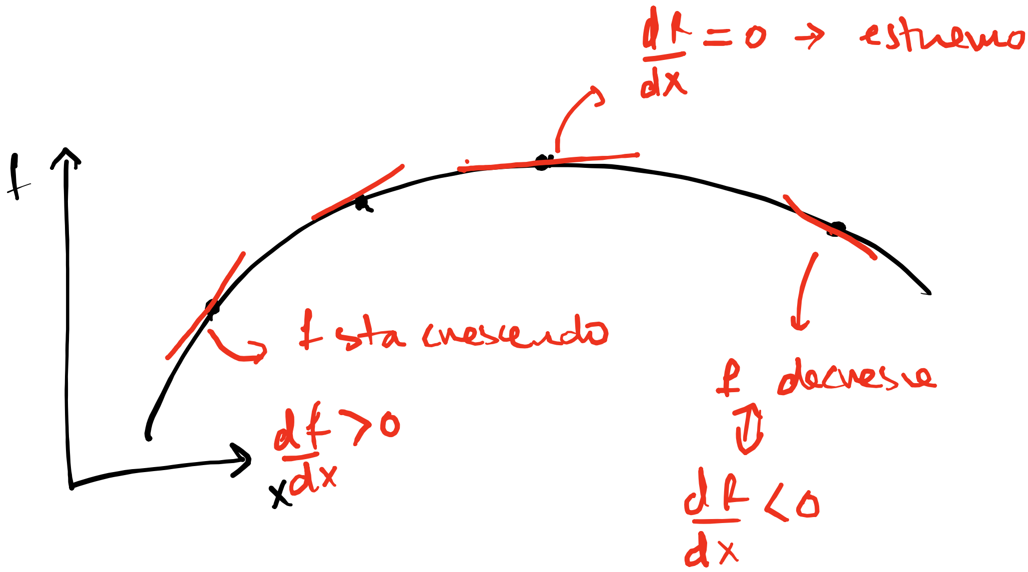






$f'(x) = 0$ negli estremi di f

$$\left(\begin{array}{l} f''(x) > 0 \rightarrow \text{minimo} \\ f''(x) < 0 \rightarrow \text{massimo} \end{array} \right)$$



Esempio 1: funzione lineare $f(x) = ax+b$

dove $a, b \in \mathbb{R}$

$$\frac{d}{dx} f(x) = \frac{d}{dx}(ax+b)$$

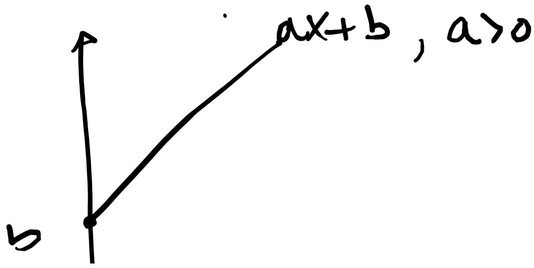
$$\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

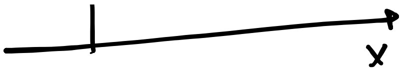
$$\lim_{\Delta \to 0} \frac{(a(x+\Delta)+b)-(ax+b)}{\Delta} =$$

$$\lim_{\Delta \to 0} \frac{\cancel{ax} + a\Delta + \cancel{b} - \cancel{ax} - \cancel{b}}{\Delta}$$

$$\lim_{\Delta \to 0} a \frac{\Delta}{\Delta} = a$$

$$\frac{d}{dx}(ax+b) = a$$





Esempio 2 funzione quadratica

$$f(x) = ax^2$$

$$f'(x) = \frac{df}{dx} = \lim_{\Delta \to 0} \frac{a(x+\Delta)^2 - ax^2}{\Delta}$$

$$= \lim_{\Delta \to 0} \frac{\cancel{ax^2} + 2ax\Delta + a\Delta^2 - \cancel{ax^2}}{\Delta}$$

$$= \lim_{\Delta \to 0} 2ax + a\Delta$$

$$= 2ax + \underbrace{\lim_{\Delta \to 0} a\Delta}_{0}$$

$$\frac{d}{dx}(ax^2) = 2ax$$

Esempio 3: funzione monomiale di grado p

$$f(x) = ax^p$$

$$\frac{d}{dx}(ax^p) = \lim_{\Delta \to 0} \frac{a(x+\Delta)^p - ax^p}{\Delta}$$

Binomio di Newton

$$= \lim_{\Delta \to 0} \frac{\cancel{ax^p} + apx^{p-1}\Delta + \ldots - \cancel{ax^p}}{\Delta}$$

termini di ordine Δ^q, $q > 1$: Δ^2 , Δ^3 Δ

$$= apx^{p-1}$$

Binomio di Newton

$$(a+b)^p = \sum_{q=0}^{p} \frac{p!}{q!(p-q)!} a^{p-q} b^q$$

$$\frac{d}{dx}(ax^p) = a p x^{p-1}$$

Esempio 4

$$\cos x$$
$$\downarrow$$

$$\frac{d}{dx}\cos x = \lim_{\Delta \to 0} \frac{\cos(x+\Delta) - \cos x}{\Delta}$$

Usare

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\frac{d}{dx}\cos x = \lim_{\Delta \to 0} \frac{\cos x \cos \Delta - \sin x \sin \Delta - \cos x}{\Delta}$$

$$\cos \Delta \simeq 1 \qquad = \lim_{\Delta \to 0} \frac{\cancel{\cos x}(1+\dots) - \sin x \sin \Delta - \cancel{\cos x}}{\Delta}$$

$$= -\sin x \underbrace{\lim_{\Delta \to 0} \frac{\sin \Delta}{\Delta}}_{1} = -\sin x$$

$$\frac{d}{dx}\cos x = -\sin x$$

nello stesso modo $\quad \frac{d}{dx}\sin x = \cos x$

Esempio 5: $\quad f(x) = e^{ax}, \quad a \in \mathbb{R}$

$$\frac{d}{dx}e^{ax} = \lim_{\Delta \to 0} \frac{e^{a(x+\Delta)} - e^{ax}}{\Delta} \qquad e^y = \sum_{n \geq 0} \frac{y^n}{n!}$$

$$= \lim_{\Delta \to 0} e^{ax} \frac{(e^{a\Delta} - 1)}{\Delta}$$

$$= e^{ax} \lim_{\Delta \to 0} (\cancel{1} + a\Delta + \dots - \cancel{1}) / \Delta$$

$$= a e^{ax}$$

$$\frac{d}{dx}\left(e^{ax}\right) = a e^{ax}$$

<u>Proprietà</u>:

- $\frac{d}{dx}\left(f(x)\, g(x)\right) = f'(x)\, g(x) + f(x)\, g'(x)$
- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)}{g(x)} - \frac{f(x)\, g'(x)}{g^2(x)}$
- $\frac{d}{dx} f(g(x)) = \frac{df}{dg} \frac{dg}{dx}$ → regola della catena

Derivate & Cinematica

Studio del moto:

- Posizione $\vec{r}$
- velocità $\vec{v}$
- accelerazione $\vec{a}$

velocità è la tassa di cambiamento di $\vec{r}$ con il tempo

$$\vec{v} = \frac{d\vec{r}}{dt}$$

acc → cambiamento di $\vec{v}$

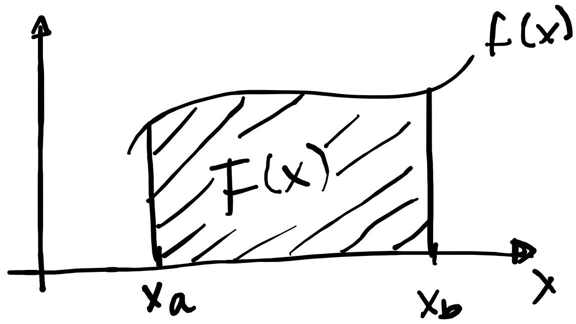
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d}{dt}\vec{r}\right) \equiv \frac{d^2}{dt^2}\vec{r}$$

Integrali

↳ "inverso" di derivata

$$F(x) = \int_{x_a}^{x_b} f(x)\,dx$$

Integrale di f tra x_a e x_b



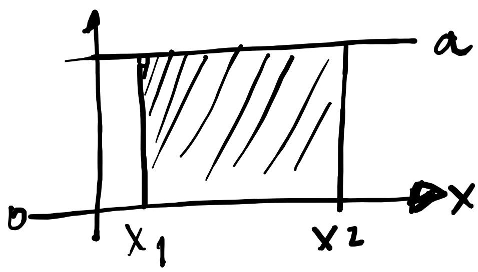
$$\frac{d}{dx}F(x) = f(x)$$

$F(x)$ è la funzione che derivata fa $f(x)$

Esempio 0: funzione costante

$$f(x) = a \quad , \quad a \in \mathbb{R}$$

$$F(x_1, x_2) = \int_{x_1}^{x_2} a\,dx = a\int_{x_1}^{x_2} dx = a(x_2 - x_1)$$

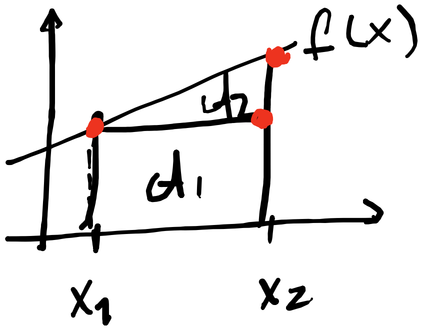


$$\int dx = x$$

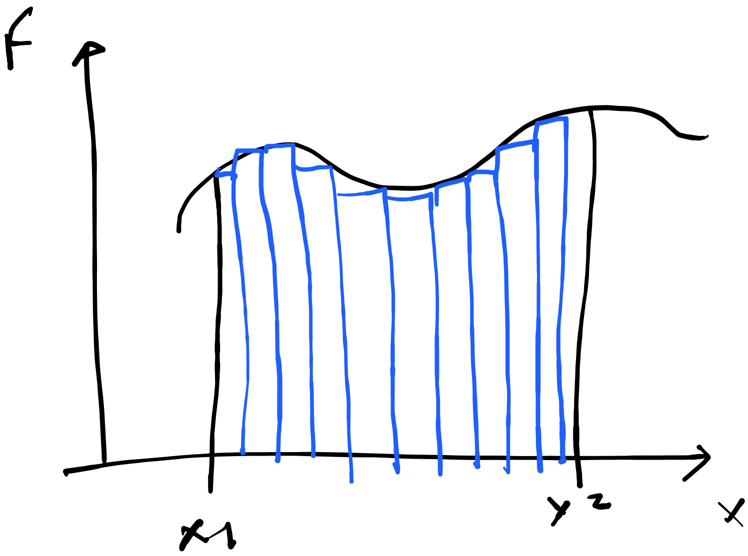
Esempio 1: $f(x) = ax + b$, $a, b \in \mathbb{R}$

$$F(x_1,x_2) = \int_{x_1}^{x_2} (ax+b)\,dx = a\underbrace{\int_{x_1}^{x_2} x\,dx}_{\left.\frac{x^2}{2}\right|_{x_1}^{x_2}} + b\underbrace{\int_{x_1}^{x_2} dx}_{x_2-x_1}$$

$$F(x_1,x_2) = \frac{a}{2}\left(x_2^2 - x_1^2\right) + b\left(x_2 - x_1\right)$$



$$F(x_1,x_2) = A_1 + A_2$$



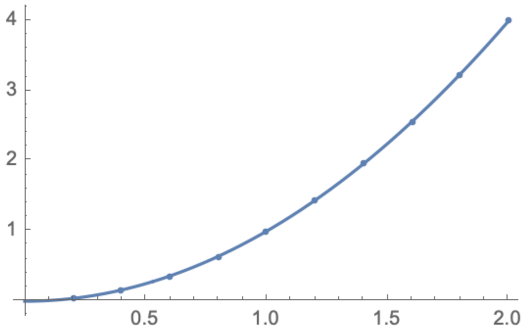
$[x_1, x_2]$ $\quad N \to \infty$

```
(*=================*)
f = x ^ 2
```

```
xMin = 0;
xMax = 2;
nPoints = 10;
(*================*)


xPoints = Table[xMin + (xMax - xMin)/nPoints i, {i, 1, nPoints}];
Δ = (xMax - xMin)/nPoints;

Plot[f, {x, xMin, xMax}];
Table[{xPoints[[i]], f /. {x → xPoints[[i]]}}, {i, 1, nPoints}] // ListPlot;
Show[%%, %]
"risultato approssimato"
Sum[Δ * f /. {x → xPoints[[i]]}, {i, 1, nPoints - 1}] // N
"risultato esatto "

Integrate[x^2, {x, xMin, xMax}] // N
```

t[102]= x^2

t[110]=



t[111]= risultato approssimato

t[112]= 2.28

t[113]= risultato esatto

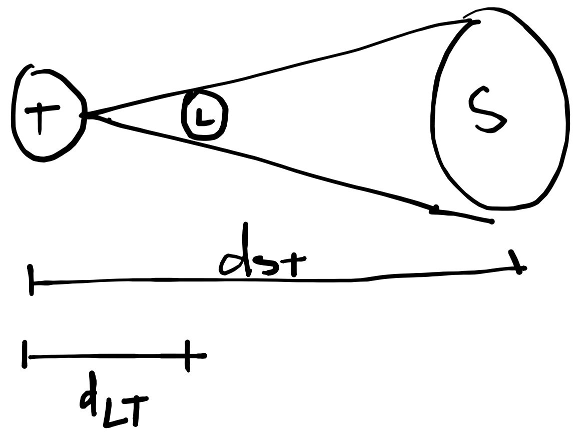
t[114]= 2.66667

Capitolo 1: Le misure

Problemi $\boxed{2}$

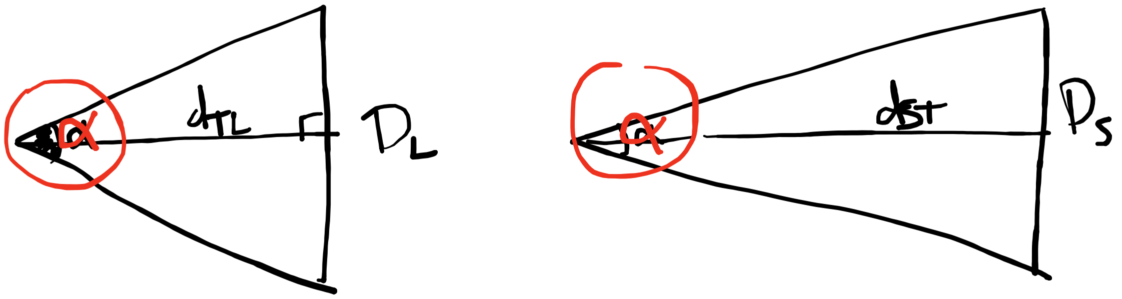
D. D.

L ; S



$d_{ST} = 390\, d_{LT}$

2 triangoli



a) Calcolare D_S/D_L

$$\operatorname{tg}(\alpha/2) = \frac{D_L/2}{d_{TL}} \qquad \operatorname{tg}(\alpha/2) = \frac{D_S/2}{d_{ST}}$$

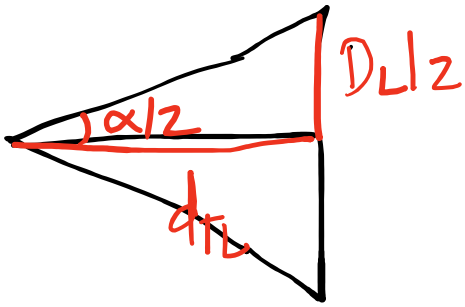
$$\frac{D_L}{d_{TL}} = \frac{D_S}{d_{ST}} \Leftrightarrow \frac{D_L}{D_S} = \frac{d_{TL}}{d_{ST}} = \frac{1}{390}$$

b) V_{SOL}/V_{LUN}

$$V = \frac{4}{3}\pi R^3 \rightarrow \frac{V_{sol}}{V_{Lun}} = \frac{\frac{4\pi}{3} R_{sol}^3}{\frac{4\pi}{3} R_{Lun}^3} = \left(\frac{R_{soL}}{R_{Lun}}\right)^3$$

c)

$\alpha = 0.52^\circ$, $d_{TL} = 3.82 \times 10^5 \, km$ $\quad D_L$?



$$\operatorname{tg} \alpha/2 = \frac{D_L/2}{d_{TL}}$$

$$D_L = 2\, d_{TL} \operatorname{tg}(\alpha/2)$$

$\downarrow$

$$D_L \simeq 3.47 \times 10^3 \, km$$