$$f(x,y) = x^3 y + y^2 - 3xy.$$

$$\begin{cases} \partial_x f = 3x^2 y - 3y = 3y(x^2-1) = 0 \\ \partial_y f = x^3 + 2y - 3x = 0 \end{cases}$$

Dalla prima equazione: $y = 0$, $x = \pm 1$

- Se $y = 0$ allora $x^3 - 3x = 0 \Rightarrow x = 0, \pm\sqrt{3}$

  Dunque $(0,0)$, $(\sqrt{3}, 0)$ e $(-\sqrt{3}, 0)$ sono critici

- Se $x = 1 \Rightarrow 1 + 2y - 3 = 0 \Rightarrow y = +1$

  Se $x = -1 \Rightarrow -1 + 2y + 3 = 0 \Rightarrow y = -1$

  Dunque ho i punti $(1,1)$ e $(-1,-1)$

$$Hf(x,y) = \begin{bmatrix} 6xy & 3(x^2-1) \\ 3(x^2-1) & 2 \end{bmatrix}.$$

Pertanto

$$Hf(0,0) = \begin{bmatrix} 0 & -3 \\ -3 & 2 \end{bmatrix}$$ (ha det. negativo $\Rightarrow (0,0)$ di sella)

$$Hf(\sqrt{3}, 0) = \begin{bmatrix} 0 & 6 \\ 6 & 2 \end{bmatrix}$$ sella (come sopra)

$$Hf(-\sqrt{3}, 0) = \begin{bmatrix} 0 & 6 \\ 6 & 2 \end{bmatrix}$$ ancora punto di sella

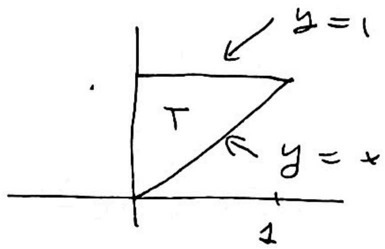
$$Hf(1,1) = \begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix}$$ punto di minimo (matrice positiva)

$$Hf(-1,-1) = \begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix}$$ ancora di minimo (matrice positiva)

Essendo $f(-2,1) = -2$ e $\nabla f(-2,1) = (9,0)$ l'equazione cercata

è $z = -2 + 9(x+2) + 0(y-1) = -2 + 9(x+2)$

$$\int_T x^2\cos(\pi xy)\,dx\,dy$$



$$= \int_0^1 \left(\int_x^1 x^2\cos(\pi xy)\,dy\right)dx =$$

$$= \int_0^1 \left[\frac{x}{\pi}\sin(\pi xy)\right]_{y=x}^{y=1} dx =$$

$$= \int_0^1 \left(\frac{x}{\pi}\sin(\pi x) - \frac{x}{\pi}\sin(\pi x^2)\right)dx =$$

$$= \int_0^1 \frac{x}{\pi}\sin(\pi x)\,dx - \int_0^1 \frac{x}{\pi}\sin(\pi x^2)\,dx$$

$$= \left[-\frac{\cos(\pi x)}{\pi}\cdot\frac{x}{\pi}\right]_0^1 + \int_0^1 \frac{\cos(\pi x)}{\pi}\cdot\frac{1}{\pi}\,dx + \left[\frac{1}{2\pi^2}\cos(\pi x^2)\right]_0^1$$

$$= \left[\frac{-x}{\pi^2}\cos(\pi x) + \frac{\sin(\pi x)}{\pi^3} + \frac{1}{2\pi^2}\cos(\pi x^2)\right]_0^1$$