PART IV: Constraint-Based Scheduling

Scheduling

- Ordering resource-requiring tasks over time.
- A very important (and tough) problem class.
- Many practical applications.

Some Scheduling Problems

- Project planning and scheduling.
  - Software project planning.
- Machine scheduling.
  - Allocation of jobs to computational resources.
- Scheduling of flexible assembly systems.
  - Car production.
- Employee scheduling.
  - Nurse rostering.
- Transport scheduling.
  - Gate assignment for flights.
- Sports scheduling.
  - Schedule for NHL, world cup, olympics.
- Educational timetabling.
  - Timetables at schools.

Resource Constrained Project Scheduling Problem (RCPSP)

- Given:
  - a set of resources with fixed capacities,
  - a set of tasks with given durations and resource requirements,
  - a set of temporal constraints between tasks,
  - and a performance metric,

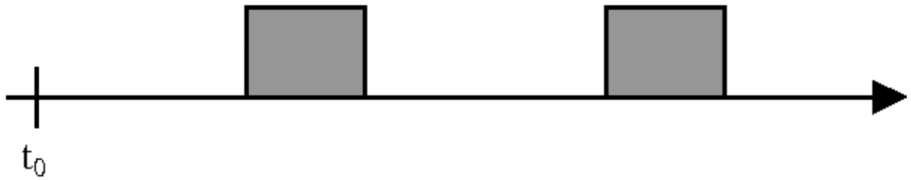
  RCPSP consists of deciding:
  - when to execute each task so as to optimize the performance metric, subject to temporal and resource constraints.

A Constraint-Based Model

- Tasks → (activity) variables.
- Resource constraints →
  - unary/disjunctive/sequential resource;
  - cumulative/parallel resource.
- Temporal constraints.
- Performance metric → schedule dependent cost function.

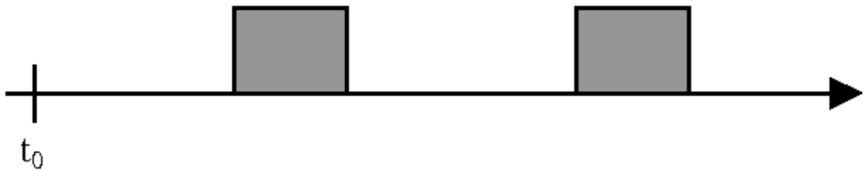
(Activity) Variables

- Correspond to the operations to be performed. E.g.,
  - processing an order,
  - executing a job,
  - working in a shift,
  - performing a loading operation.
- Need to decide the operation positions in the timeline of the schedule.



- Main variables of the scheduling problem.

(Activity) Variables



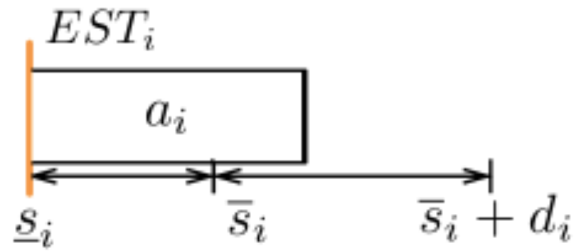
- Activity a_i
  - Start Time S_i
    - Starting time variable of an activity a_i, with domain $D(S_i)$
      - min(S_i) is the earliest start time (release date), also referred to as $EST_{i.}$
      - max(S_i) is the latest start time, also referred to as $LST_{i.}$
  - Duration d_i
    - usually assumed to be known.
  - End Time E_i
    - Ending time variable of an activity $a_{i,}$ with domain $D(E_i)$
      - max(E_i) is the latest end time (deadline), also referred to as $LET_{i.}$
      - min(E_i) is the earliest end time, also referred to as $EET_{i.}$

(Activity) Variables

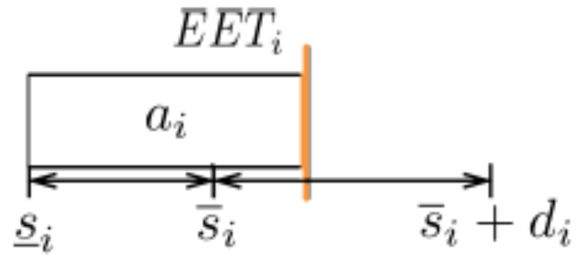
- Preemptive activity a_i
  - Can be interrupted at any time.
  - $S_i + d_i \leq E_i$
- Non-preemptive activity a_i
  - Cannot be interrupted at any time.
  - $S_i + d_i = E_i$
- We focus on non-preemptive activities.

Non-Preemptive Activity

Earliest Start Time: $\boldsymbol{EST_i = \underline{s}_i}$

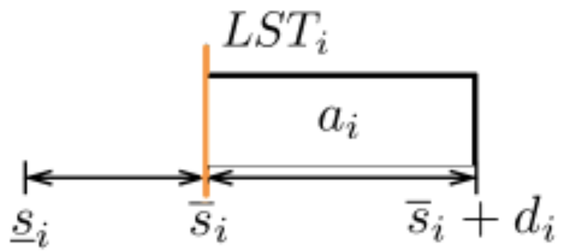


Earliest End Time: $\boldsymbol{EET_i = \underline{s}_i + d_i}$

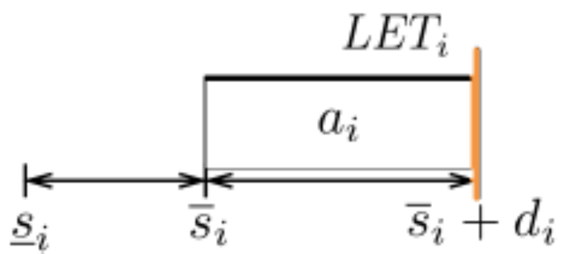


Non-Preemptive Activity

Latest Start Time: $LST_i = \bar{s}_i$



Latest End Time: $LET_i = \bar{s}_i + d_i$



Resources

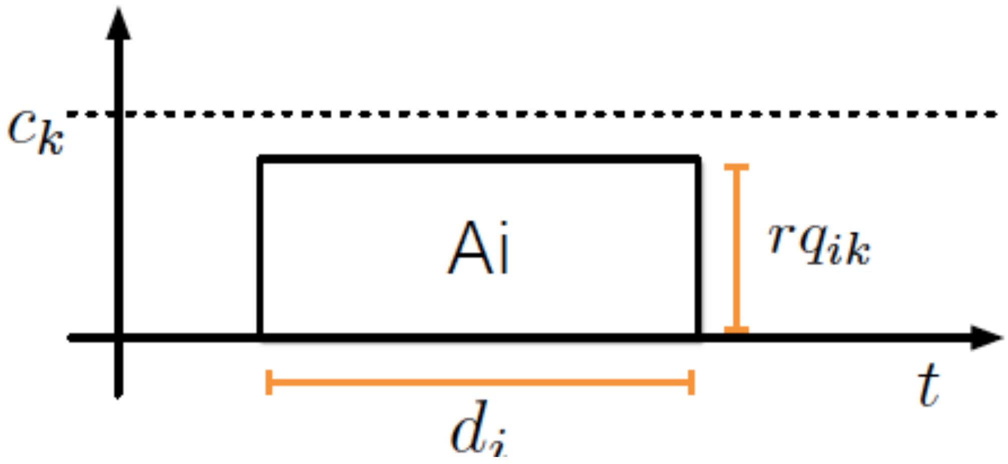
- A resource corresponds to an asset available to execute the operations. E.g.,
  - the capacity of a machine,
  - the volume of a truck,
  - the number of seats in a classroom,
  - number of available workers.

Cumulative/Parallel Resource

- Can execute multiple activities in parallel.
  - Activities can overlap in time!
  - E.g., a group of identically skilled workers, a delivery truck, a multi-core CPU.

Cumulative/Parallel Resource

- A resource r_k is associated to a capacity c_k.
- Each a_i requires some amount $rq_{ik} \geq 0$ of each resource r_k during its execution.
- During the execution of the schedule, the total usage of r_k by the activities a_i should not exceed c_k.



Cumulative/Parallel Resource

- Any two activity requiring the same resource is related by a cumulative constraint.
  - for all $r_k \in R$ with the capacity c_k:

cumulative([S_1, S_2, ..., S_n], [d_1, d_2, ..., d_n], [rq_{1k}, rq_{2k}, ..., rq_{nk}], c_k)

iff $\sum_{i | S_i \leq u < S_i + d_i} rq_{ik} \leq c_k$ for all u in D

- RCPSP resources are cumulative.

Another Cumulative Example

- You are moving house. You have 4 people to do the move and you must move in 1 hour. Piano must be moved before bed.

Item	Time	No. of people
piano	30 min	3
chair	10 min	1
bed	20 min	2
table	15 min	2

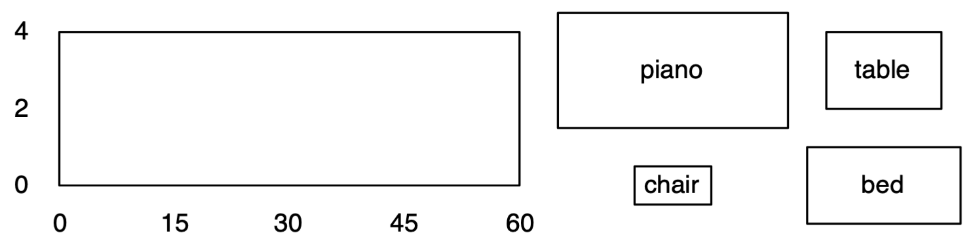
Another Cumulative Example

cumulative resource with capacity

- You are moving house. You have 4 people to do the move and you must move in 1 hour. Piano must be moved before bed.

timeline/horizon

Item	Time	No. of people
piano	30 min	3
chair	10 min	1
bed	20 min	2
table	15 min	2



activities durations resource requirements

Another Cumulative Example

cumulative resource with capacity

- You are moving house. You have 4 people to do the move and you must move in 1 hour. Piano must be moved before bed.

timeline/horizon

Item	Time	No. of people
piano	30 min	3
chair	10 min	1
bed	20 min	2
table	15 min	2

activities durations resource requirements

D(P) = D(C) = D(B) = D(T) = [0..60]
P + 30 ≤ 60, C + 10 ≤ 60,
B + 15 ≤ 60, T+ 15 ≤ 60,
P + 30 ≤ B,
cumulative([P,C,B,T], [30,10,20,15], [3,1,2,2], 4)

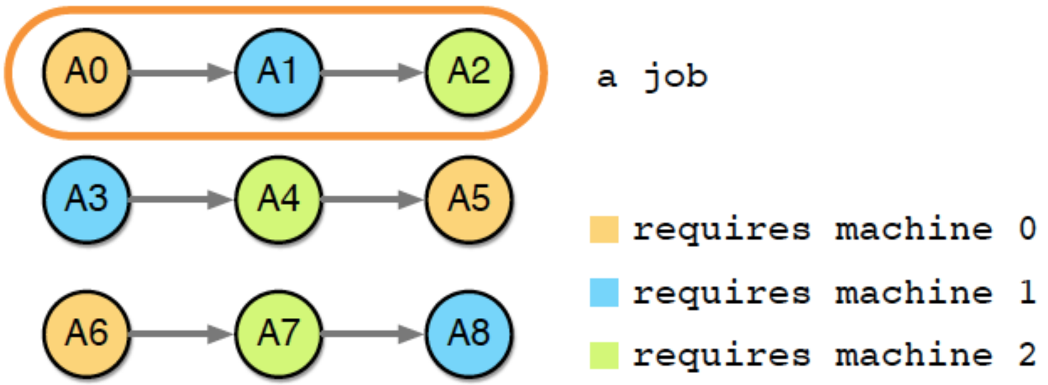
Unary/Disjunctive/Sequential Resource

- Can execute one activity at a time.
  - Activities cannot overlap in time independently of the resource capacity!
  - E.g., a classroom, a train track segment, a crane on a construction site.
- Any two activity is related by a disjunctive (noOverlap) constraint.
  - disjunctive($[S_1, S_2, ..., S_n], [d_1, d_2, ..., d_n]$) iff

    $S_i + d_i \leq S_j \lor S_j + d_j \leq S_i$ for all $1 \leq i < j \leq n$

A Disjunctive Example

- Job shop scheduling problem
  - A job is a sequence of activities (e.g., manufacture of an automobile).
  - Only disjunctive resources (machines).
  - Activities in a job require distinct machines.
  - There are as many activities as machines.

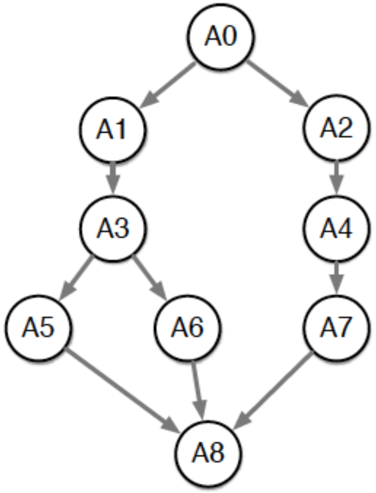


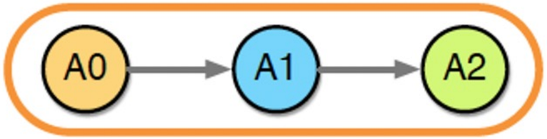
Temporal Constraints

- Precedence constraints
  - Forces one activity to end before another starts.
  - $a_i \rightarrow a_j$
    - $E_i \leq S_j$

Temporal Constraints

- House moving
  - Piano must be moved before bed
- RCPSP
  - Activities and precedence constraints form DAG, called Project Graph.
- Job shop scheduling problem
  - Tasks of a job are processed in a sequential order.





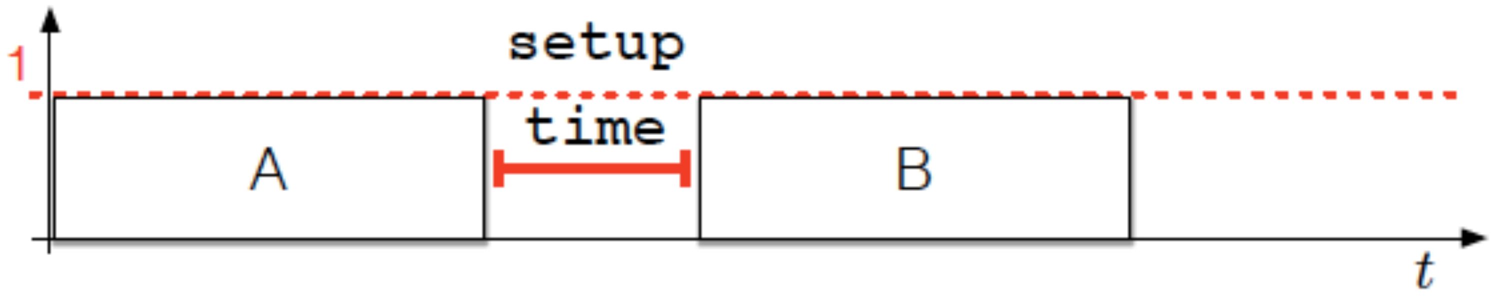
a job

Temporal Constraints

- Time-legs & Time windows
  - Bounds the difference between the end time and the start time of two activities.
  - $a_i \xrightarrow{[l_{ij},\, u_{ij}]} a_j$
    - $l_{ij} \leq S_j - E_i \leq u_{ij}$
  - Time windows are time legs from a dummy activity a_0 with $S_0 = 0$ and $d_0 = 0$.
    - $l_j \leq S_j \leq u_j$

Temporal Constraints

- Sequence-dependent set up times
  - Defined for unary resources.
  - If a_i and a_j are scheduled in a sequence, then they must obey a separation constraint.
    - $E_i \leq S_j \rightarrow E_i + d_{ij} \leq S_j$



Cost Function

- A common cost function: makespan
  - Completion time of the last activity.
  - Optimum makespan is the minimum makespan.
  - RCPSP and job shop scheduling cost functions are makespan.
- Minimum makespan can be modeled in different ways.
  - minimize max($[S_1+d_1,\dots, S_n+d_n]$)
  - Alternatively:
    - Introduce a dummy activity a_{n+1}, with $d_{n+1} = 0$ and constrain it to have the lowest precedence in the schedule:
      - $a_i \rightarrow a_{n+1}$ for all i
    - minimize S_{n+1}

Other Cost Functions

- (Weighted) Tardiness costs:

$$\sum_{a_i \in A} w_i \cdot \max(0, E_i - LET_i)$$

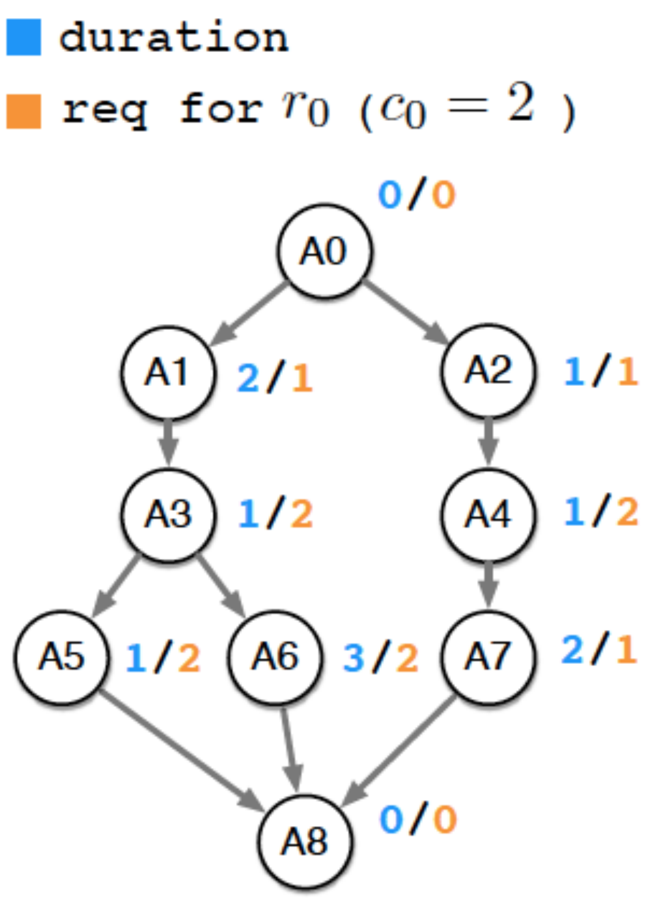
- (Weighted) Earliness costs:

$$\sum_{a_i \in A} w_i \cdot \max(0, LET_i - E_i)$$

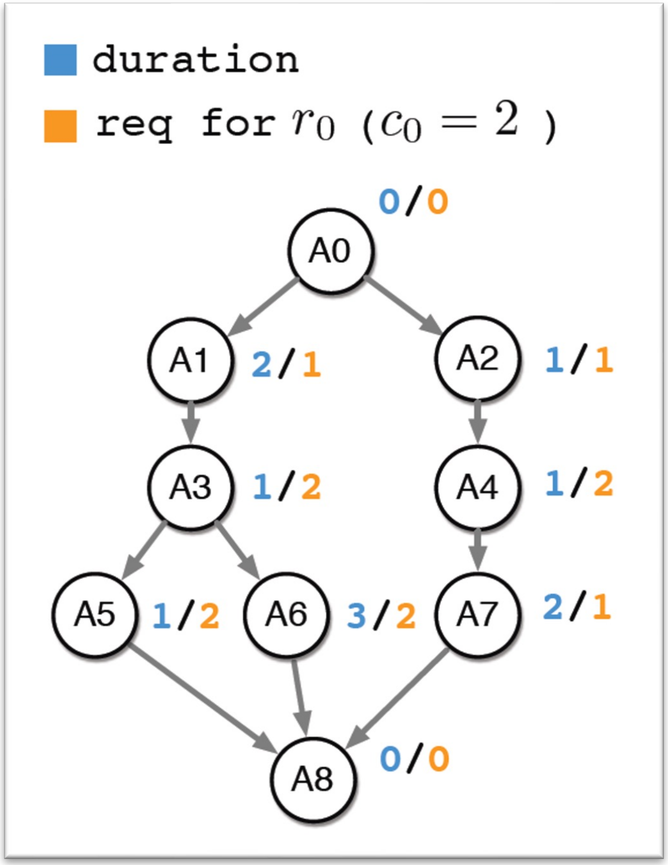
- The peak resource utilization.
- The sum of set up times and costs.

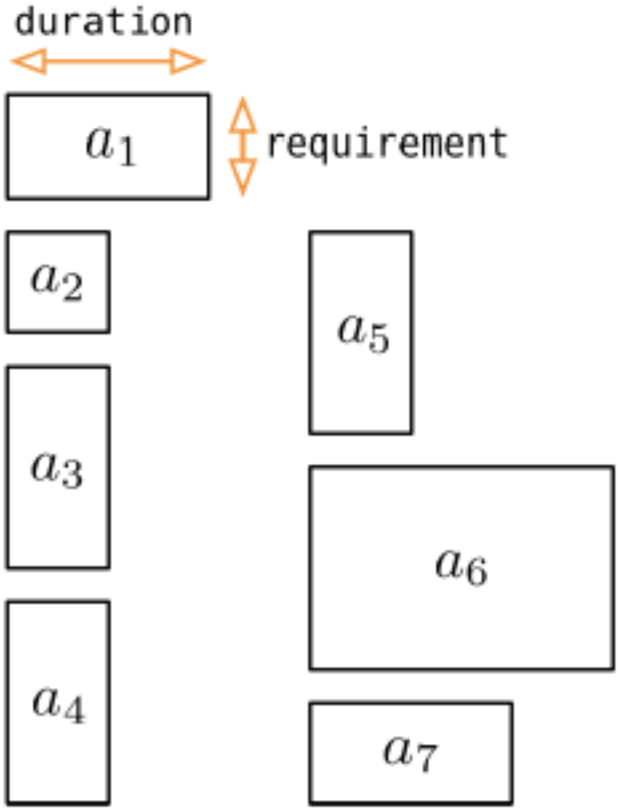
A Sample RCPSP

- A Project Graph $\langle A, E \rangle$
- A is the set of activities a_i
- E is a set of activity pairs (a_i, a_j), representing the precedence constraints
- Activity durations d_i
- Set R of resources r_k , with capacity c_k
- All resource requirements r_{ik}



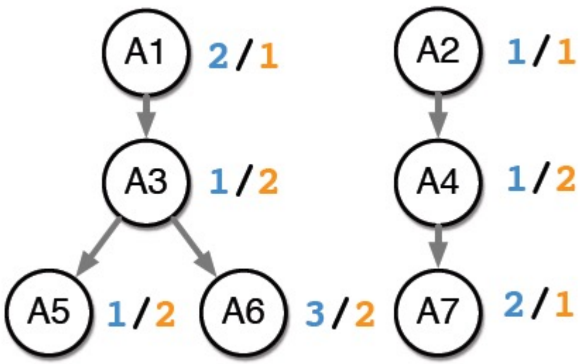
A Sample RCPSP



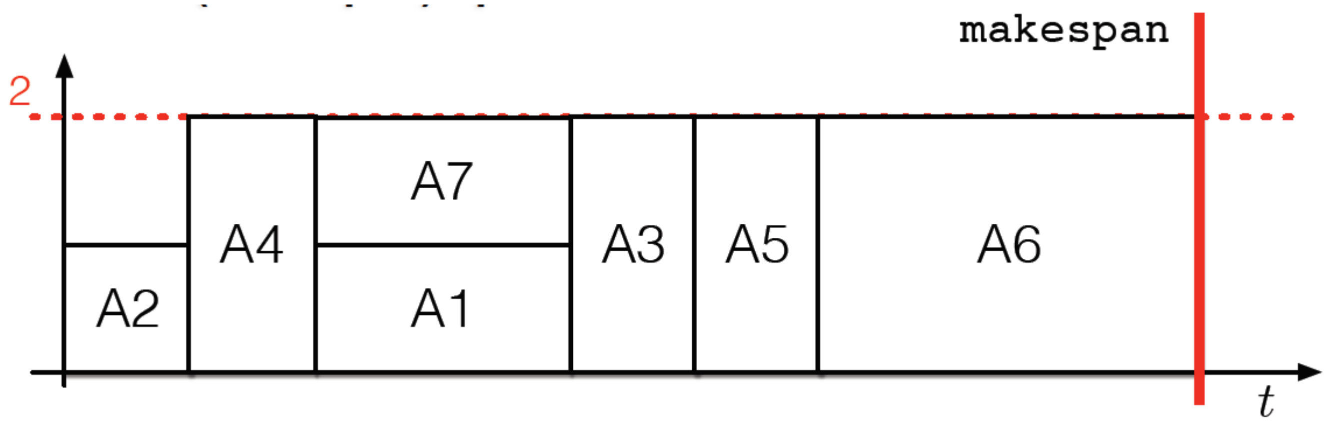


A Sample RCPSP

- The source and sink activities are fake and can be disregarded.



- Makespan optimal schedule:



Search Heuristics

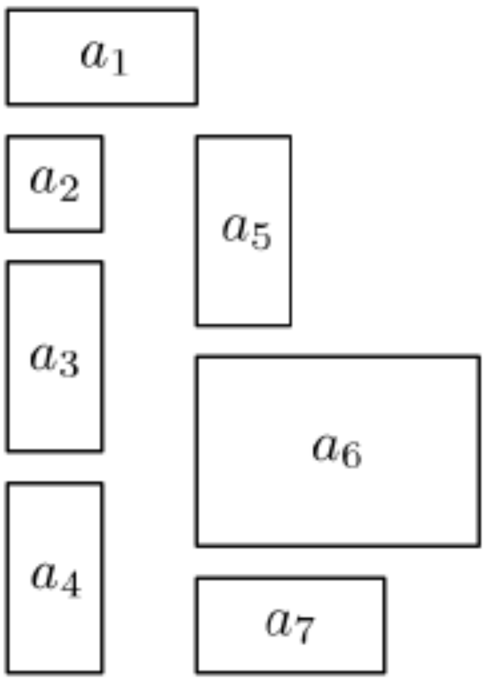
- Typical instances have large domains.
  - S_i and E_i domains are as large as the horizon.
- Which variable to pick next?
- Which value to assign?

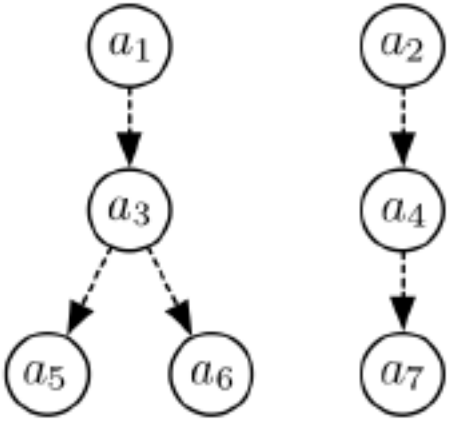
Value Selection for the RCPSP

- The objective is to minimize the makespan.
- Increasing an S_i value (with other S_j untouched) cannot improve the makespan.
  - → Select EST_i.
- This is true not only for the RCPSP.
  - Many scheduling problems have so-called regular cost metrics.
  - Regular = increasing a single S_i cannot improve the cost.

Variable Selection for the RCPSP

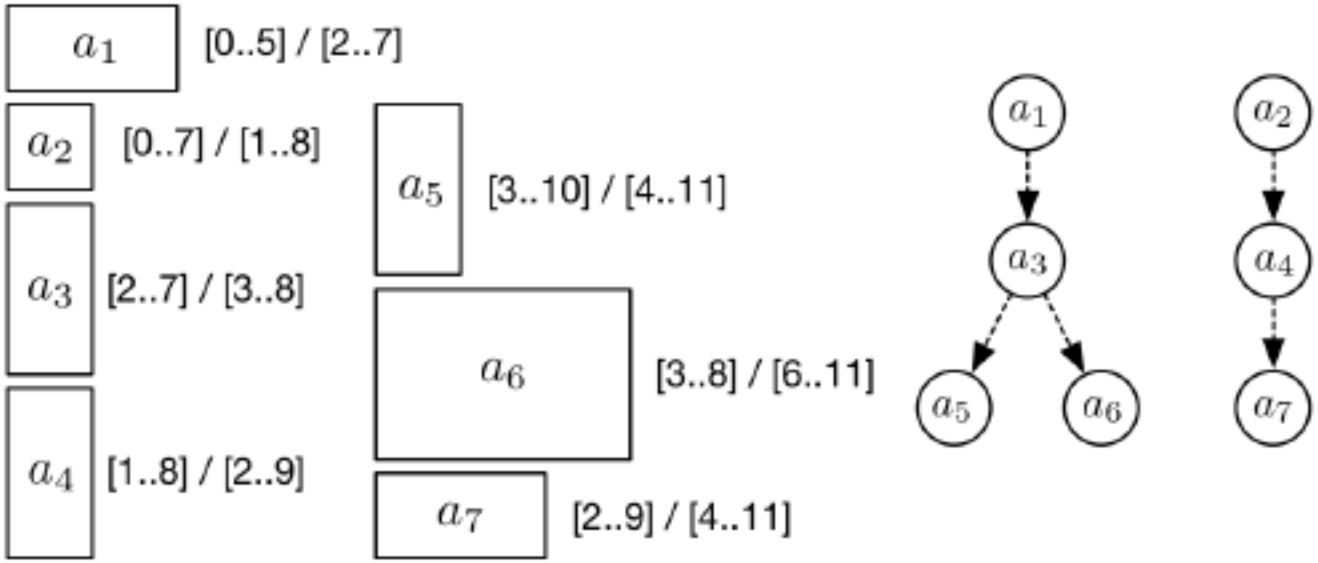
- Example





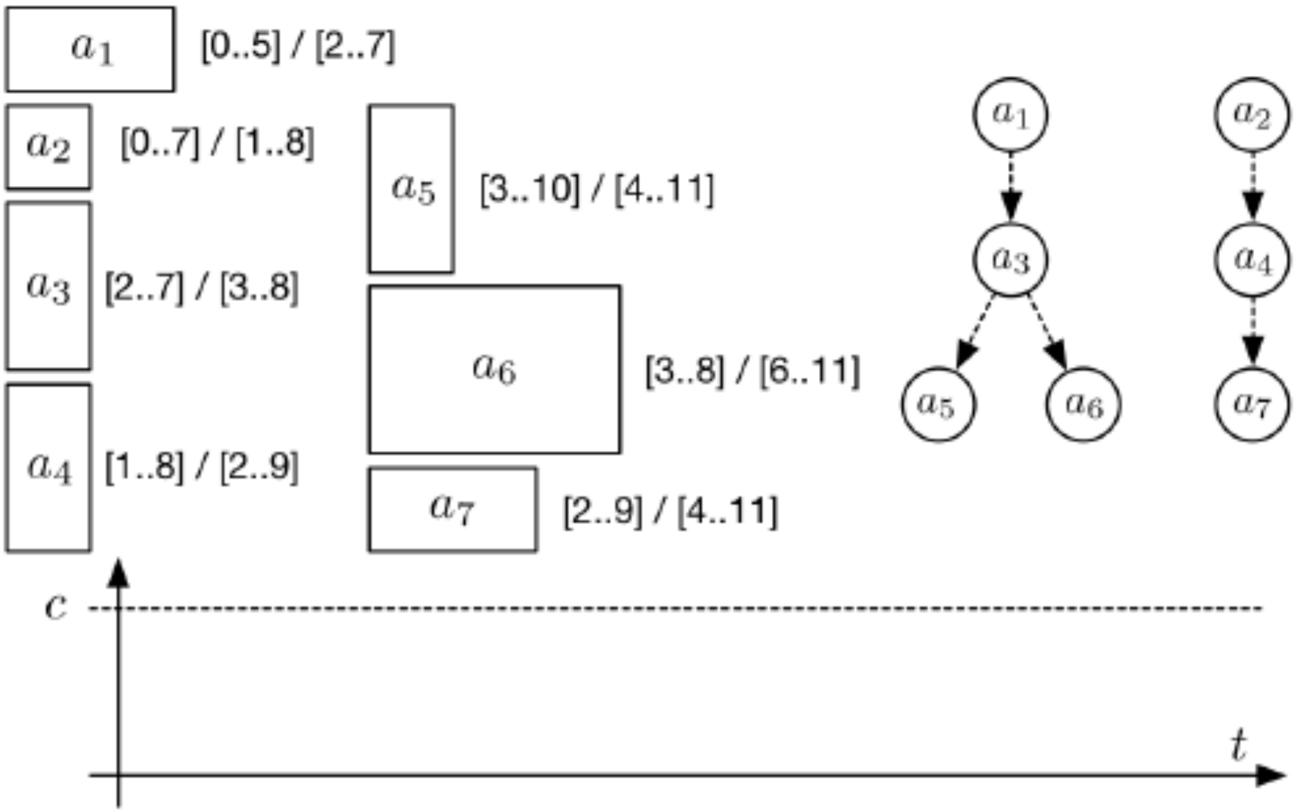
Variable Selection for the RCPSP

- Assume we have the following domains after propagating the precedence constraints:



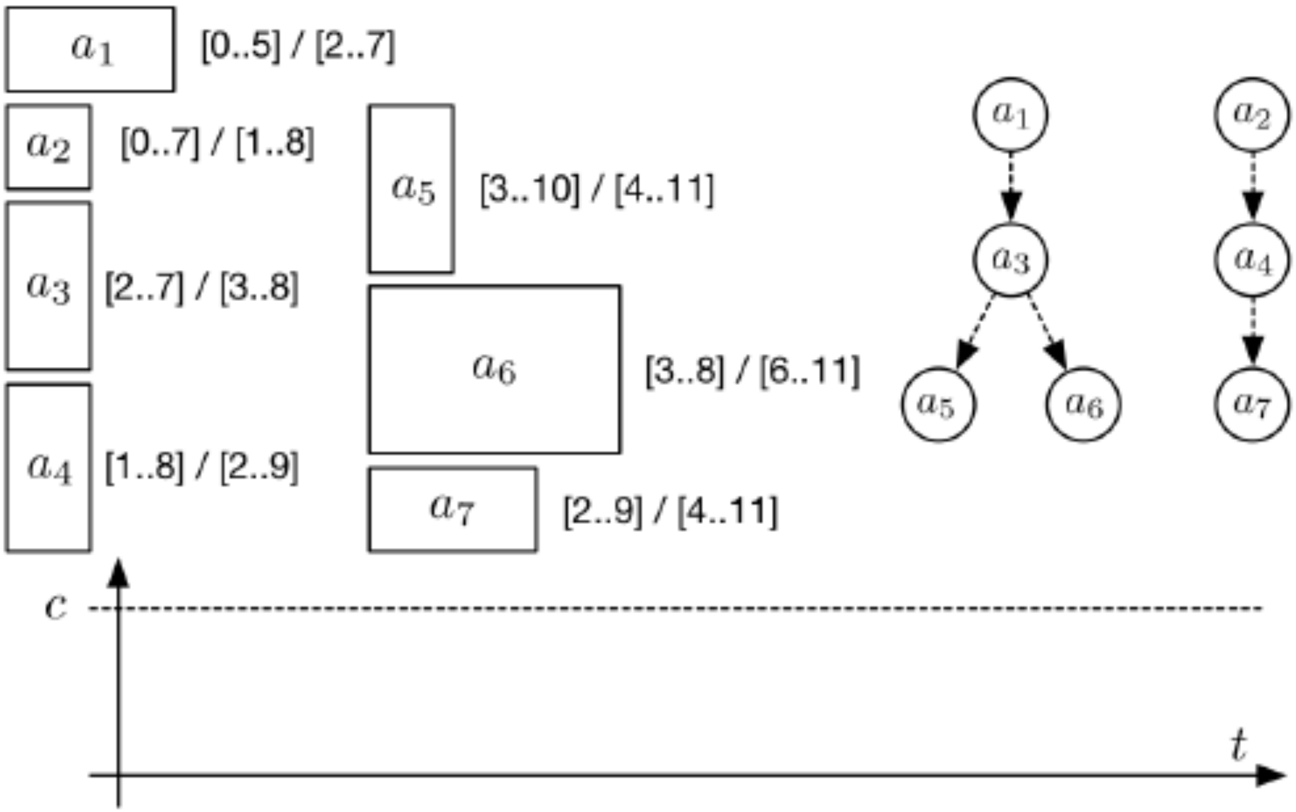
- Notation: $[EST_i..LST_i]/[EET_i..LET_i]$

Variable Selection for the RCPSP



- Which variable first?

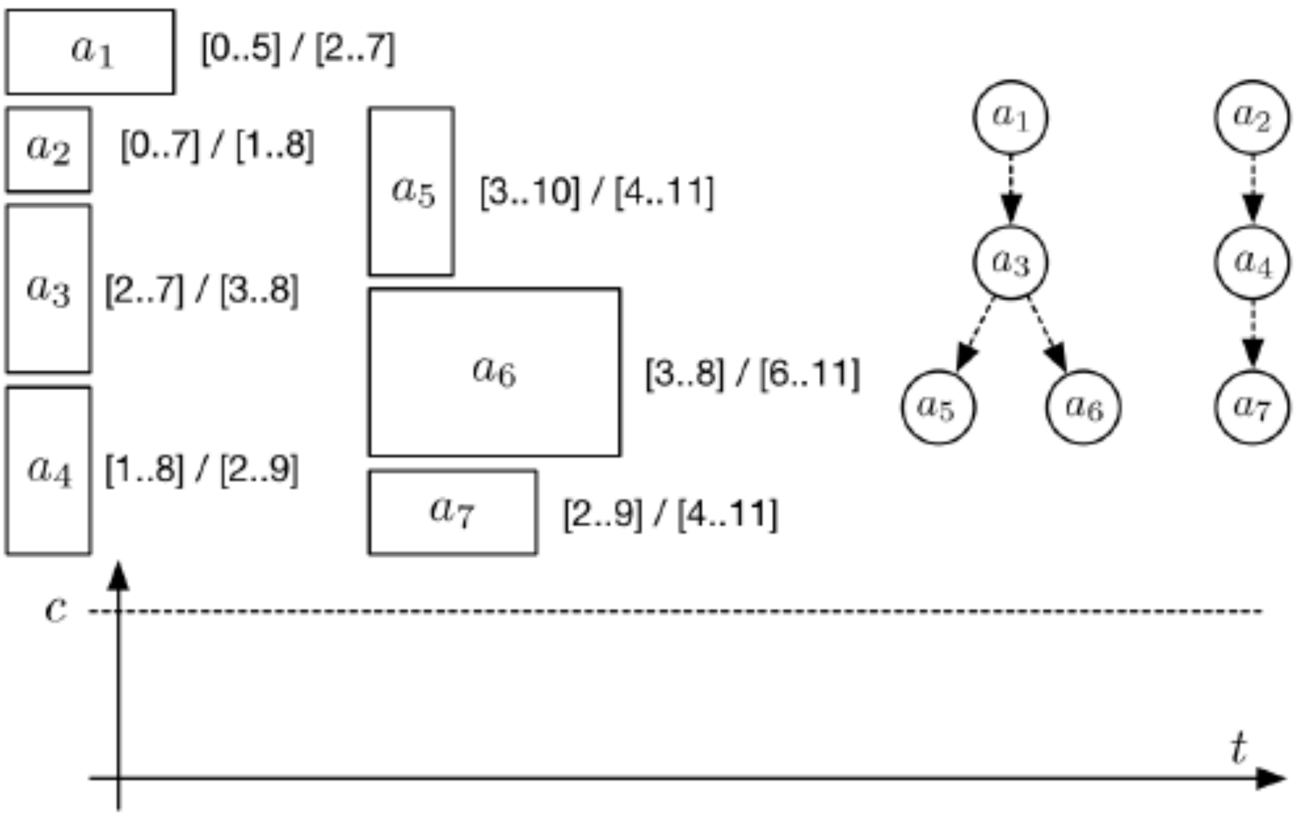
Variable Selection for the RCPSP



- A sensible criterion: minimum EST_i.

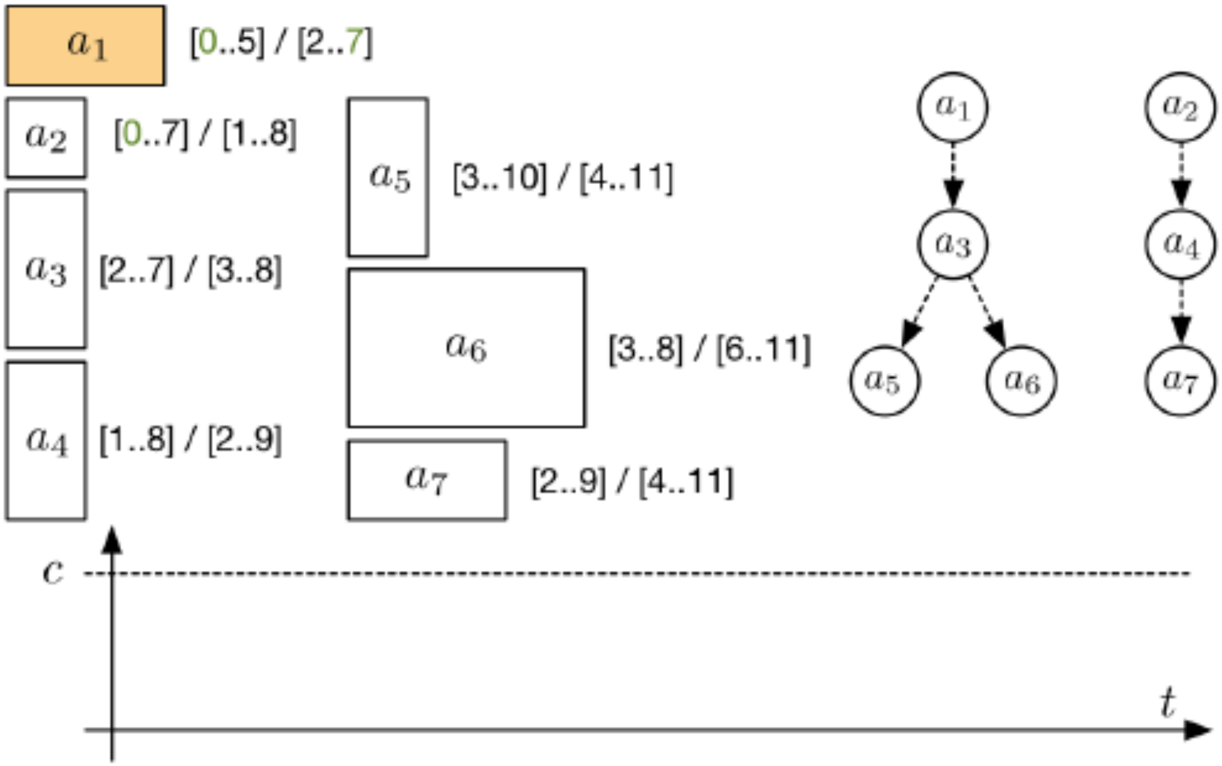
Variable Selection for the RCPSP

- How to break ties?



Variable Selection for the RCPSP

- How to break ties?
  - Tightest deadline, i.e. minimum LET_i.



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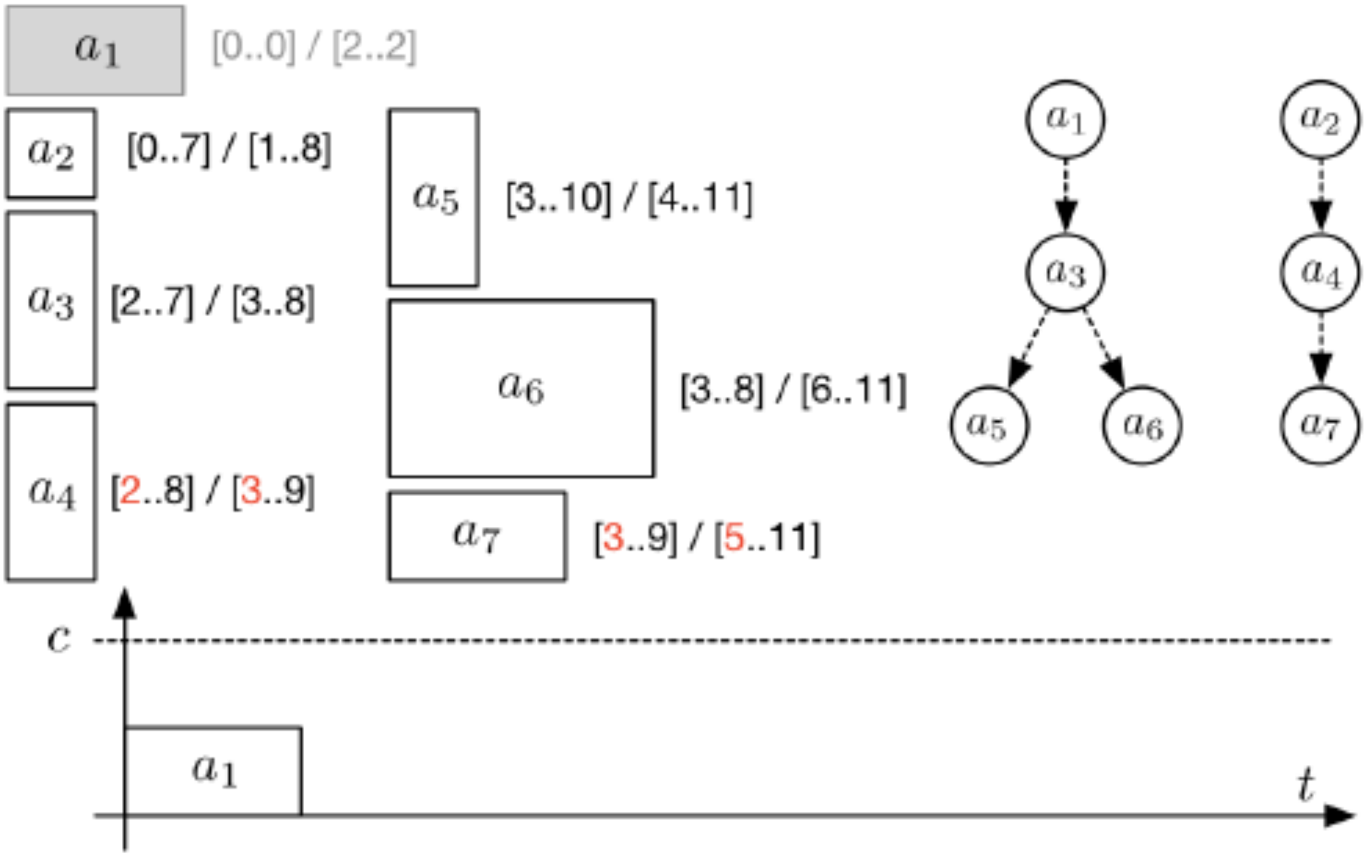

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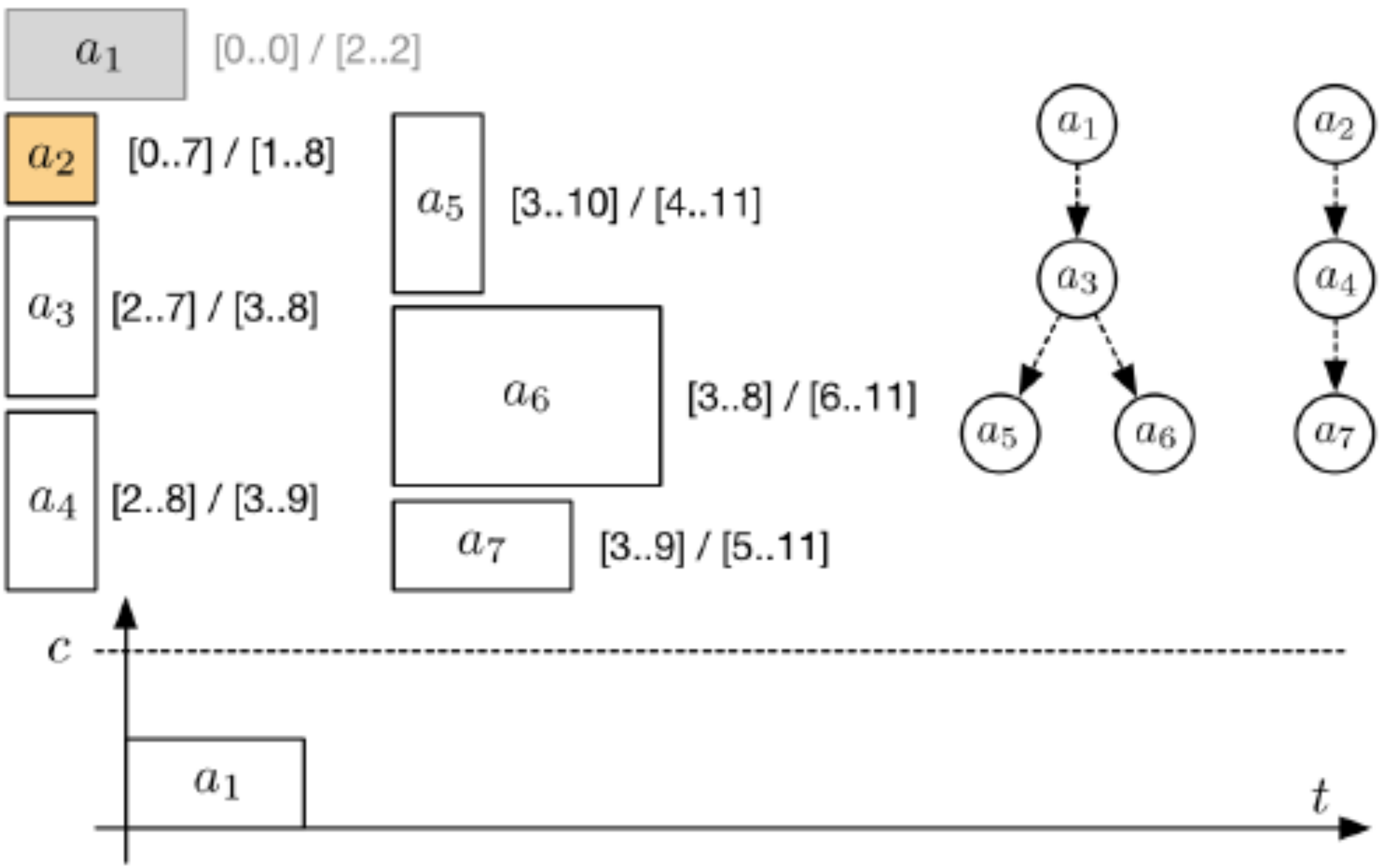

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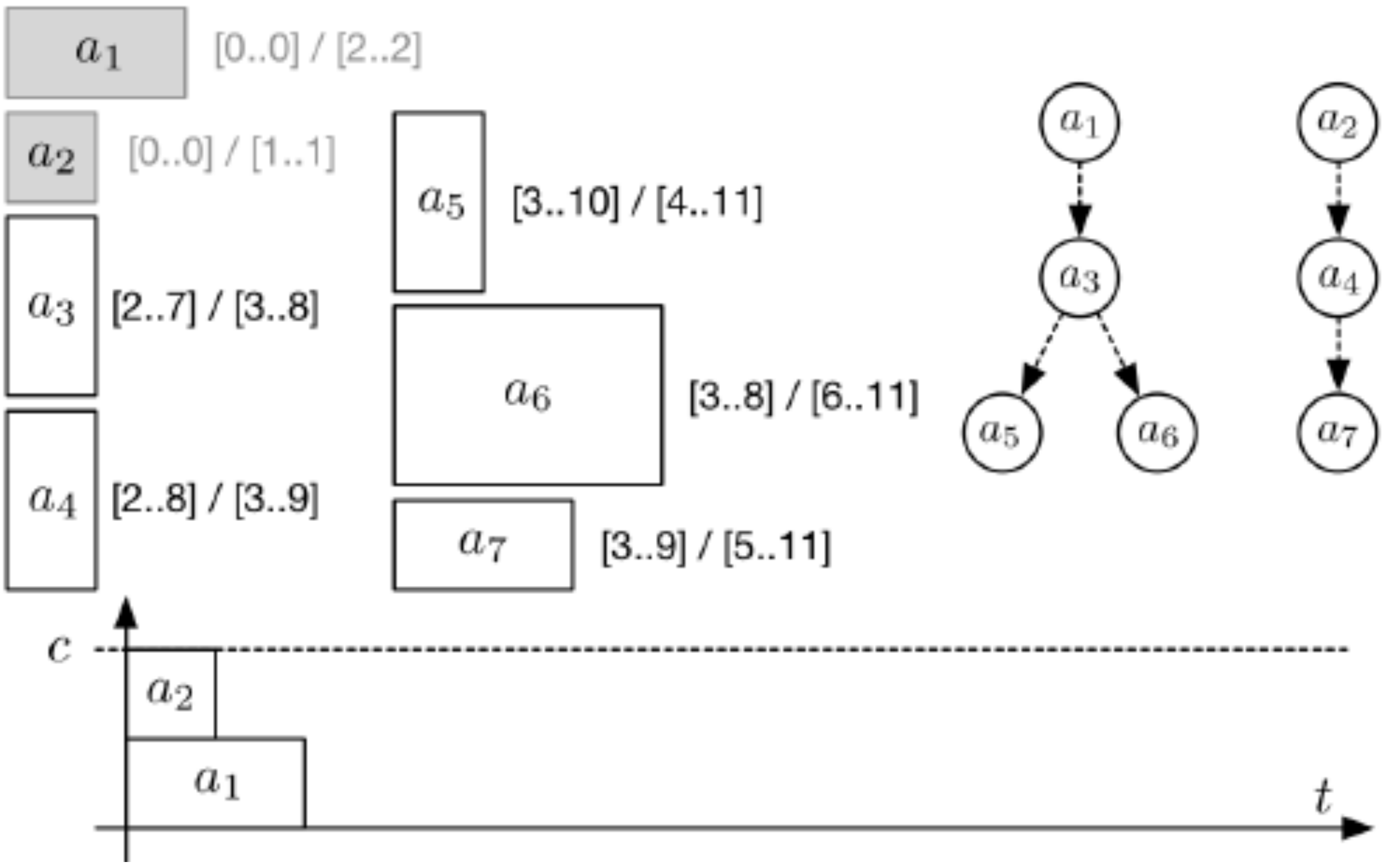

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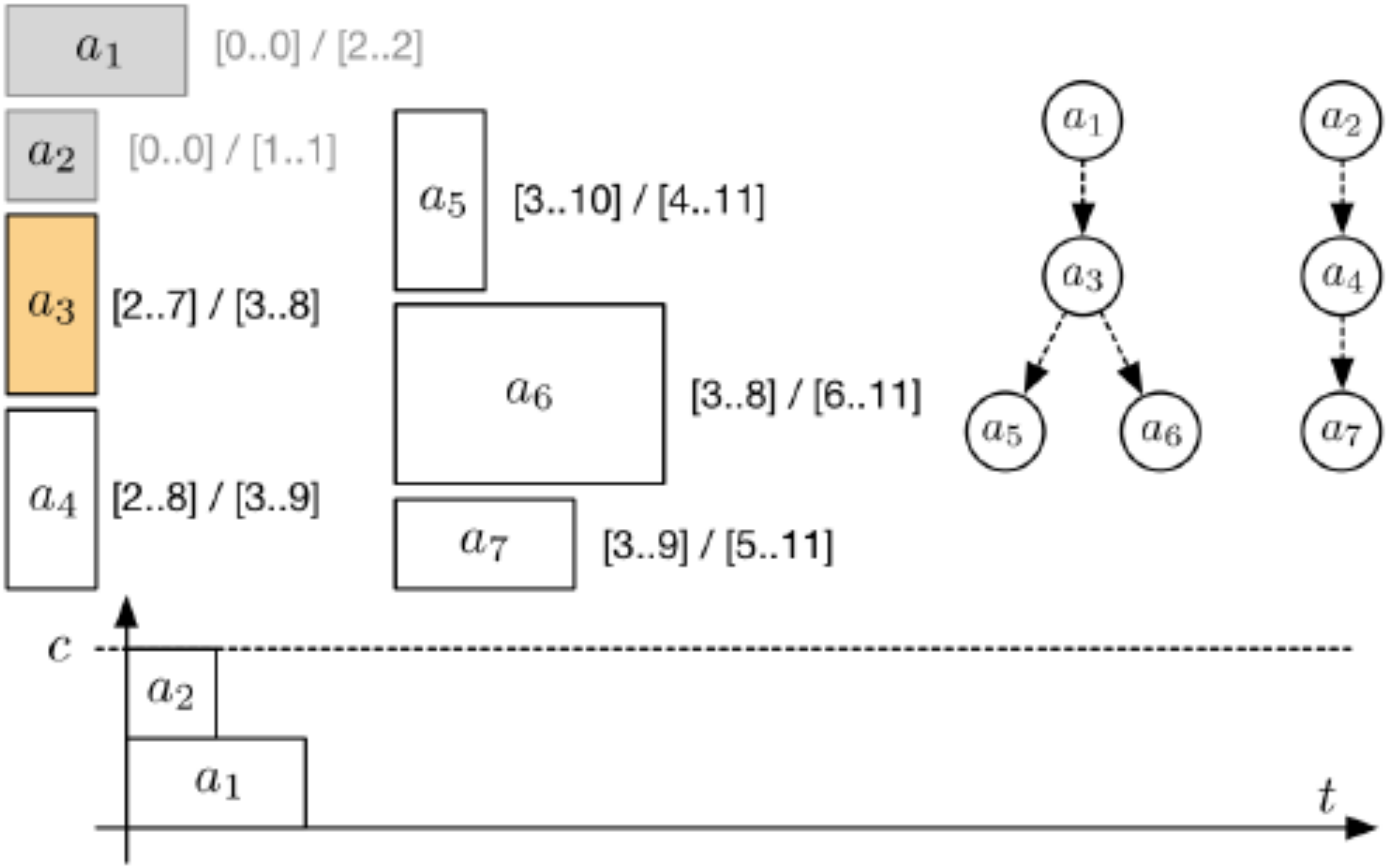

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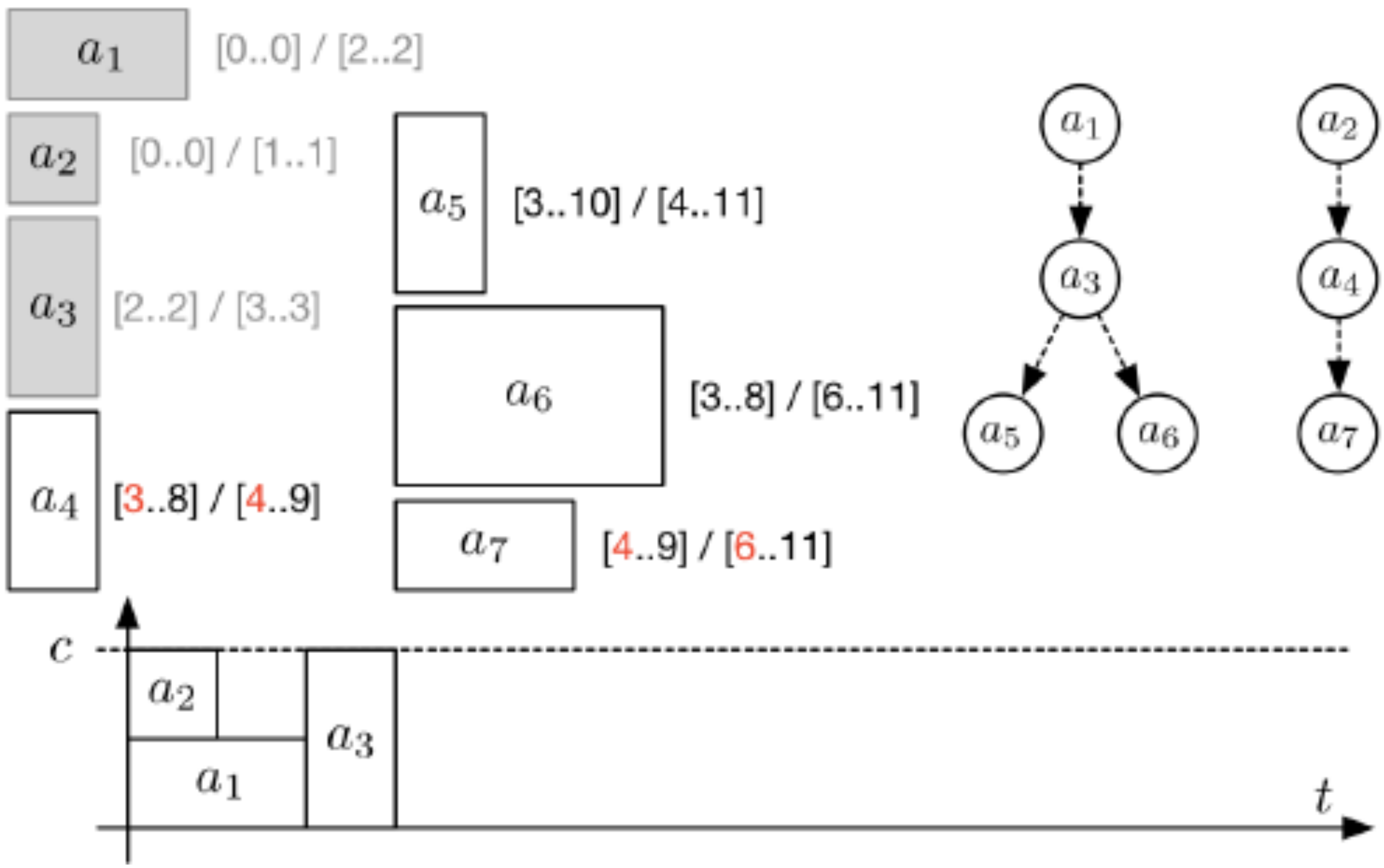

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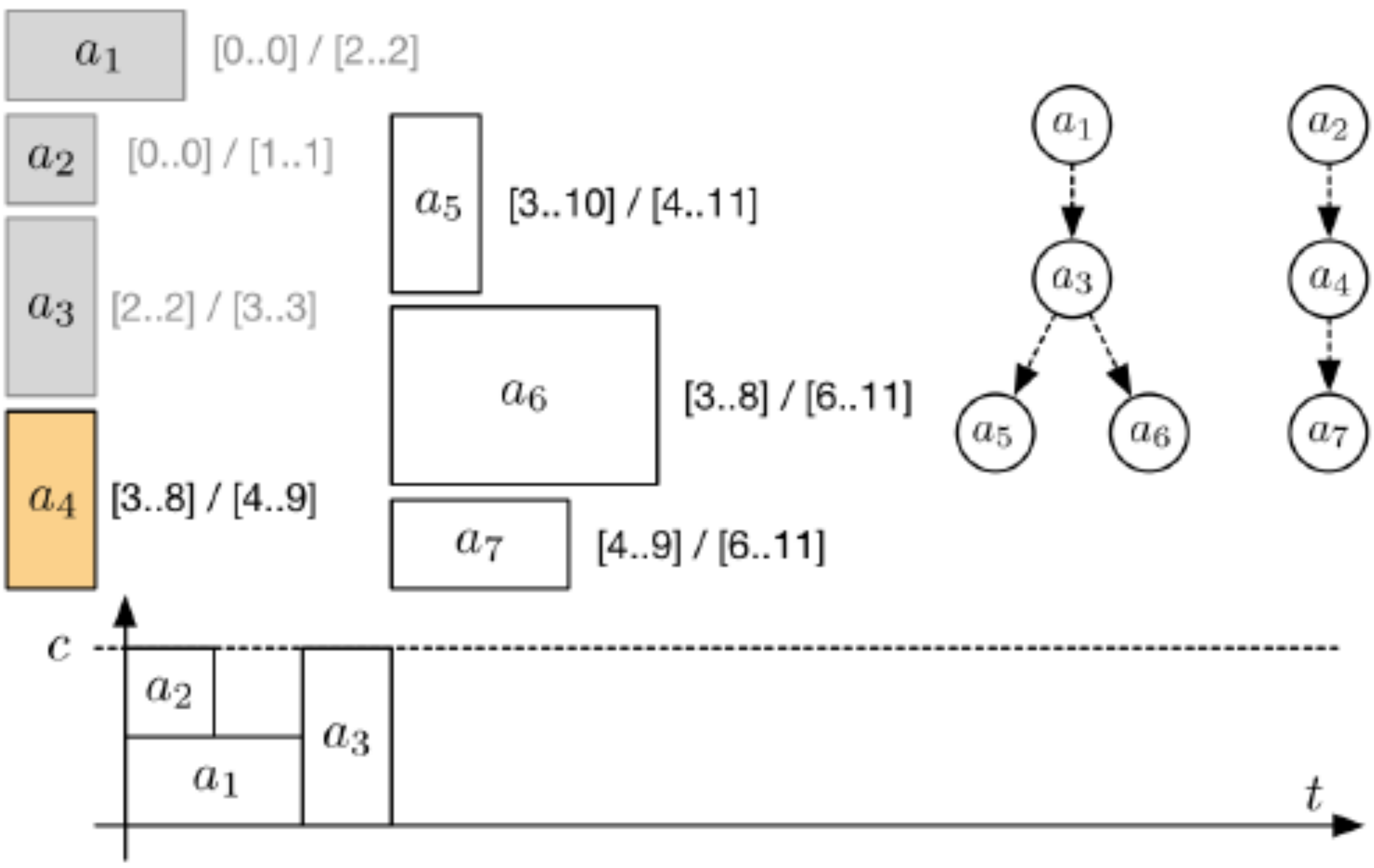

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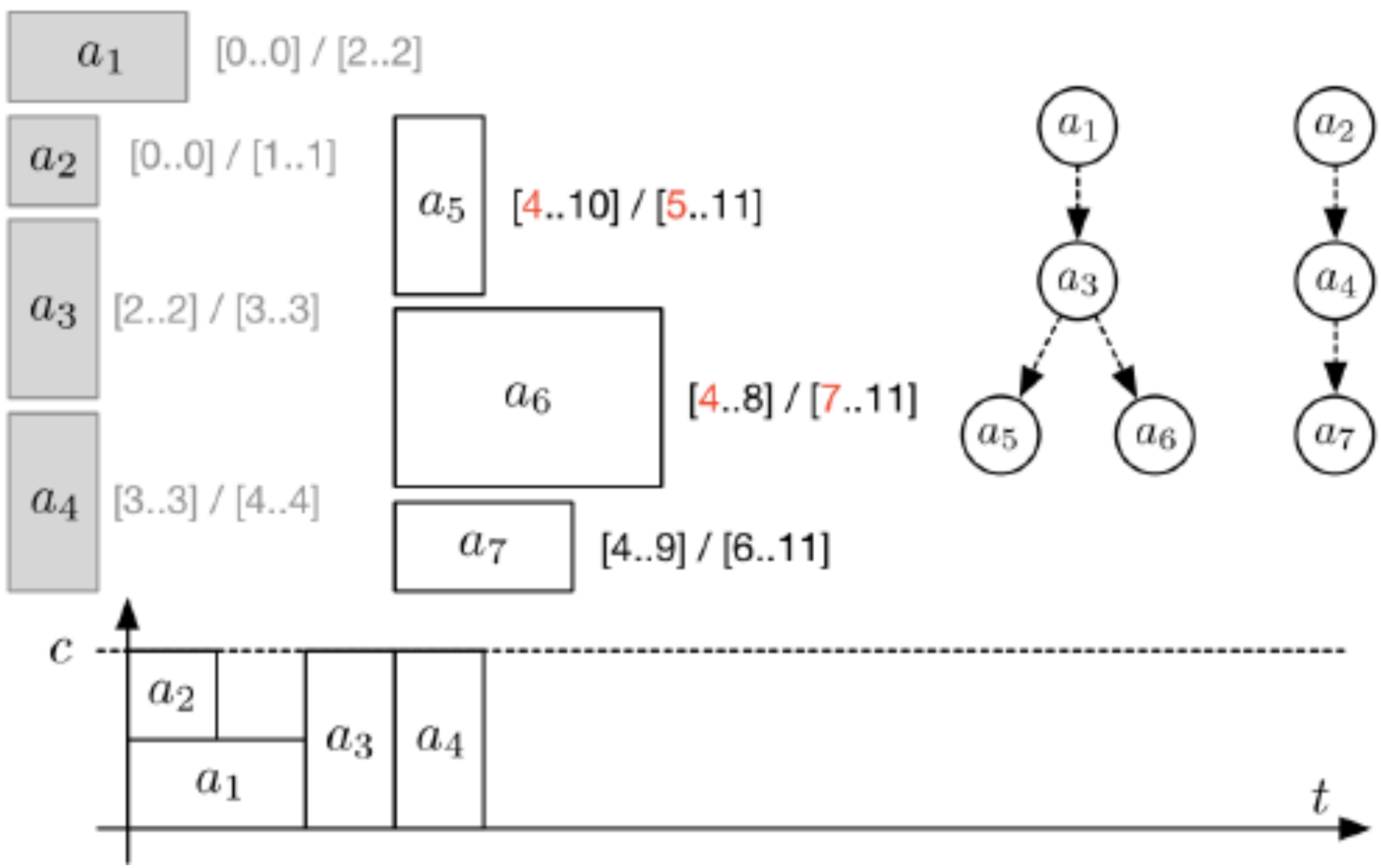

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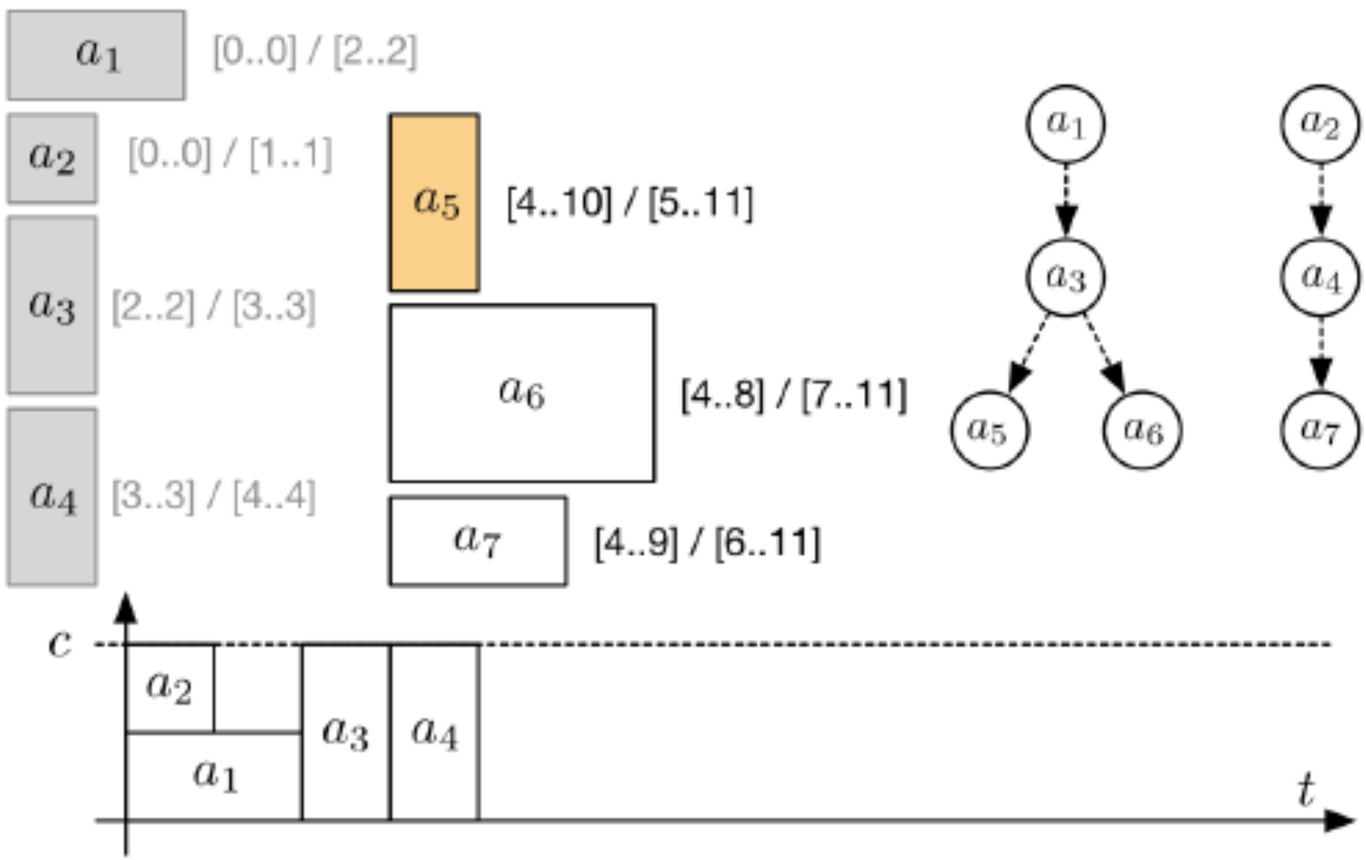

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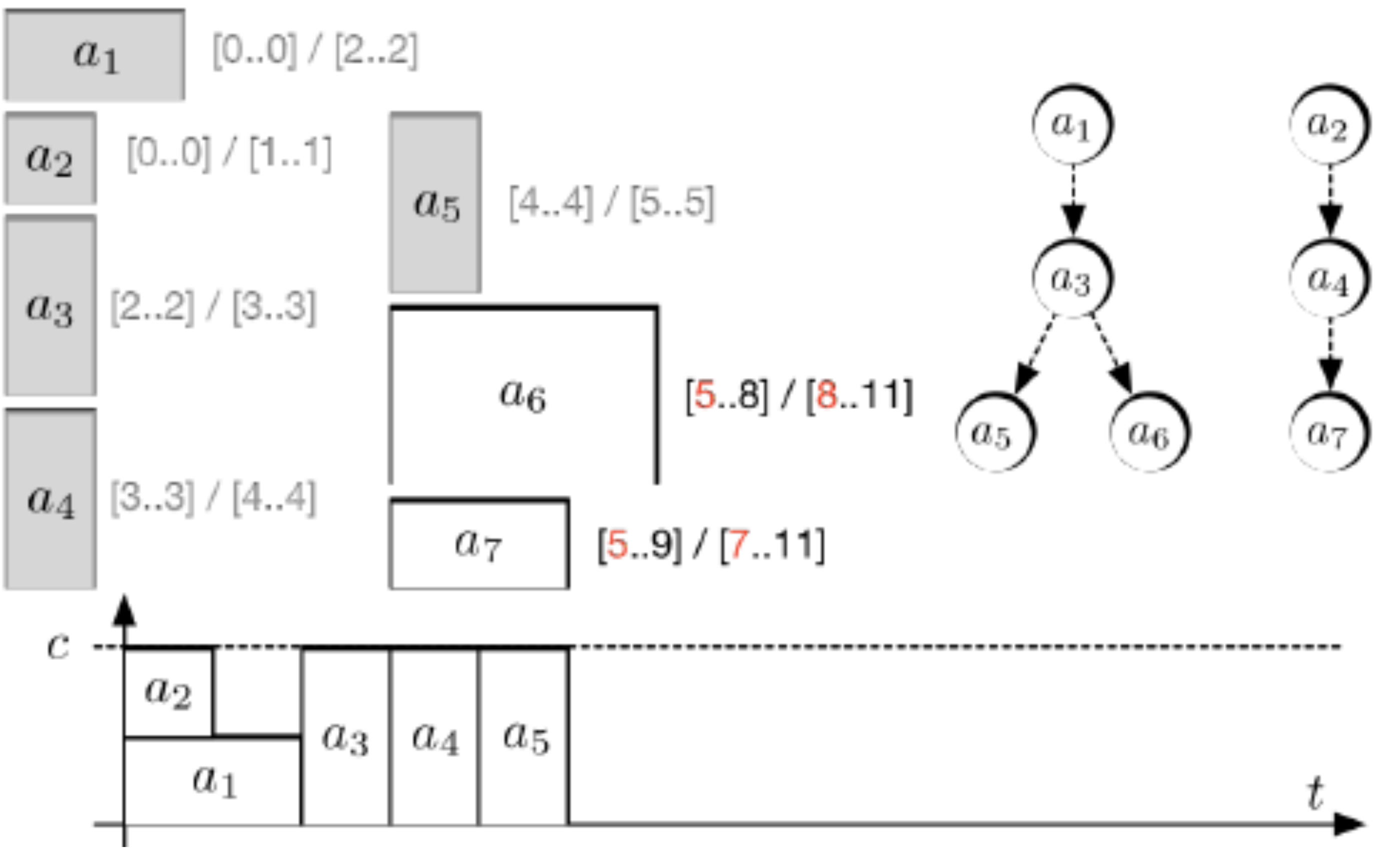

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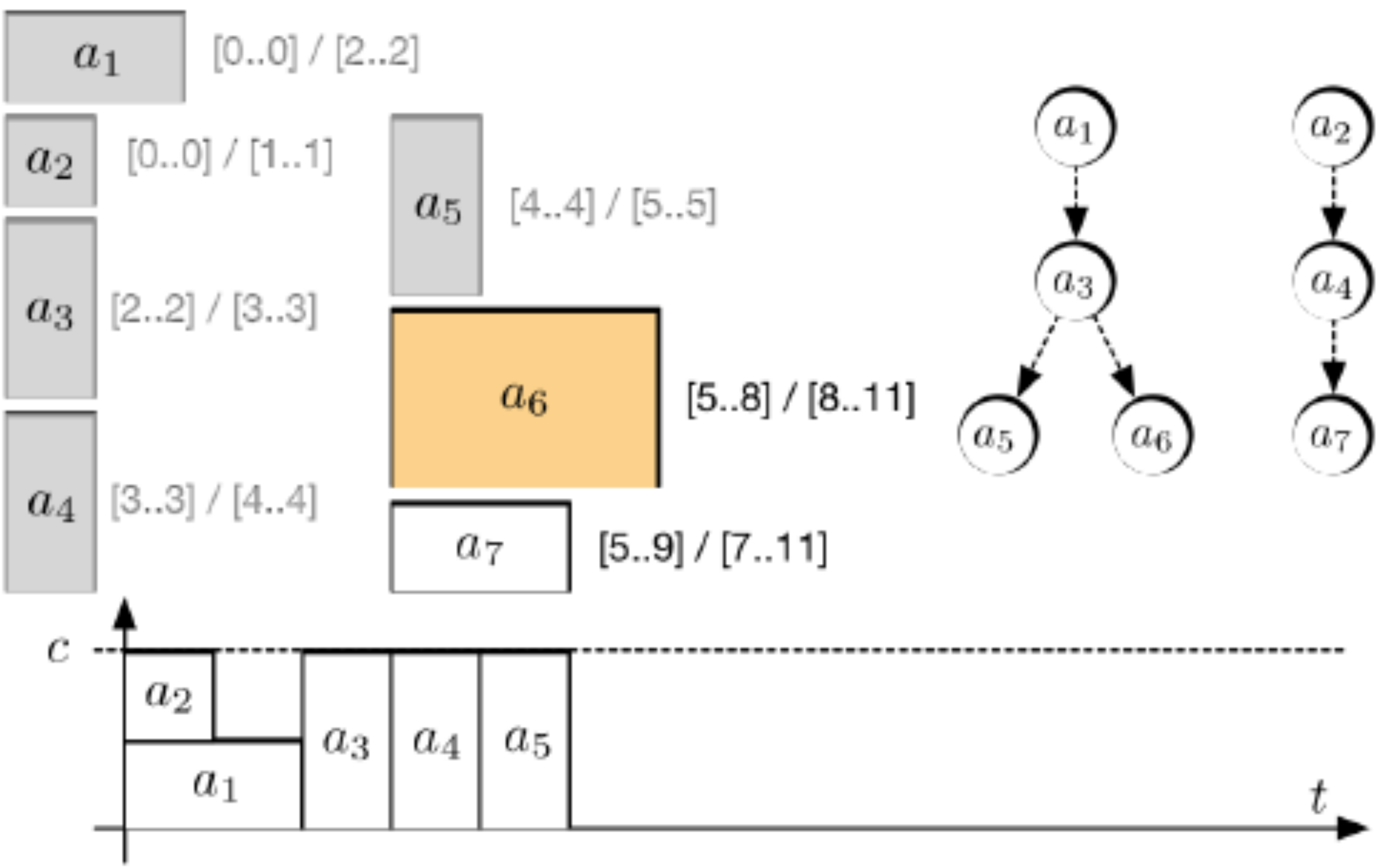

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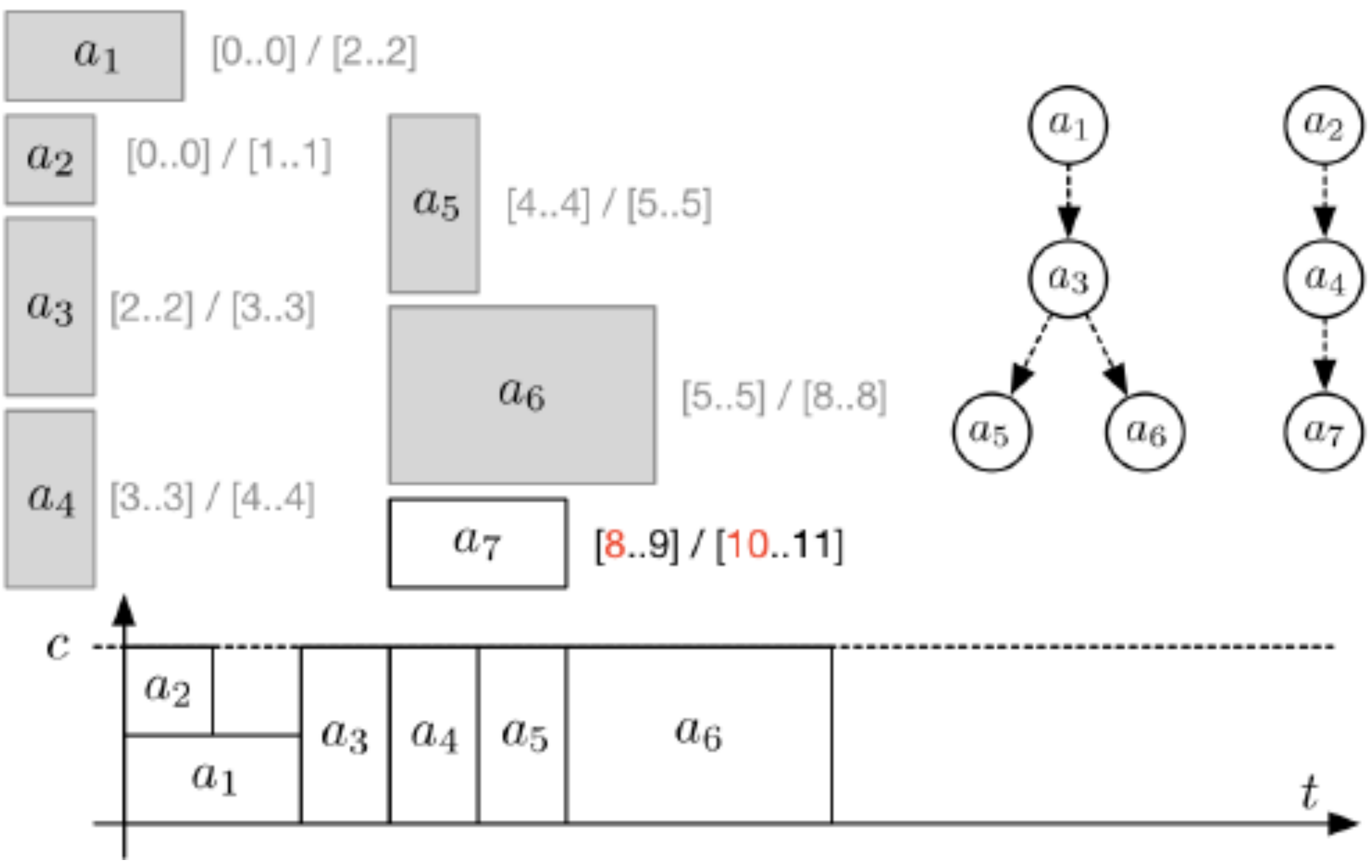
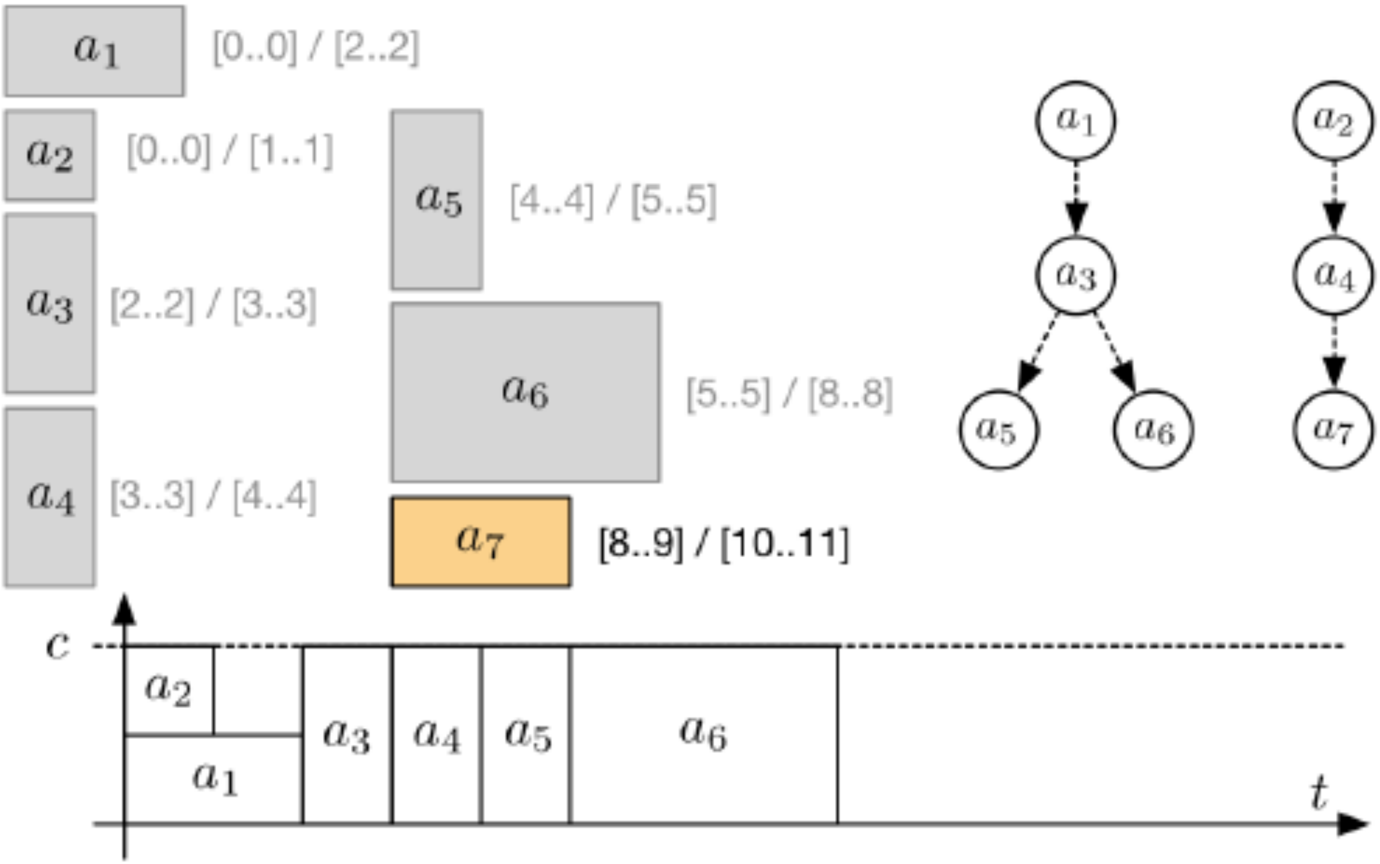


Variable Selection for the RCPSP



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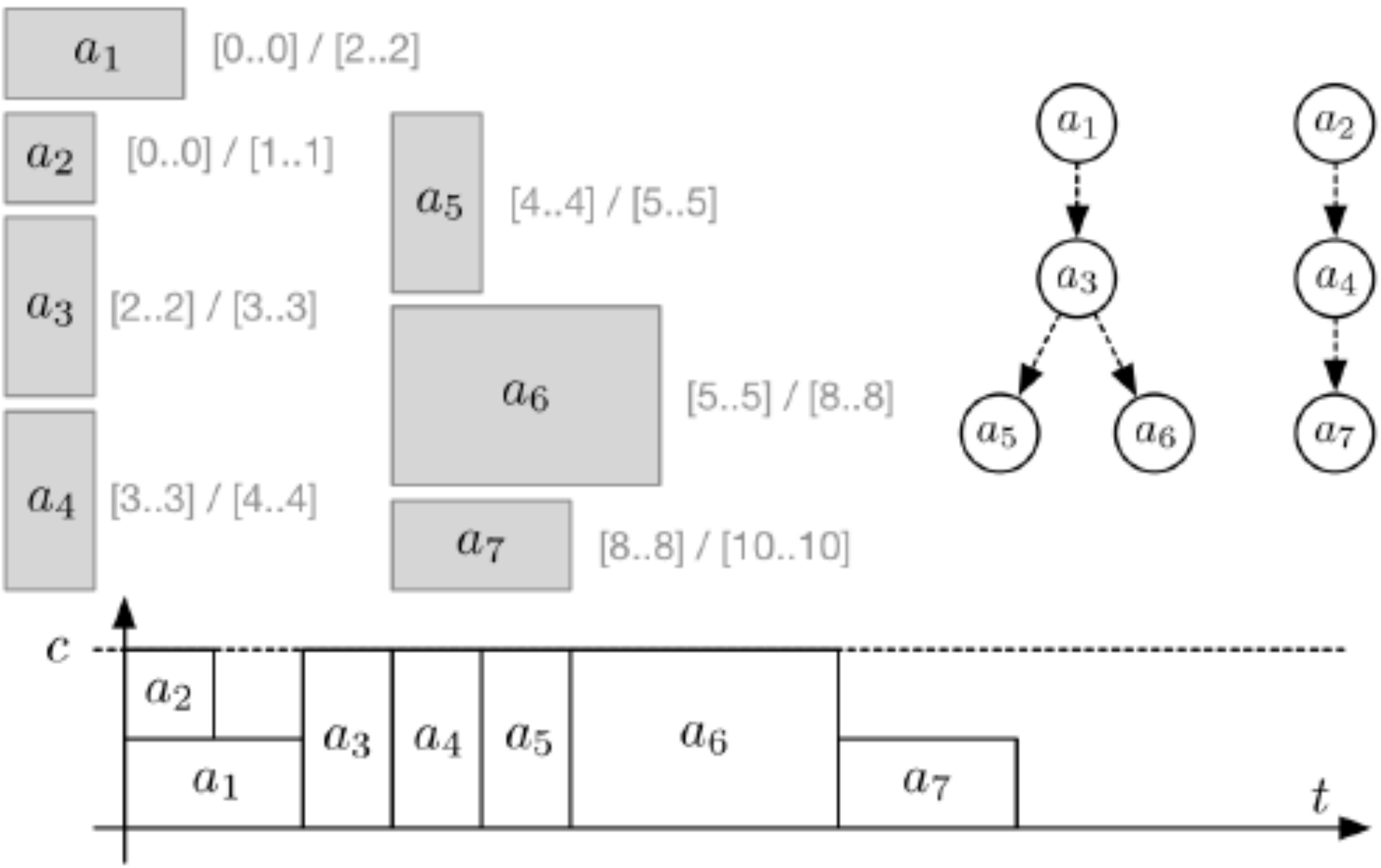
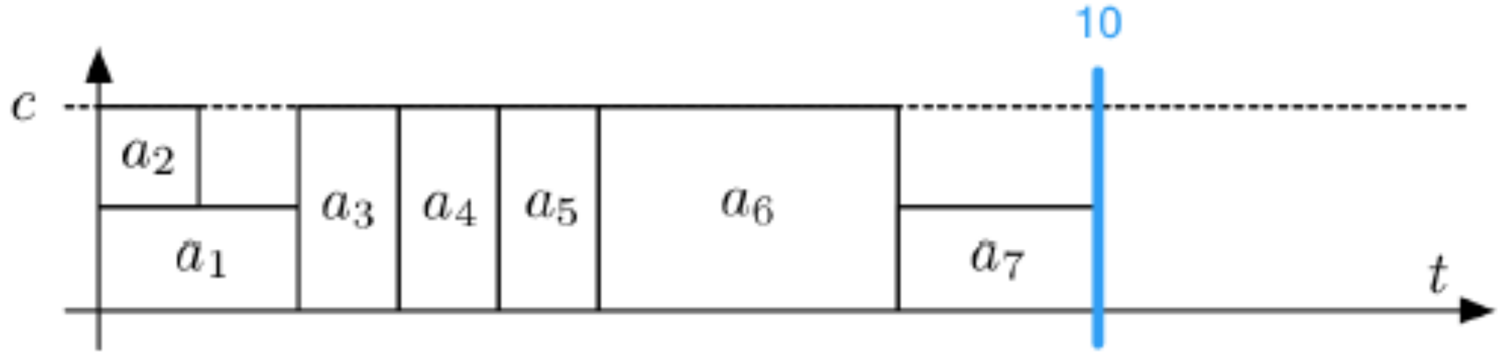


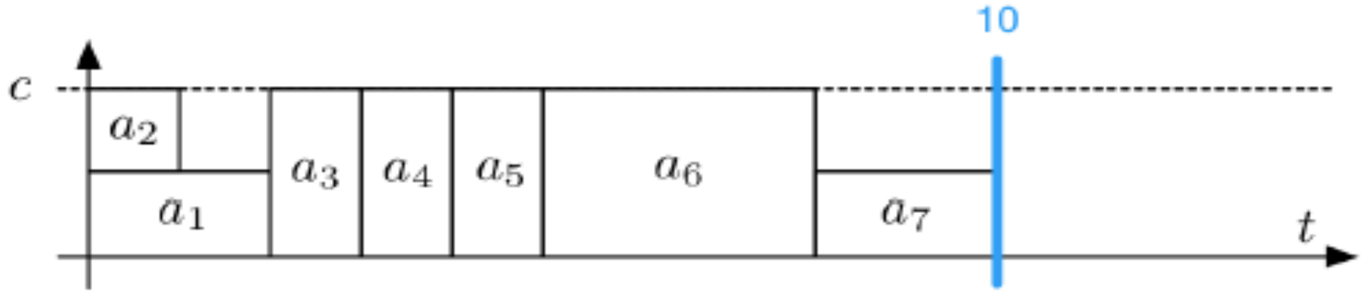
Priority Rule-Based Scheduling

- A simple greedy solution approach.
- Works well in many cases.

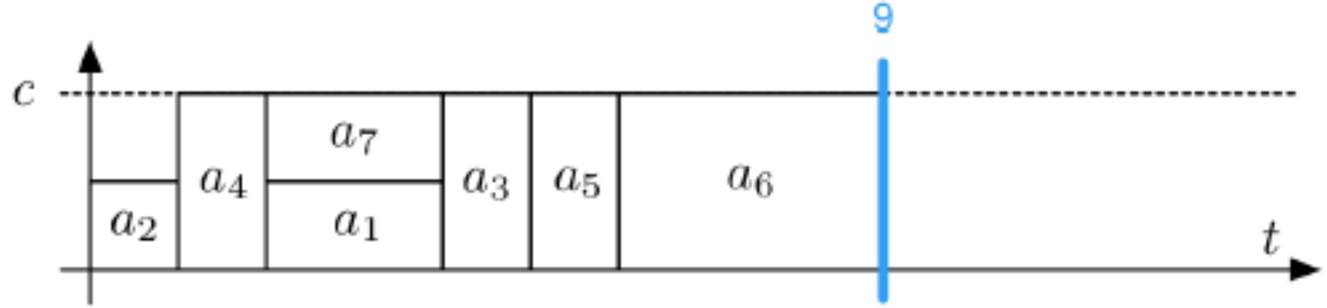


Priority Rule-Based Scheduling

- May not give the optimal solution.
- A PRB solution.

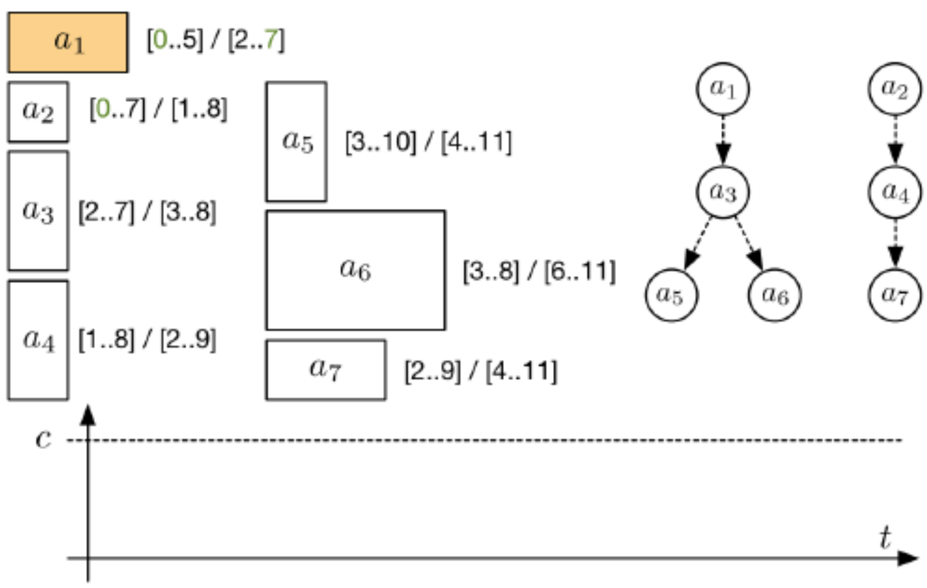


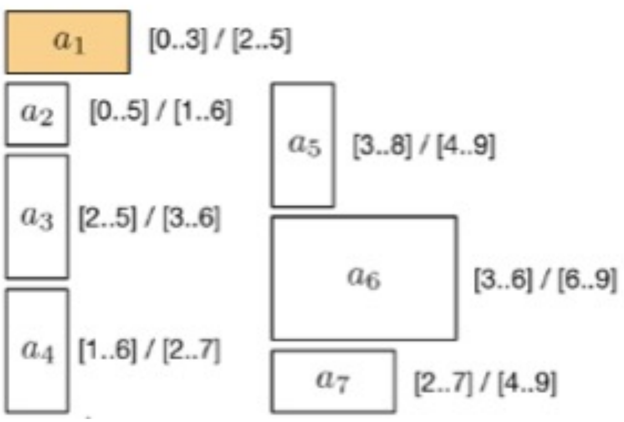
- An optimal solution.



Backtracking for Proving Optimality

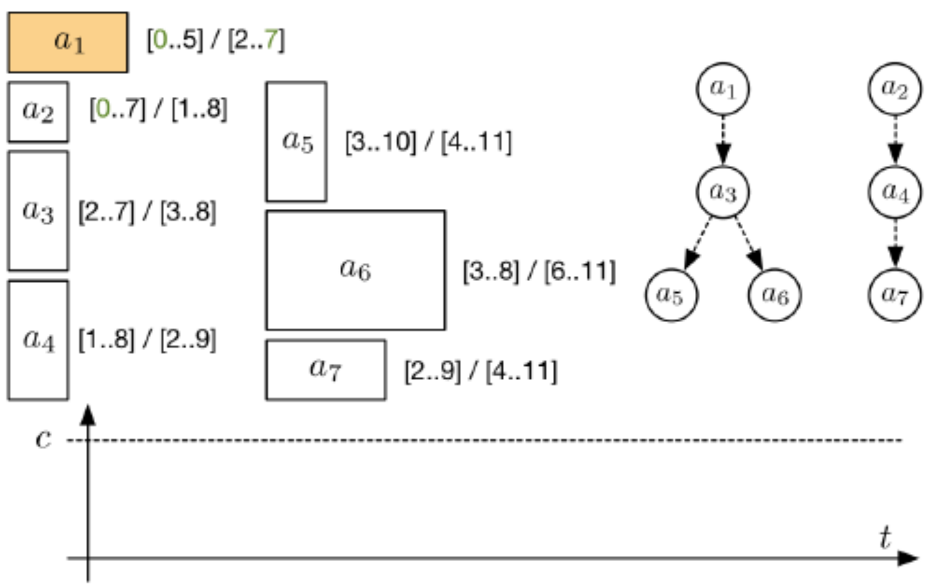
- Need to go back to the root node after posting $S_{n+1}<10$.

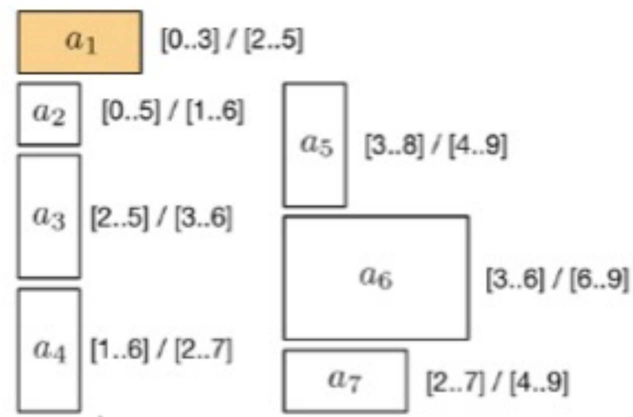




Backtracking for Proving Optimality

- Need to go back to the root node after posting $S_{n+1}<10$.
  - $S_1 \neq 0$?

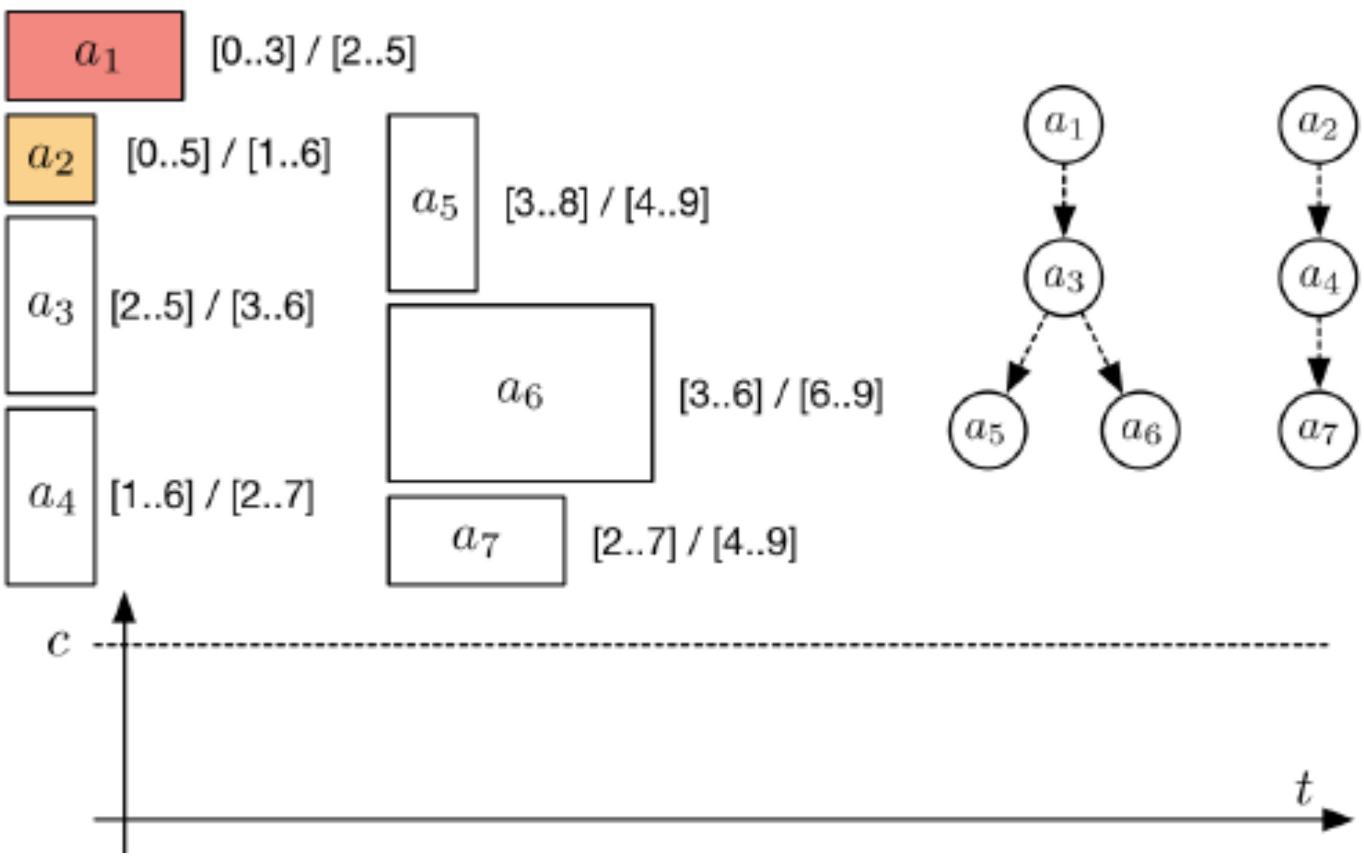




Backtracking for Proving Optimality

- $S_1 \neq 0$
  - It is weak, since S_i domains tend to be very large.
- **Alternative**: mark activity **i** as postponed.
  - A postponed activity cannot be selected for branching until its EST_i changes.
- **Rationale**: we want to explore a different branching decision.
  - We always schedule activities at their EST_i.
  - The scheduling decision changes when EST_i changes.

Backtracking for Proving Optimality



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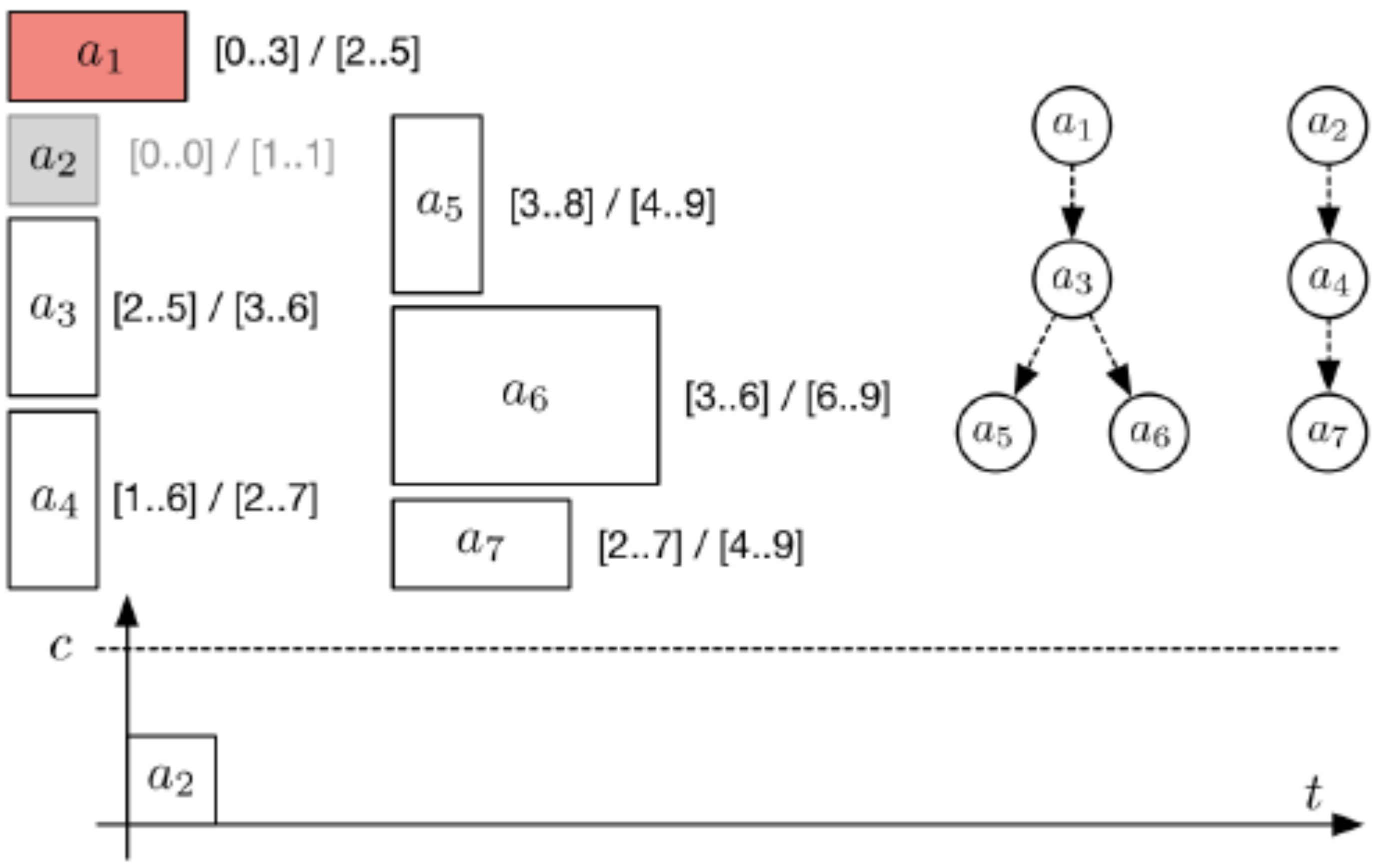

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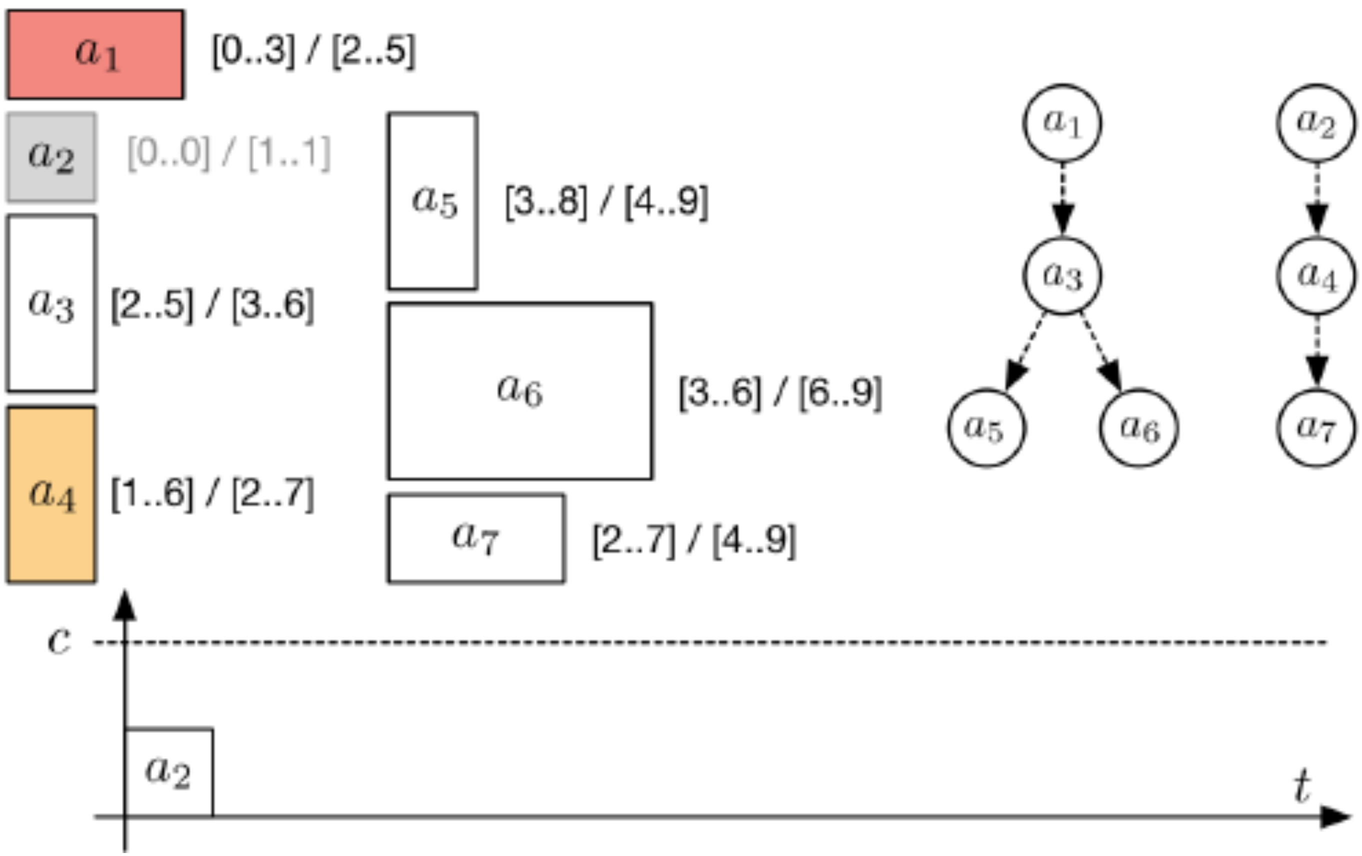
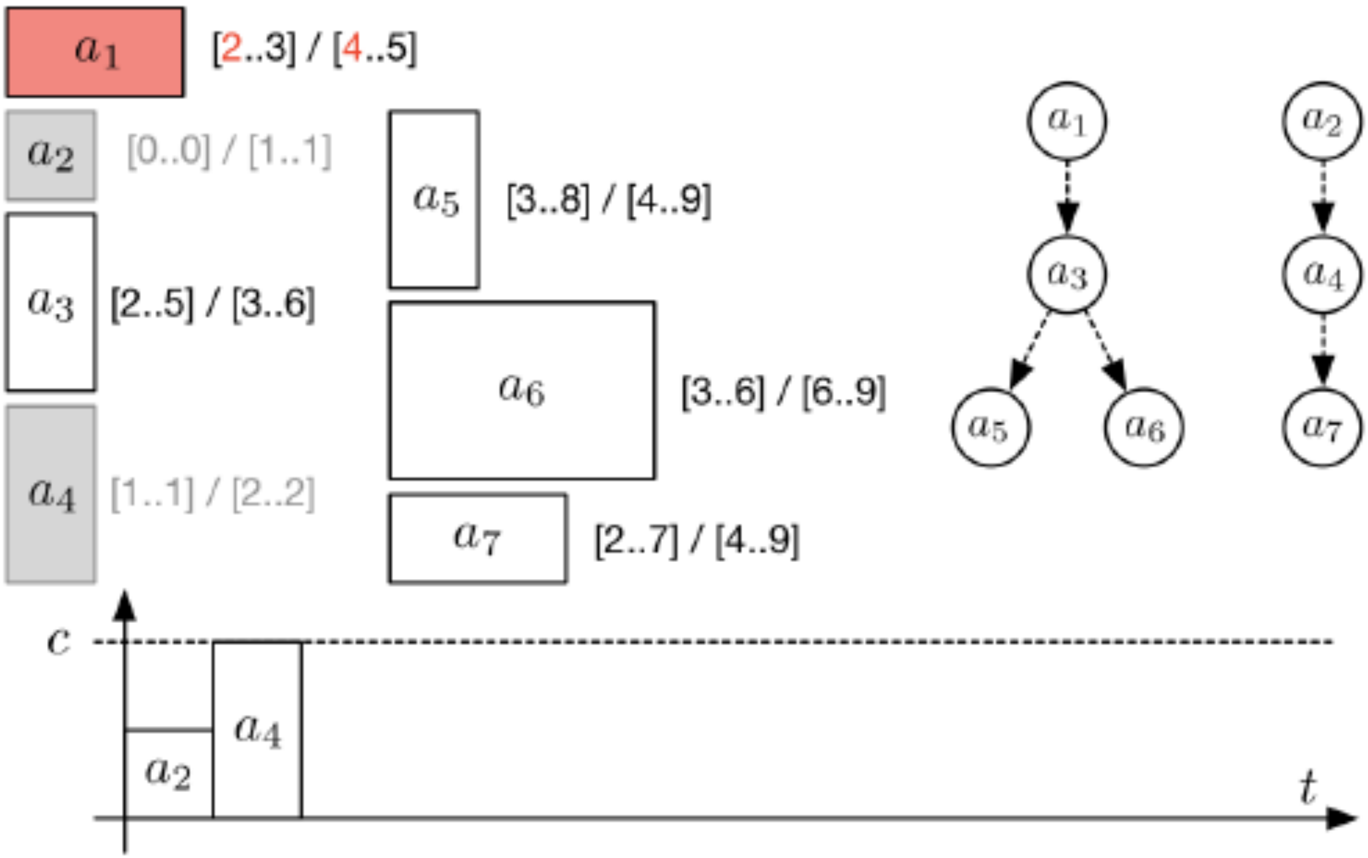
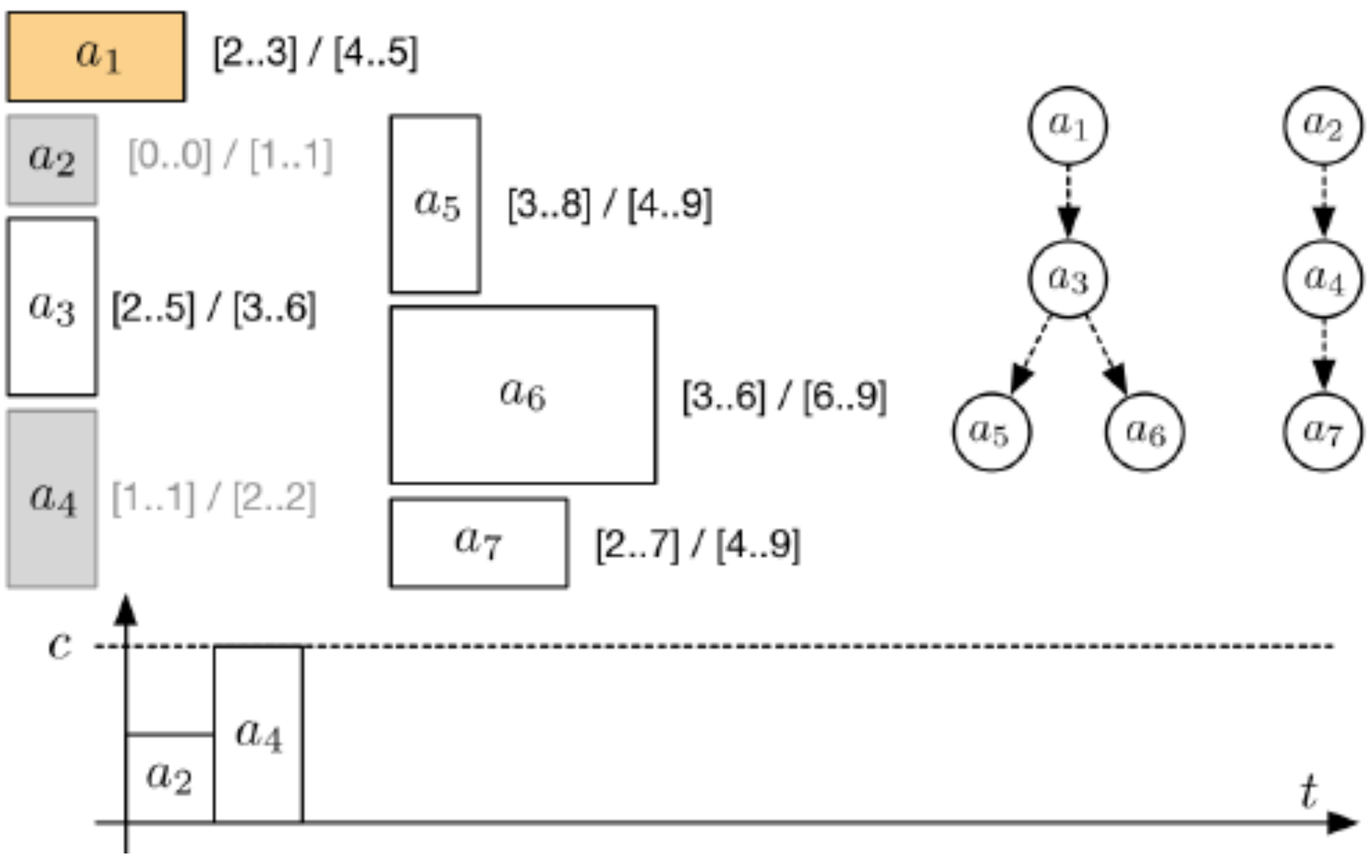


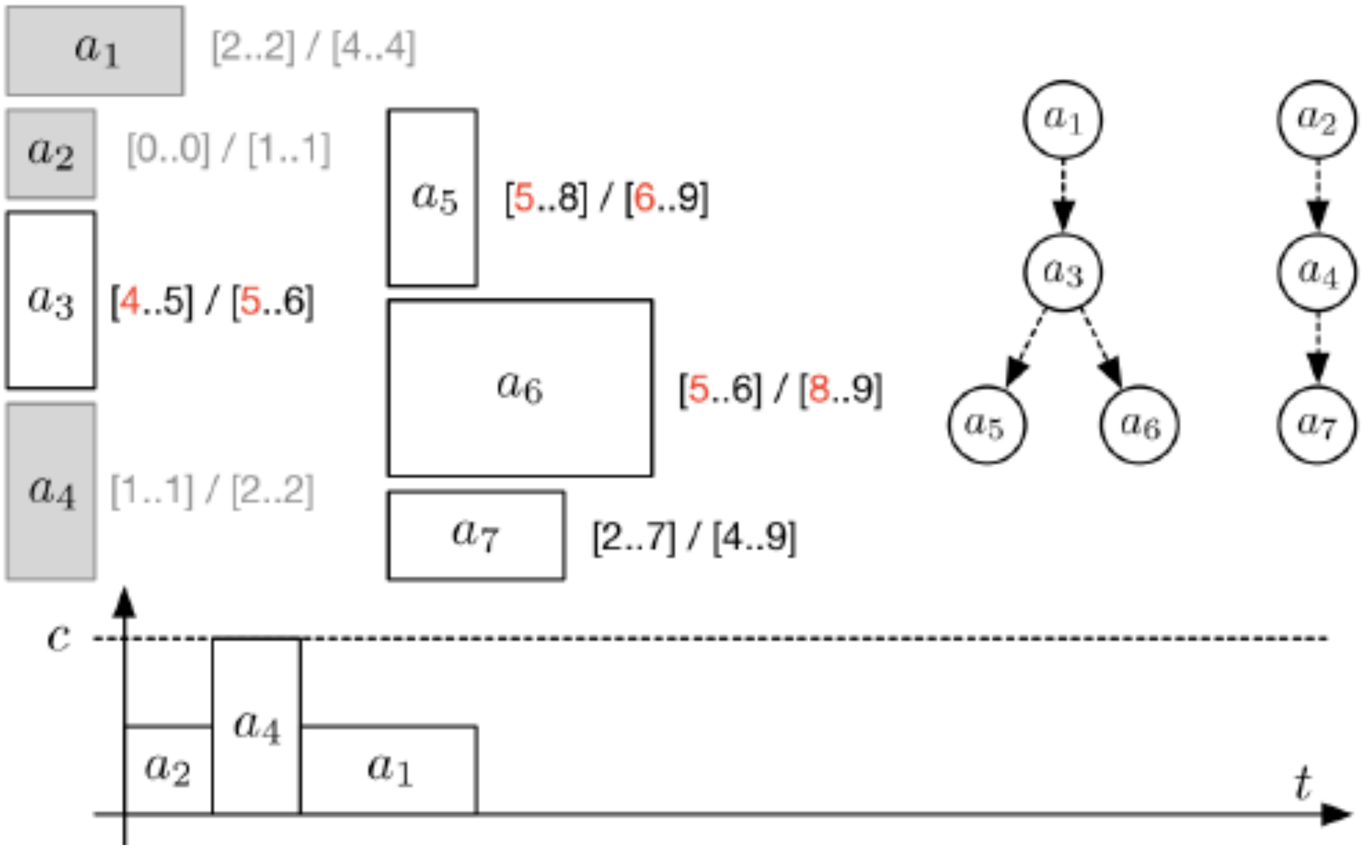
Backtracking for Proving Optimality



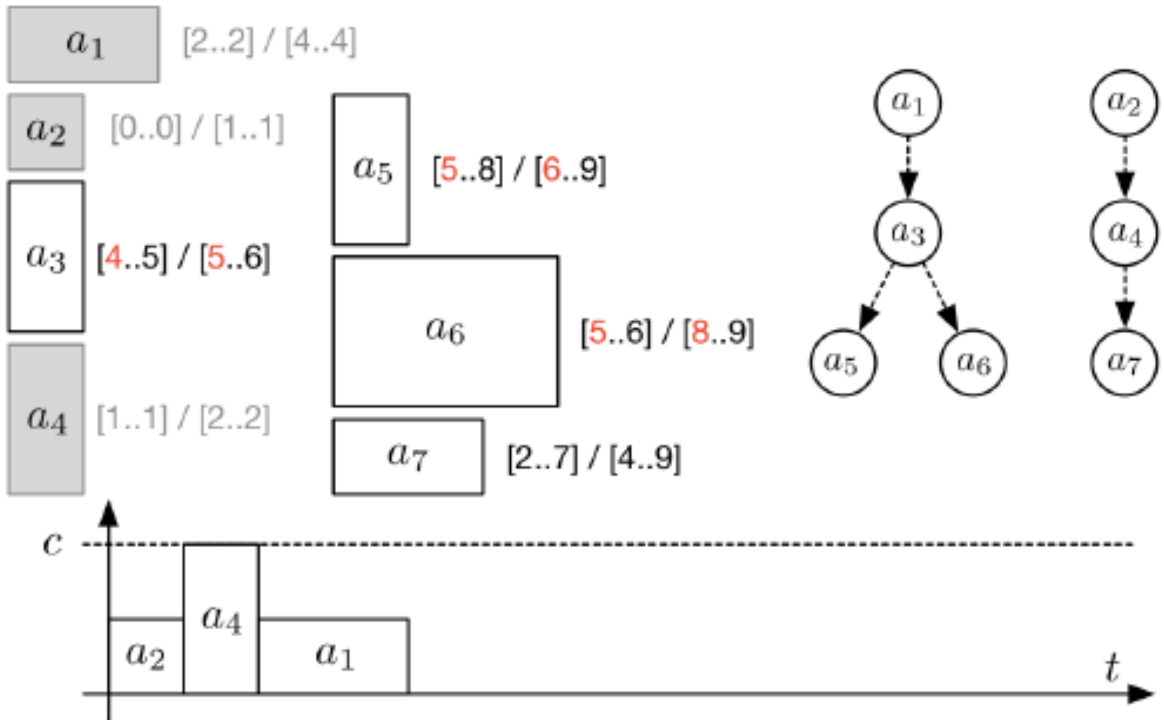
Backtracking for Proving Optimality



Backtracking for Proving Optimality

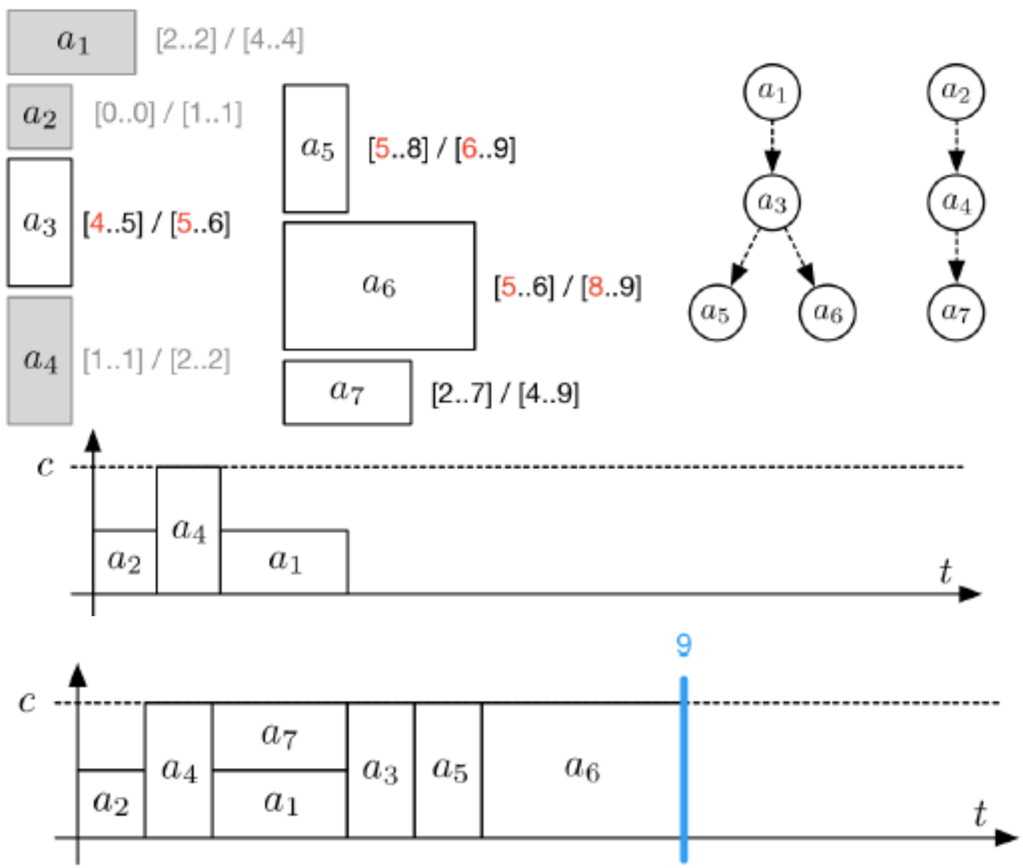


Backtracking for Proving Optimality



- By proceeding along this branch, we will find the optimal solution.

Backtracking for Proving Optimality



SetTimes Search Strategy

- Main idea
  - On the first branch schedule an activity a_i with minimum EST_i, schedule it at its EST_i.
    - Break ties according to any rule.
  - On backtracking, postpone a_i.
    - When propagation updates EST_i, schedule a_i.

SetTimes Search Strategy

- A very effective search strategy.
  - Based on PRB scheduling: finds good solutions early.
  - Effective branching choices (much better than posting $S_i \neq v$).
- Incomplete search strategy
  - At choice points, we do not partition the search space.
  - Either we schedule an activity **i** at EST_i or we make it wait.

SetTimes Search Strategy

- Why does it work?
  - The cost function is regular.
  - There is no point in not scheduling activities at their EST_i unless they are delayed by previous activities.
- When doesn't it work?
  - Non-regular cost functions.
    - E.g., costs for starting activities too early.
  - Side constraints that alter the problem structure.
    - E.g., maximal time legs.
- Other strategies are becoming more popular. E.g., domain splitting.