1

$$\lim_{x \to 0} \frac{(x+1)^{x+1} - 1 - x - x^2}{x^3}$$

$$(x+1)^{x+1} = e^{(x+1)\ln(1+x)}$$

$$(x+1)\ln(1+x) = (x+1)\left(x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)\right)$$

$$= x^2 - \frac{x^3}{2} + x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

$$= x + \frac{x^2}{2} - \frac{x^3}{6} + o(x^3)$$

$$\left((x+1)\ln(1+x)\right)^2 = \left((x+1)\left(x - \frac{x^2}{2} + o(x^2)\right)\right)^2$$

$$= \left(x^2 + x - \frac{x^2}{2} + o(x^2)\right)^2 = \left(x + \frac{x^2}{2} + o(x^2)\right)^2$$

$$= x^2 + x^3 + o(x^3)$$

$$\left((x+1)\ln(1+x)\right)^3 = \left((x+1)(x+o(x))\right)^3 =$$

$$= (x+o(x))^3 = x^3 + o(x^3)$$

$$(x+1)^{x+1} = e^{(x+1)\ln(1+x)}$$

$$= 1 + x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{1}{2}(x^2 + x^3) + \frac{1}{3!}x^3 + o(x^3)$$

$$= 1 + x + \frac{x^2}{2} - \cancel{\frac{x^3}{6}} + \frac{x^2}{2} + \frac{x^3}{2} + \cancel{\frac{x^3}{6}} + o(x^3)$$

$$= 1 + x + x^2 + \frac{x^3}{2} + o(x^3)$$

$$(x+1)^{x+1} - 1 - x - x^2 =$$

$$= \cancel{1} + \cancel{x} + \cancel{x^2} + \frac{x^3}{2} - \cancel{1} - \cancel{x} - \cancel{x^2} + o(x^3) =$$

$$= + \frac{x^3}{2} + o(x^3)$$

$$\lim_{x \to 0} \frac{(x+1)^{x+1} - 1 - x - x^2}{x^3} =$$

$$= \lim_{x \to 0} \frac{+ \frac{x^3}{2} + o(x^3)}{x^3} = + \frac{1}{2}$$

2

$$f(x) = x \ln^2 x$$

$$D(f) = \{x > 0\}$$

$$\lim_{x \to 0+} x \ln^2 x = \lim_{x \to 0+} \frac{\ln^2 x}{\frac{1}{x}} \overset{H}{=}$$

$$\overset{H}{=} \lim_{x \to 0+} \frac{2 \cdot \ln x \cdot \frac{1}{x}}{-\frac{1}{x^2}} =$$

$$= \lim_{x \to 0+} 2 \cdot \frac{\ln x}{-\frac{1}{x}} \overset{H}{=} \lim_{x \to 0+} \frac{\frac{1}{x}}{\frac{1}{x^2}} =$$

$$= \lim_{x \to 0+} x = 0$$

$$\lim_{x \to +\infty} x \ln^2 x = +\infty$$

$$f' = \ln^2 x + \cancel{x} \cdot 2 \cdot \ln x \cdot \frac{1}{\cancel{x}} =$$

$$= \ln^2 x + 2\ln x =$$

$$= \ln x \cdot (\ln x + 2)$$

$\ln x > 0$

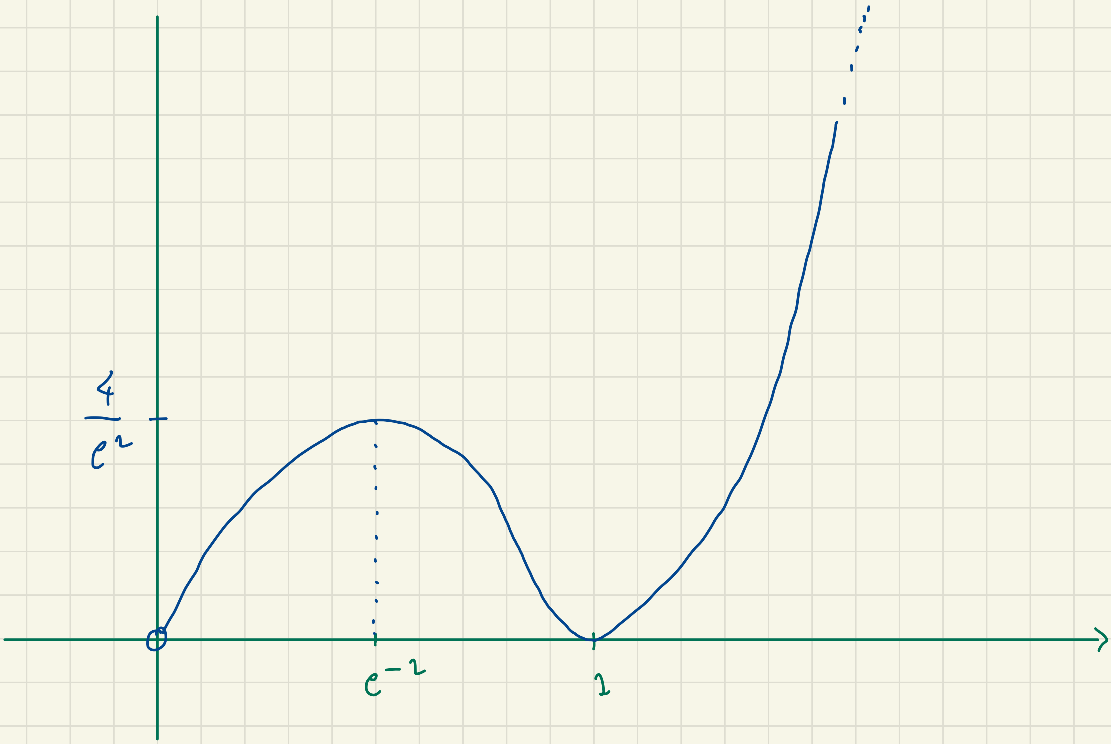
$\ln x + 2 > 0$

$\Updownarrow$

$x > e^{-2}$

	0		e^{-2}		1	
$\ln x > 0$		$-$		$-$		$+$
$\ln x + 2 > 0$		$-$		$+$		$+$
f'		$+$		$-$		$+$
f		$\nearrow$		$\searrow$		$\nearrow$

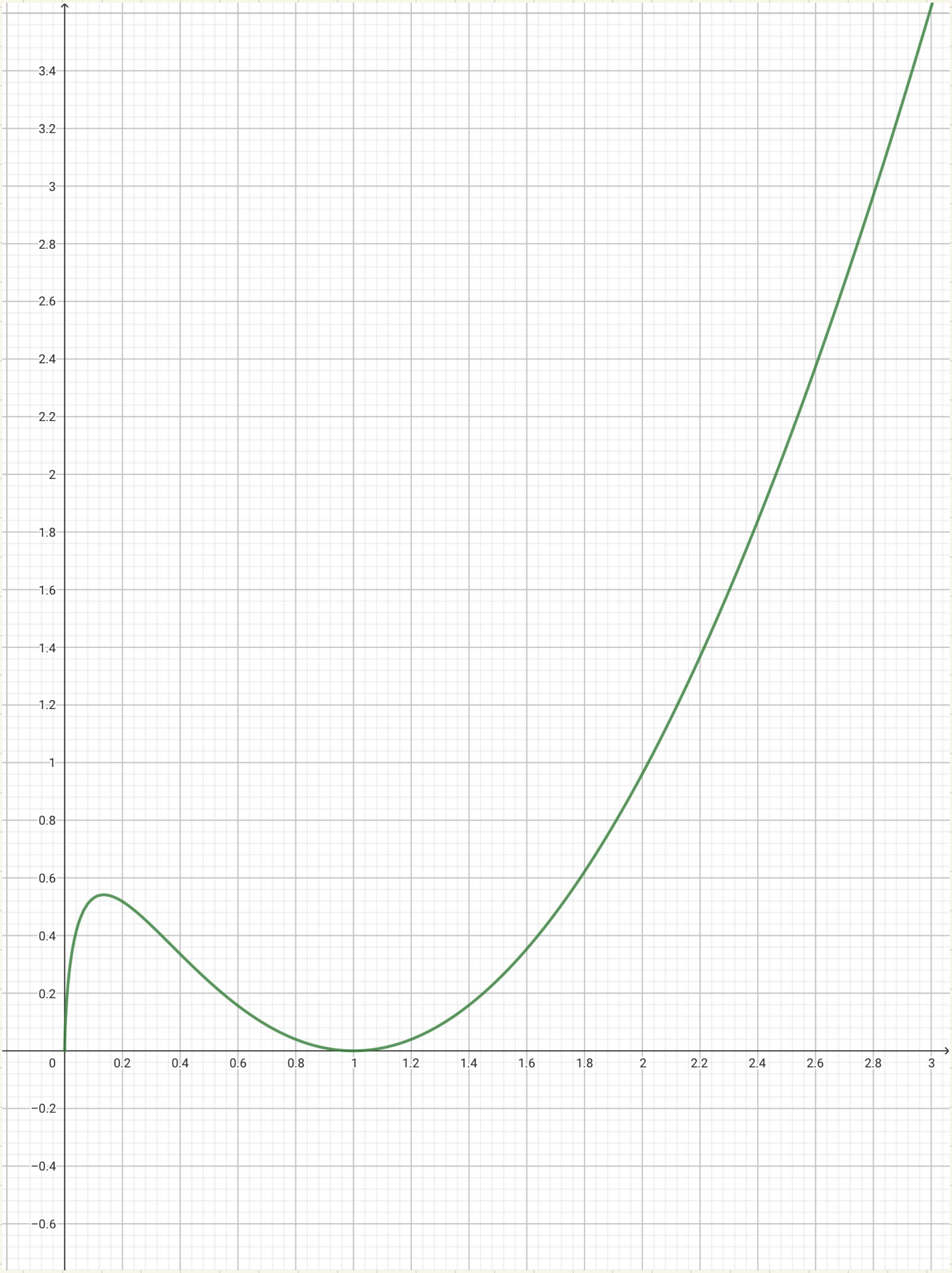
$$f(e^{-2}) = e^{-2} \cdot (-2)^2 = \frac{4}{e^2} \qquad f(1) = 0$$



$\text{Im} f = [0, +\infty[$

$f(x) = K$ ha 1 soluzione se

$K = 0$ o $K > \frac{4}{e^2}$



3.4
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
−0.2
−0.4
−0.6