

RSA Key Extraction via Low-Bandwidth Acoustic Cryptanalysis

Matteo Berti

matteo.berti11@studio.unibo.it

LM Informatica

A.A. 2018/2019

Despite there are **many side-channels attacks** (electromagnetic, power-monitoring, timing, optical, acoustic, ...), **this research** is interesting because it is the **only** available **source** on acoustic cryptanalysis **of a cryptosystem**.

We will cover the following sections:

- **Introduction**
- **Foundations of the attack**
- **Further detail on the cryptanalysis**
- **Problems**
- **Error detection**
- **Attack mitigation**

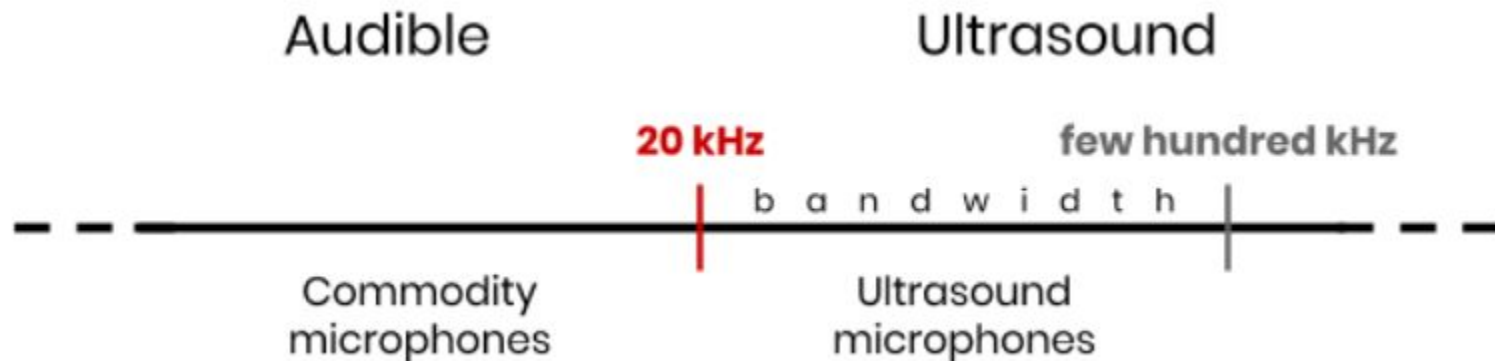
Introduction

CPUs change power according to the **type of operations** they perform.



Electronic components in the computers **generate vibrations**.

The **bandwidth** of these signals is **very low**:



Introduction

GnuPG operations can be **identified** by their **acoustic frequency spectrum**.



GPG RSA **secret keys** can be **distinguished** by the **sound** they made.



Therefore, the attack requires **ciphertexts** adaptively **chosen** by the **attacker**:

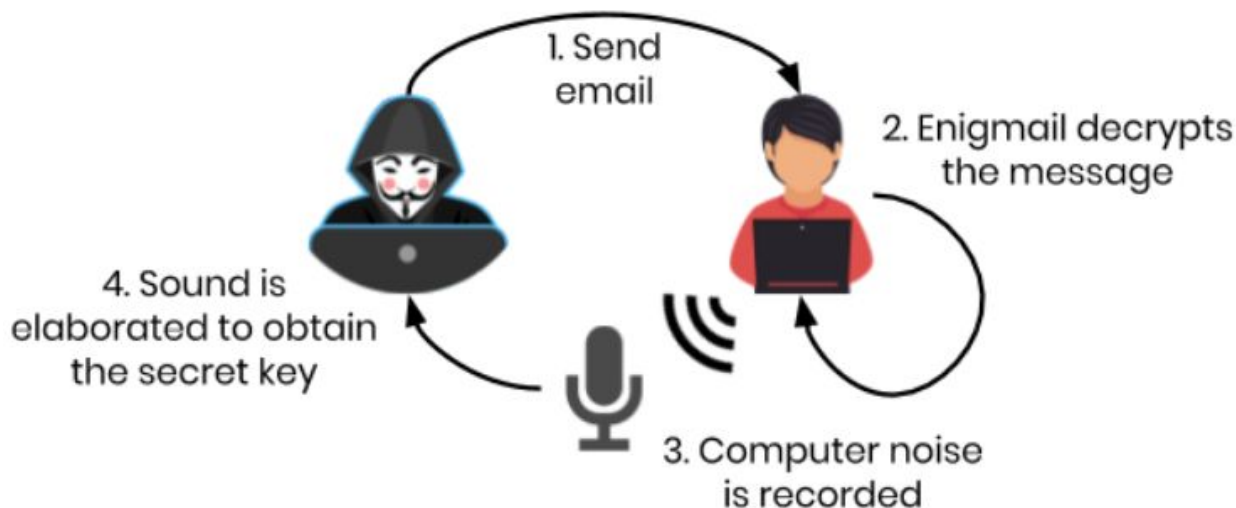
Chosen-ciphertext channel by email.

Introduction

A suitable ciphertext **attack vector** is:

OpenPGP encrypted email messages.

Enigmail: Thunderbird **plugin** that **automatically decrypts incoming** email for notification purposes.



Introduction

Other ways to eavesdrop secret keys:

- A **mobile device** remotely **compromised**, which record the target computer noise.



- The **target computer** if compromised may **spy on itself**.



Introduction

Three levels of recording accuracy:

- **Lab-grade setup: Brüel&Kjær condenser microphones** with 3 capsules (350kHz, 40kHz, 21kHz):



- **Portable setup:** same **Brüel&Kjær capsules** as before but replaced some components to **fit in a briefcase** (100kHz).



Introduction

Three levels of recording accuracy:

- **Mobile-phone setup**: were used several **Android smartphones** (24kHz).



Distant acquisition:

- **Parabolic microphones**: increase effective range from 1 meter to 4 meter.



Foundations of the attack

Recall on RSA cryptosystem:

- 2 large random primes p and q
- 2 numbers e and d such that $ed = 1 \bmod \varphi(n)$ and $n = pq$

Encryption: $m^e \bmod n$

Decryption: $c^d \bmod n$

public key

$$pk = (n, e)$$

secret key

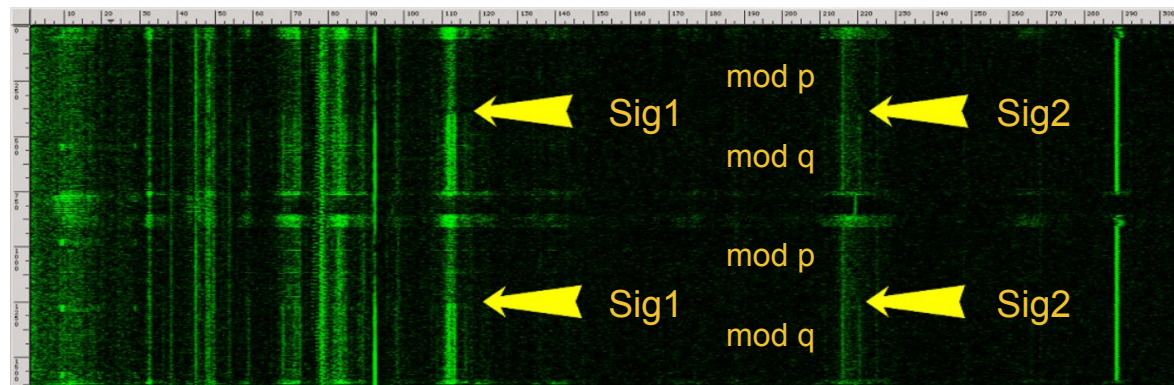
$$sk = (d, p, q)$$

The *signature* is computed:

$$m^{d \bmod (p-1)} \bmod p$$

$$m^{d \bmod (q-1)} \bmod q$$

$$s = m^d \bmod n$$



Each signature has a unique spectral signature (2 signatures and 4 modules above).



Foundations of the attack

The attack exposes the **secret factor q** one bit at a time, from MSB to LSB.

q 1 0 0 1 0 1 1 0 ...

For each bit q_i we **assume** that $q_{2048} \dots q_{i+1}$ were correctly **recovered**, and **check** if q_i is **0 or 1**.

0 1
1 0 0 ? 0 1 1 0 ...
2048 q_i

Eventually, we **learn all of q** and recover the factorization of n .

q 1 0 0 1 0 1 1 0 ...
p ?????????? ... $\rightarrow n/q = p$

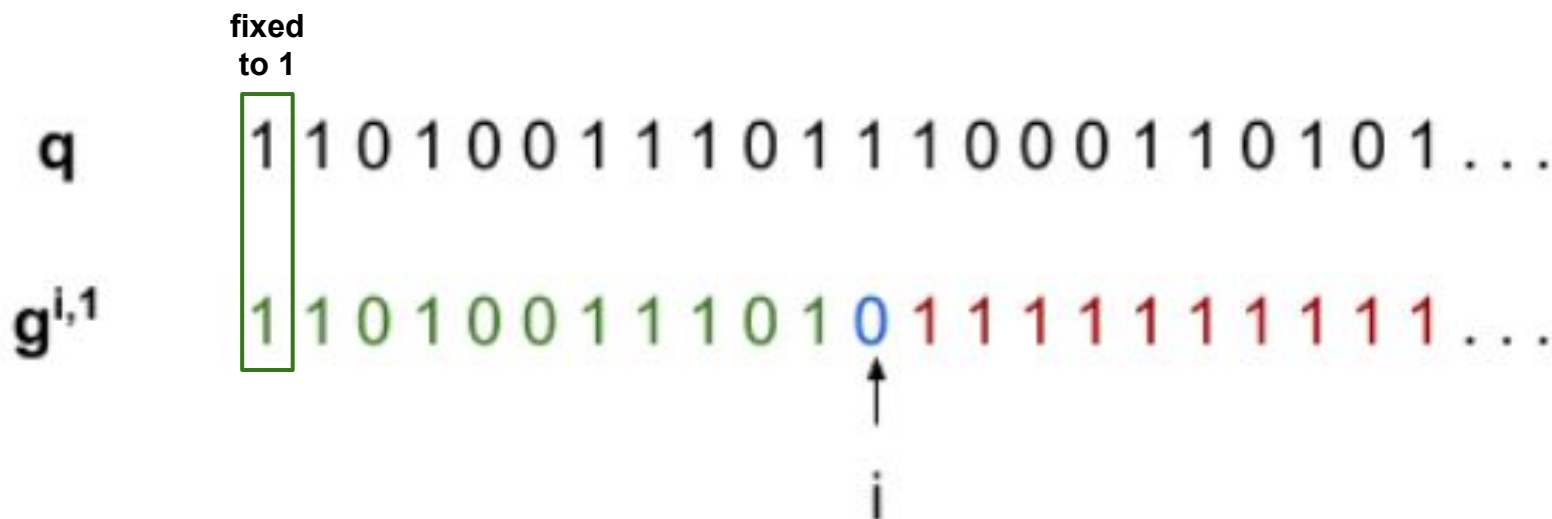
The same technique applies to p , but **q** has a **better signal**.



Foundations of the attack

Let $g^{i,1}$ be the **ciphertext** whose **topmost $i-1$ bits** are **correctly recovered from q** , the **i -th bit is 0**, and the **remaining (low) bits are 1**.

Moreover **RSA keys** in GPG have **MSB** of q is **set**: $q_{2048} = 1$





Foundations of the attack

Algorithm 1 GnuPG's modular exponentiation (see function `mpi_powm` in `mpi/mpi-pow.c`).

Input: Three integers c , d and q in binary representation such that $d = d_n \cdots d_1$.

Output: $m = c^d \pmod q$.

```
1: procedure MODULAR_EXPONENTIATION( $c, d, q$ )
2:   if SIZE_IN_LIMBS( $c$ ) > SIZE_IN_LIMBS( $q$ ) then
3:      $c \leftarrow c \pmod q$ 
4:    $m \leftarrow 1$ 
5:   for  $i \leftarrow n$  downto 1 do
6:      $m \leftarrow m^2$ 
7:     if SIZE_IN_LIMBS( $m$ ) > SIZE_IN_LIMBS( $q$ ) then
8:        $m \leftarrow m \pmod q$ 
9:     if SIZE_IN_LIMBS( $c$ ) < KARATSUBA_THRESHOLD then ▷ defined as 16
10:       $t \leftarrow \text{MUL\_BASECASE}(m, c)$  ▷ Compute  $t \leftarrow m \cdot c$  using Algorithm 3
11:    else
12:       $t \leftarrow \text{MUL}(m, c)$  ▷ Compute  $t \leftarrow m \cdot c$  using Algorithm 5
13:    if SIZE_IN_LIMBS( $t$ ) > SIZE_IN_LIMBS( $q$ ) then
14:       $t \leftarrow t \pmod q$ 
15:    if  $d_i = 1$  then
16:       $m \leftarrow t$ 
17:  return  $m$ 
18: end procedure
```

A **limb** is the part of a multi-precision number that fits in a **single machine word**, normally a limb is **32** or **64 bits**.



Foundations of the attack

When we **decrypt** $g^{i,1}$, i -th bit of q could be:

- $q_i = 1$ then $g^{i,1} < q$

$$\begin{array}{r} q \quad 1\ 1\ 0\ \boxed{1}\ 0\ 0\ 1\ 1 = 211 \\ g^{i,1} \quad 1\ 1\ 0\ \boxed{0}\ 1\ 1\ 1\ 1 = 207 \end{array}$$

If we assume **line 2** of Alg1 is **removed**.

- Line 3: $c \leftarrow c \bmod q$ **returns c** because $c = g^{i,1} < q$

```
1: procedure MODULAR_EXPONENTIATION( $c, d, q$ )
2: if SIZE_IN_LIMBS( $c$ ) > SIZE_IN_LIMBS( $q$ ) then
3:    $c \leftarrow c \bmod q$ 
4:    $m \leftarrow 1$ 
```



Foundations of the attack

When we **decrypt** $g^{i,1}$, i -th bit of q could be:

- $q_i = 0$ then $g^{i,1} \geq q$

$$\begin{array}{r} q \quad 1\ 1\ 0\ \boxed{0}\ 1\ 0\ 0\ 1 = 201 \\ g^{i,1} \quad 1\ 1\ 0\ \boxed{0}\ 1\ 1\ 1\ 1 = 207 \end{array}$$

If we assume **line 2** of Alg1 is **removed**.

- Line 3: $c \leftarrow c \bmod q$ **returns** $c - q$ because $q \leq g^{i,1} < 2q$

```
1: procedure MODULAR_EXPONENTIATION( $c, d, q$ )
2:   if SIZE_IN_LIMBS( $c$ ) > SIZE_IN_LIMBS( $q$ ) then
3:      $c \leftarrow c \bmod q$ 
4:    $m \leftarrow 1$ 
```

The occurrence or not of **this reduction** will lead us to **distinguish** if the **bit** of q is **1** or **0**.




Foundations of the attack

If we enable again line 2 of Alg1, we see **line 3 is never taken**.

This happens because $\mathbf{g}^{i,1}$ and \mathbf{q} have the **same number of limbs** (64 each).

`SIZE_IN_LIMS(c) > SIZE_IN_LIMBS(q) → 64 > 64 → FALSE`

```
1: procedure MODULAR_EXPONENTIATION(c, d, q)
2:   if SIZE_IN_LIMBS(c) > SIZE_IN_LIMBS(q) then
3:     ✘ c ← c mod q
```



But **we need the reduction** to distinguish $q_i = 0$ from $q_i = 1$

This can be solved in either of two way.

Foundations of the attack

1. It could be **added leading zero limbs** to $g^{i,1}$, so **line 3** will be **always taken**.

X But the **algorithm** could be **changed** to not allocate leading zero limb.

q 1 1 0 0 1 0 0 1
 $g^{i,1}$ 0 0 0 0 1 1 0 0 1 1 1 1
padding

2. It could be **decrypted** the 128 limb **number** $g^{i,1} + n$ (the result would be the same) so **line 3** will be **always taken**.



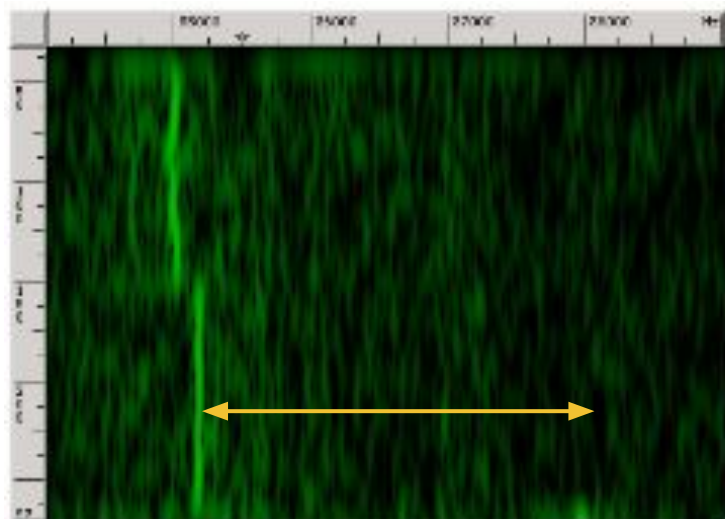
DECRYPT($g^{i,1} + n$)

$\#limbs(g^{i,1} + n) = 128 > \#limbs(q) = 64$

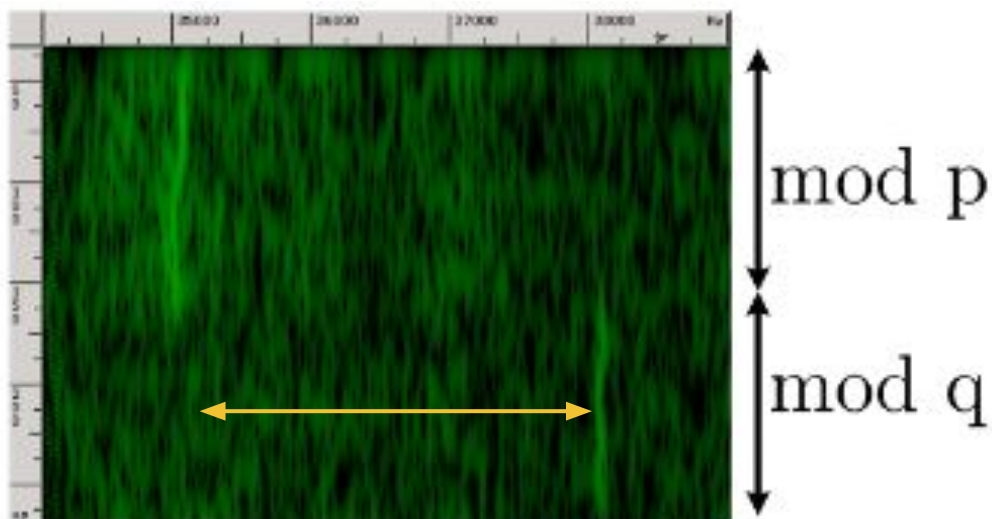
```
1: procedure MODULAR_EXPONENTIATION( $c, d, q$ )  
2:   if  $SIZE\_IN\_LIMBS(c) > SIZE\_IN\_LIMBS(q)$  then  
3:     😊  $c \leftarrow c \bmod q$ 
```


Foundations of the attack

As we can see in the figures when $q_i = 0$ the **frequency** of the modular exponentiation is **lower than** when $q_i = 1$.



Attacked bit is 0



Attacked bit is 1

Foundations of the attack

To sum up what we have seen thus far:

1. **Decrypt $c (= g^{i,1} + n)$** on the target machine.

decrypt($c = g^{i,1} + n$)



2. **Measure acoustic leakage** during decryption.



3. **Recognize the difference between the two leakage patterns.**



This can be done sending **email messages** with the **chosen ciphertext** backdated or marked as **spam**.



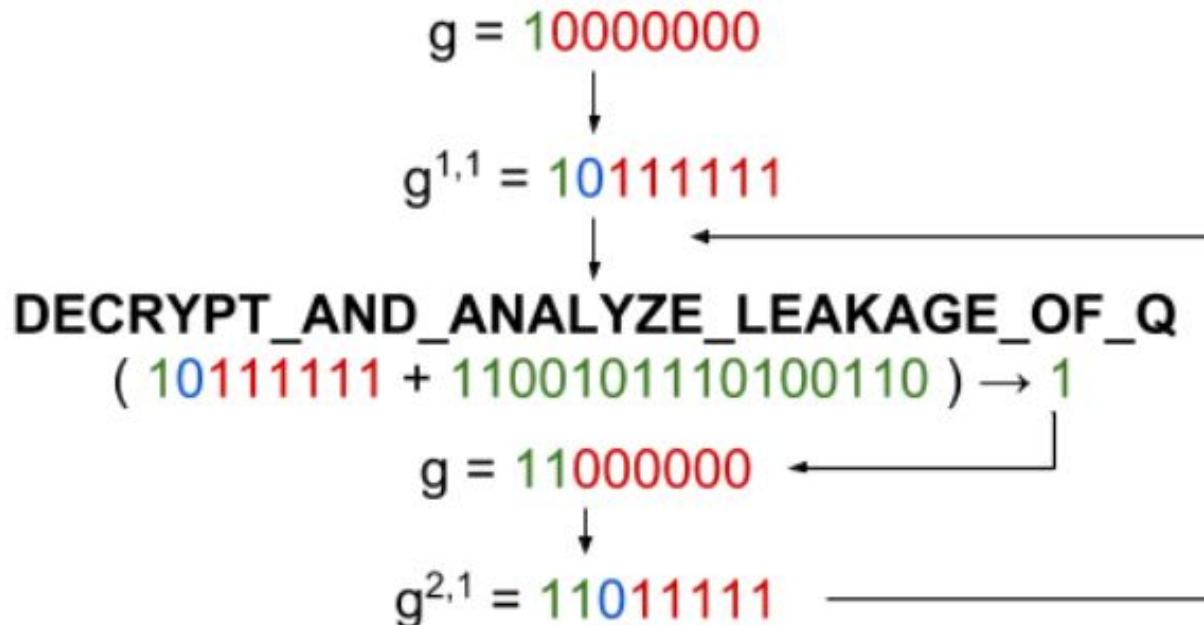
Foundations of the attack

Algorithm 2 Top loop of the (simplified) attack on GnuPG's RSA decryption.

Input: An RSA public key $pk = (n, e)$ such that $n = pq$ where n is an m bit number.

Output: The factorization p, q of n .

```
1: procedure SIMPLIFIEDATTACK(pk)
2:    $g \leftarrow 2^{(m/2)-1}$  ▷  $g$  is a  $m/2$  bit number of the form  $g = 10 \dots 0$ 
3:   for  $i \leftarrow m/2 - 1$  downto 1 do
4:      $g^{i,1} \leftarrow g + 2^{i-1} - 1$  ▷ set all the bits of  $g$  starting from  $i - 1$ -th bit to be 1
5:      $b \leftarrow \text{DECRYPT\_AND\_ANALYZE\_LEAKAGE\_OF\_Q}(g^{i,1} + n)$  ▷ obtain the  $i$ -th bit of  $q$ 
6:      $g \leftarrow g + 2^{i-1} \cdot b$  ▷ update  $g$  with the newly obtained bit
7:    $q \leftarrow g$ 
8:    $p \leftarrow n/q$ 
9:   return  $(p, q)$ 
10: end procedure
```





Further detail on the cryptanalysis

But exactly what makes the **difference** in the **acoustic frequency** when the bit attacked is 1 or 0?

To understand this we need to go **deeper** in the **modular exponentiation** algorithm.

The algorithm consists of **two main** multiplication **routines**:

- A **basic schoolbook** multiplication routine (for short ciphertexts).
- A **recursive Karatsuba** multiplication algorithm (for large ciphertexts).

MODULAR_EXPONENTIATION

```
9:   if SIZE_IN_LIMBS(c) < KARATSUBA_THRESHOLD then                                ▷ defined as 16
10:      t ← MUL_BASECASE(m, c)                                                    ▷ Compute  $t \leftarrow m \cdot c$  using Algorithm 3
11:   else
12:      t ← MUL(m, c)                                                            ▷ Compute  $t \leftarrow m \cdot c$  using Algorithm 5
```



Further detail on the cryptanalysis

Algorithm 3 GnuPG's basic multiplication code (see functions `mul_n_basecase` and `mpihelp_mul` in `mpi/mpih-mul.c`).

Input: Two numbers $a = a_k \cdots a_1$ and $b = b_n \cdots b_1$ of size k and n limbs respectively.

Output: $a \cdot b$.

```
1: procedure MUL_BASECASE( $a, b$ )  for short ciphertexts
2:   if  $b_1 \leq 1$  then
3:     if  $b_1 = 1$  then
4:        $p \leftarrow a$ 
5:     else
6:        $p \leftarrow 0$ 
7:   else
8:      $p \leftarrow \text{MUL\_BY\_SINGLE\_LIMB}(a, b_1)$  ▷  $p \leftarrow a \cdot b_1$ 
9:   for  $i \leftarrow 2$  to  $n$  do
10:    if  $b_i \leq 1$  then
11:      if  $b_i = 1$  then ▷ (and if  $b_i = 0$  do nothing)
12:         $p \leftarrow \text{ADD\_WITH\_OFFSET}(p, a, i)$  ▷  $p \leftarrow p + a \cdot 2^{32 \cdot i}$ 
13:      else
14:         $p \leftarrow \text{MUL\_AND\_ADD\_WITH\_OFFSET}(p, a, b_i, i)$  ▷  $p \leftarrow p + a \cdot b_i \cdot 2^{32 \cdot i}$ 
15:   return  $p$ 
16: end procedure
```



Further detail on the cryptanalysis

Algorithm 5 GnuPG's multiplication code (see function `mpihelp_mul_karatsuba_case` in `mpi/mpih-mul.c`).

Input: Two numbers $a = a_k \cdots a_1$ and $b = b_n \cdots b_1$ of size k and n limbs respectively.

Output: $a \cdot b$.

```
1: procedure MUL( $a, b$ ) for long ciphertexts
2:   if  $n < \text{KARATSUBA\_THRESHOLD}$  then                                     ▷ defined as 16
3:     return MUL_BASECASE( $a, b$ )                                         ▷ multiply using Algorithm 3
4:    $p \leftarrow 0$ 
5:    $i \leftarrow 1$ 
6:   while  $i \cdot n \leq k$  do
7:      $t \leftarrow \text{KARATSUBA\_MUL}(a_{i \cdot n} \cdots a_{(i-1) \cdot n + 1}, b)$    ▷ multiply  $n$  limb numbers using Algorithm 4
8:      $p \leftarrow \text{ADD\_WITH\_OFFSET}(p, t, (i-1) \cdot n)$                  ▷  $p \leftarrow p + t \cdot 2^{32 \cdot (i-1) \cdot n}$ 
9:      $i \leftarrow i + 1$ 
10:  if  $i \cdot n > k$  then
11:     $t \leftarrow \text{MUL}(b, a_k \cdots a_{(i-1) \cdot n + 1})$                  ▷ multiply the remaining limbs of  $a$  using a recursive call
12:     $p \leftarrow \text{ADD\_WITH\_OFFSET}(p, t, (i-1) \cdot n)$                  ▷  $p \leftarrow p + t \cdot 2^{32 \cdot (i-1) \cdot n}$ 
13:  return  $p$ 
14: end procedure
```




Further detail on the cryptanalysis

Karatsuba recursive algorithm is a very efficient way to perform **large integer multiplications**.

Algorithm 4 GnuPG's Karatsuba multiplication code (see function `mul_n` in `mpi/mpih-mul.c`).

Input: Two n limb numbers $a = a_n \cdots a_1$ and $b = b_n \cdots b_1$.

Output: $a \cdot b$.

```
1: procedure KARATSUBA_MUL( $a, b$ )
2:   if  $n < \text{KARATSUBA\_THRESHOLD}$  then                                     ▷ defined as 16
3:     return MUL_BASECASE( $a, b$ ) termination           ▷ multiply using Algorithm 3
4:   if  $n$  is odd then
5:      $p \leftarrow \text{KARATSUBA\_MUL}(a_{n-1} \cdots a_1, b_{n-1} \cdots b_1)$            ▷  $p \leftarrow (a_{n-1} \cdots a_1) (b_{n-1} \cdots b_1)$ 
6:      $p \leftarrow \text{MUL\_AND\_ADD\_WITH\_OFFSET}(p, a_{n-1} \cdots a_1, b_n, n)$        ▷  $p \leftarrow p + (a_{n-1} \cdots a_1) \cdot b_n \cdot 2^{32 \cdot n}$ 
7:      $p \leftarrow \text{MUL\_AND\_ADD\_WITH\_OFFSET}(p, b, a_n, n)$                    ▷  $p \leftarrow p + b \cdot a_n \cdot 2^{32 \cdot n}$ 
8:   else
9:      $h \leftarrow \text{KARATSUBA\_MUL}(a_n \cdots a_{n/2+1}, b_n \cdots b_{n/2+1})$ 
10:     $t \leftarrow \text{KARATSUBA\_MUL}(a_n \cdots a_{n/2+1} - a_{n/2} \cdots a_1, b_{n/2} \cdots b_1 - b_n \cdots b_{n/2+1})$ 
11:     $l \leftarrow \text{KARATSUBA\_MUL}(a_{n/2} \cdots a_1, b_{n/2} \cdots b_1)$ 
12:     $p \leftarrow (2^{2 \cdot 32 \cdot n} + 2^{32 \cdot n}) \cdot h + 2^{32 \cdot n} \cdot t + (2^{32 \cdot n} + 1) \cdot l$ 
13:  return  $p$ 
14: end procedure
```



Further detail on the cryptanalysis

Each bit i in q could be:

- $q_i = 1$

In this case, following the multiplication routines to the Karatsuba algorithm:

The **second operand b** of the calls to **MUL_BASECASE** resulting from the recursive calls will contain **mostly zero limbs**.

```
KARATSUBA_MUL
```

```
2:  if  $n < \text{KARATSUBA\_THRESHOLD}$  then
```

```
3:      return MUL_BASECASE( $a$ ,  $b$ )  $\rightarrow$  mostly 0s
```




Further detail on the cryptanalysis

Each bit i in q could be:

- $q_i = 0$

In this case, following the multiplication routines to the Karatsuba algorithm:

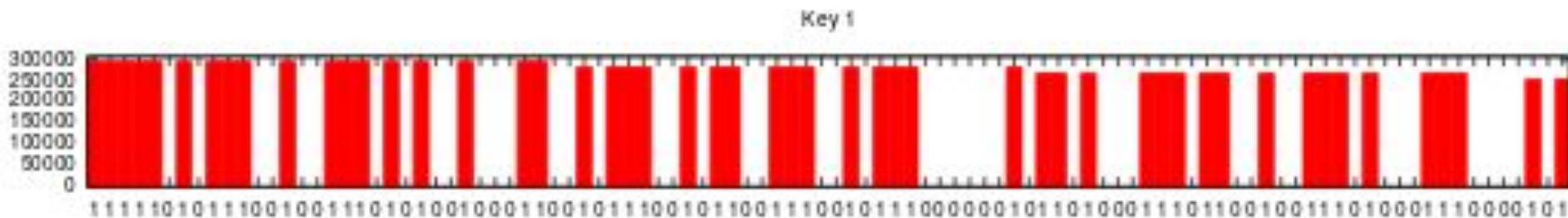
The **second operand b** of the calls to **MUL_BASECASE** resulting from the recursive calls will contain **mostly (random-looking) non-zero limbs**.

```
KARATSUBA_MUL
```

```
2:  if  $n < \text{KARATSUBA\_THRESHOLD}$  then
```

```
3:      return MUL_BASECASE( $a$ ,  $b$ ) → mostly non-0s
```

Further detail on the cryptanalysis



Axis X: attacked **bit of q**

Axis Y: **# of zero limbs** in the 2° operand of **MUL_BASECASE**

Large number of zero limbs $\rightarrow q_i = 1$

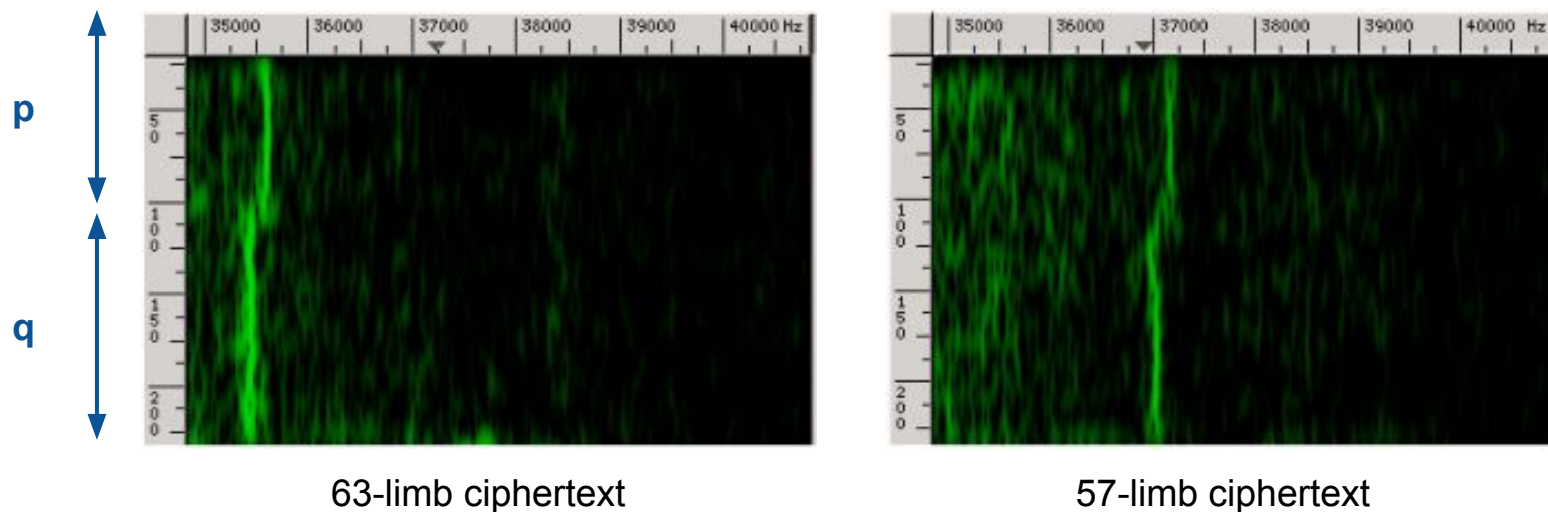
Low number of zero limbs $\rightarrow q_i = 0$

The **drastic change** in the number of **non-zero limbs** in the **second operand** of **MUL_BASECASE** is **detectable** by our **side channel measurements**.



Further detail on the cryptanalysis

Observation: generating **2 random ciphertexts** of respectively **63 limbs** and **57 limbs** (non-zero limbs):

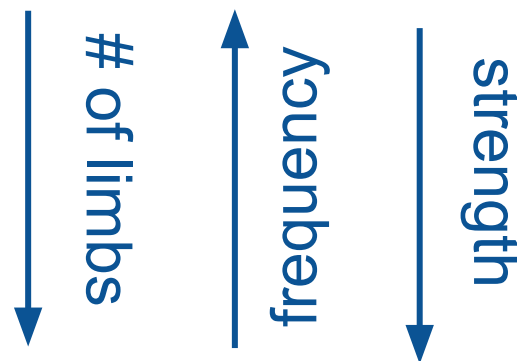
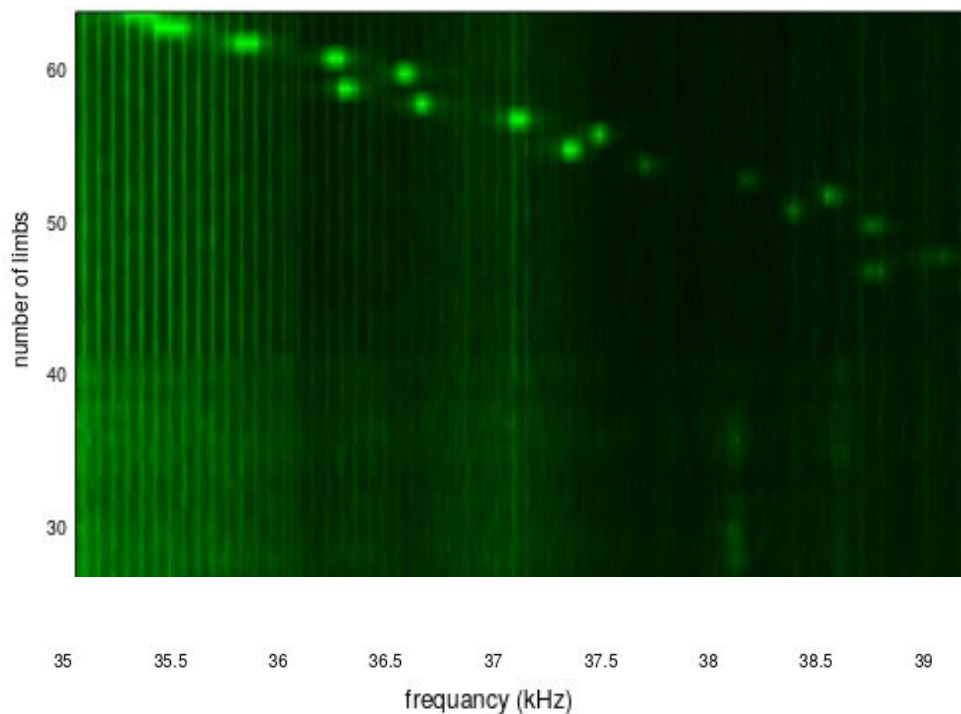


Decryption of the **63-limb** ciphertext produces a **signal** at **lower frequency** than the decryption of the **57-limb** ciphertext.

It can be found the num of limbs in the 2° operand of MUL.

Further detail on the cryptanalysis

Therefore: the **shorter** the number of **limbs** (in 2° operand) the **higher** the **frequency** of the acoustic leakage, and the **weaker** the **signal strength**.



We can **acoustically detect** when the 2° operand of MUL_BASECASE has **many non-zero limbs** ($q_i = 0$) or when it has **few non zero-limbs** ($q_i = 1$).

Problems

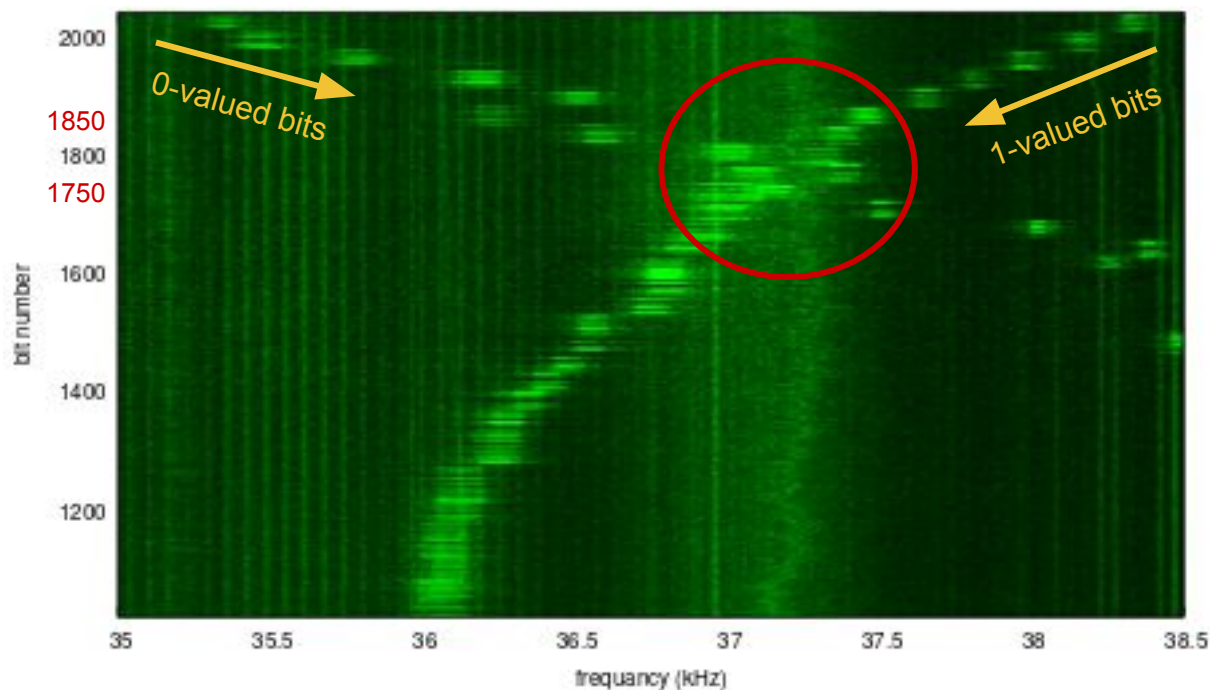
Unfortunately, there is a problem:

Distinguishing the above two cases using *side channel leakage* is particularly **hard for bits in the range of **1850–1750**.**

... 0 1 0 0 1 1 0 [1 0 1 1 0 ... 1 0 0 1 0] 1 0 0 0 0 1 1 ...
1850 1750

This complication **requires us to use additional tricks.**

Problems

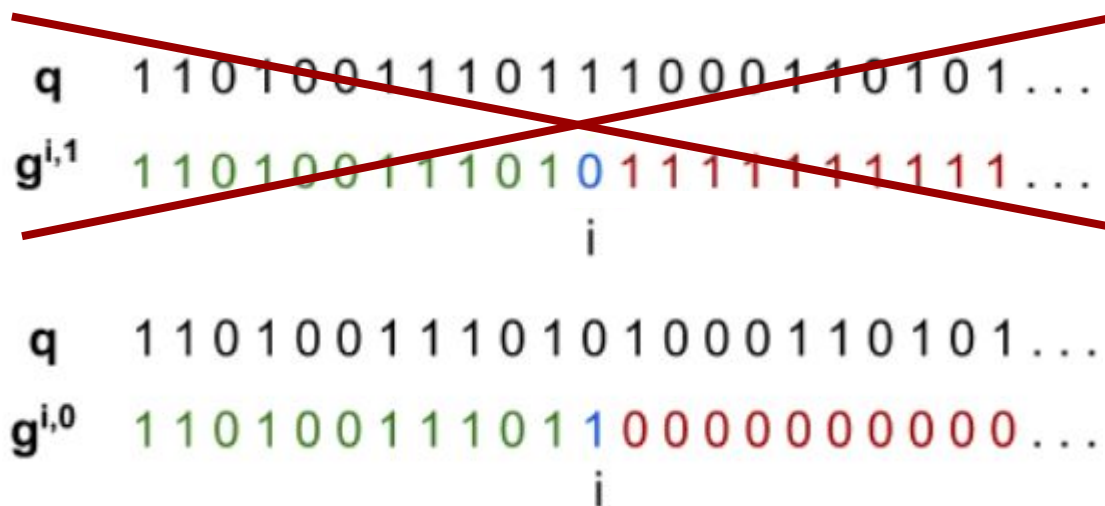


When it's used $c = g^{i,1} + n$, bits in range [1850 - 1750] emit **very similar frequencies** with a distance of nearly 200Hz.

- The **bit index** where this **crossing point** occurs **depends on the specific values of the ciphertext used!**

Problems

Let $g^{i,0}$ be **2048-bit number** whose **top $i-1$ bits** are the **same as q** , its **i -th bit is 1** and all **the rest of its bits are 0**.



Using $g^{i,0}$ it is now **possible to distinguish** the **bits** in the range of **1750–1850** thus allowing our attack to proceed.

Problems

Algorithm 6 Extracting all bits from GnuPG's implementation of 4096-bit RSA-CRT.

Input: A an RSA public key $pk = (n, e)$ such that $n = pq$ where n is an m bit number.

Output: The factorization p, q of n .

```

1: procedure ATTACKALLBITS(pk)
2:    $g \leftarrow 2^{(m/2)-1}$                                 ▷  $g$  is a  $m/2$  bit number of the form  $g = 10 \cdots 0$ 
3:   for  $i \leftarrow m/2 - 1$  downto 1 do
4:      $g^{i,1} \leftarrow g + 2^{i-1} - 1$                   ▷ set all the bits of  $g$  starting from  $i - 1$ -th bit to be 1
5:      $g^{i,0} \leftarrow g + 2^{i-1}$                        ▷ set the  $i$ -th bit of  $g$  to be 1
6:     if  $1750 \leq i \leq 1850$  then
7:        $b \leftarrow \text{decrypt\_and\_analyze\_leakage\_of\_q}(g^{i,0} + n)$     ▷ obtain the  $i$ -th bit of  $q$  using  $g^{i,0}$ 
8:     else
9:        $b \leftarrow \text{decrypt\_and\_analyze\_leakage\_of\_q}(g^{i,1} + n)$     ▷ obtain the  $i$ -th bit of  $q$  using  $g^{i,1}$ 
10:     $g \leftarrow g + 2^{i-1} \cdot b$                        ▷ update  $g$  with the newly obtained bit
11:    $q \leftarrow g$ 
12:    $p \leftarrow n/q$ 
13:   return  $(p, q)$ 
14: end procedure

```

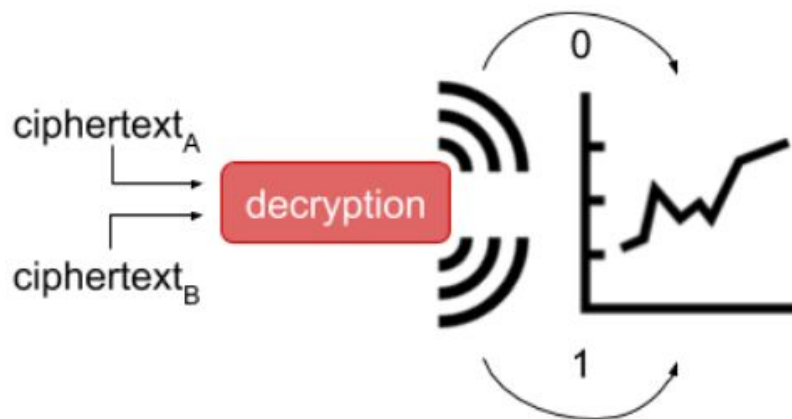

Problems

The attack proceeds in two stages:

1. Calibration stage

The attacker **generates two ciphertexts** corresponding to a **leakage of 0 and 1 bits** of q and **obtains multiple samples of their decryption**.

The attacker **generates a **template** of the leakage** caused by 0 bit and a template of the leakage caused by a 1 bit.



2. Attack stage

(2 steps)

- *Classification step*

A spectrum of an obtained leakage is **classified** using the **templates** as corresponding to **0 bit** or to a **1 bit**. This might be repeated a few times.



- *Template update step*

New templates for 0 bits and 1 bits are generated **updating** the **old ones** with the new leakages.



Error detection

If by mistake some bit $q_j = 1$ is **misclassified as 0**, successive values of both $g^{i,1}$ and $g^{i,0}$ for all $i < j$ will be **always smaller** than q .

This value will have the **same acoustic leakage** as if $q_i = 1$: next bits will result as **all 1s** regardless their actual value.



➤ Solution: when a **sequence** (ex. 20 bits) of **only 1s** is detected, the attacker can **backtrack some bits** (ex. 50 bits) and **try again**.

Error detection

If by mistake some bit $q_j = 0$ is **misclassified as 1**, successive values of both $g^{i,1}$ and $g^{i,0}$ for all $i < j$ will be always larger than q .

This value will have the **same acoustic leakage** as if $q_i = 0$: next bits will result as **all 1s** regardless their actual value.

1 0 1 0 0 0 ... → 1 0 1 0 0 1 1 0 0 0 0 0 0 ...
 1

➤ Solution: when a **sequence** (ex. 20 bits) of **only 0s** is detected, the attacker can **backtrack some bits** (ex. 50 bits) and **try again**.

Attack mitigation

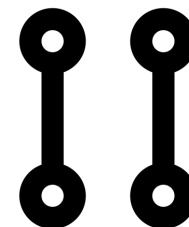
Acoustic shielding: acoustic absorbers and sound-proof enclosures could **attenuate the signals**, but **do not prevent the attack**.



Noisy environment: **noise** in a noisy environment is **below 10 kHz**, acoustic **leakage** is well **above this range**, such **noises** can be **filtered out**.



Parallel software load: perform the **computation** in **parallel** will **move the leakage frequency** from **35-38 kHz** to **32-35 kHz** (easier to detect).





Attack mitigation

Ciphertext randomization: instead of decrypting c , given a 4096-bit random value r , one can **decrypt** $r^e \cdot c$ and **multiply** the **result** by r^{-1} .



Ciphertext normalization: it can be **removed** all **leading zeros** of c and **decrypt** $c' = c \bmod n$. This value will have the **same limb** count as q , **line 2** of Alg1 will be **never taken**, making it **impossible** to use the modular reduction in order to **create a connection**.





Conclusion

- It's possible with **some version of GPG RSA (1.x)** to attack a secret key with acoustic cryptanalysis.
- It's **neither easy** nor **practical**.
- To carry out this kind of attack is required **time** and **effort**.
- It could be **mitigated**.

**BUT IT IS POSSIBLE WITH A SMARTPHONE
TO FIND AN RSA SECRET KEY!**



References

- [1] [Daniel Genkin, Adi Shamir and Eran Tromer, RSA Key Extraction via Low-Bandwidth Acoustic Cryptanalysis. CRYPTO 2014](#)

- [2] [Adi Shamir, Eran Tromer, Acoustic Cryptanalysis - On nosy people and noisy machines](#)