3-giugno-2022

1

$$f(x) = \arctan \frac{x^2+3}{x-1}$$

$$D(f) = \mathbb{R} \setminus \{1\}$$

$$\lim_{\substack{x \to -\infty \\ (+\infty)}} \arctan \underbrace{\frac{x^2+3}{x-1}}_{\substack{\downarrow \\ -\infty \\ (+\infty)}} = -\frac{\pi}{2} \quad \left(\frac{\pi}{2}\right)$$

$$\lim_{\substack{x \to 1^- \\ (1^+)}} \arctan \underbrace{\frac{x^2+3}{x-1}}_{\substack{\downarrow \\ -\infty \\ (+\infty)}} = -\frac{\pi}{2} \quad \left(\frac{\pi}{2}\right)$$

$$2x^2 - 2x - x^2 - 3$$

$$f'(x) = \frac{1}{1 + \left(\frac{x^2+3}{x-1}\right)^2} \cdot \frac{2x(x-1) - x^2 - 3}{(x-1)^2} =$$

$$= \underbrace{\frac{1}{(x-1)^2 + (x^2+3)^2}}_{>0} \cdot (x^2 - 2x - 3)$$

$$x_{1,2} = \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 4}{2} \left\langle \begin{matrix} -1 \\ 3 \end{matrix} \right.$$

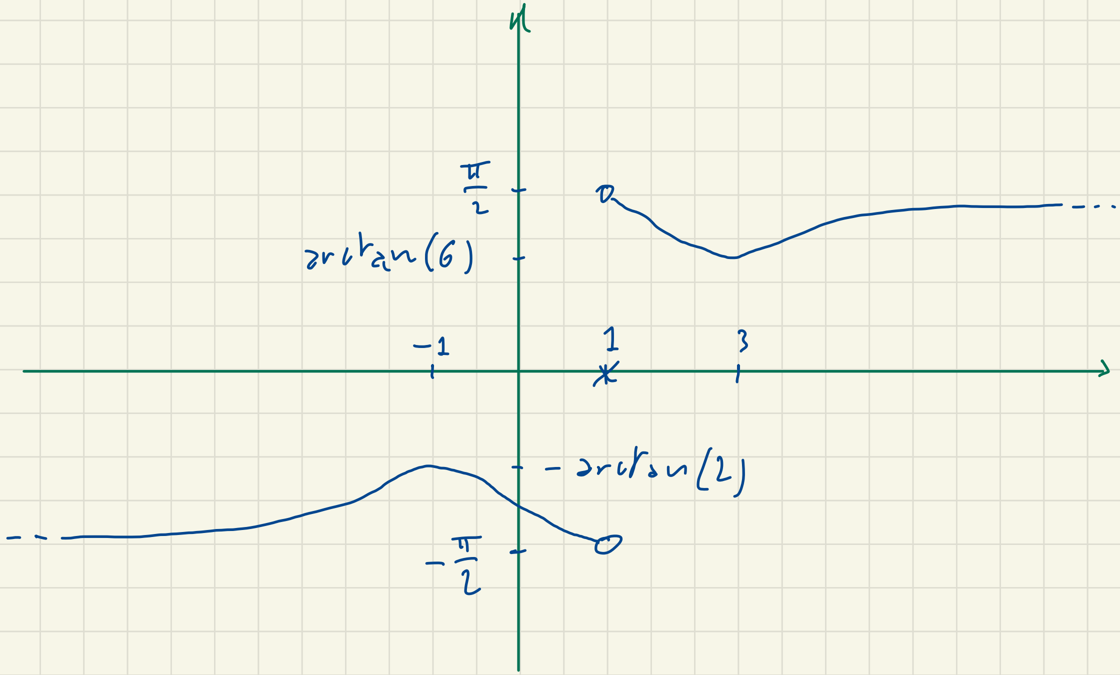
		-1		1		3	
f'	$+$	0	$-$	$\times$	$-$	0	$+$
f	$\nearrow$	0	$\searrow$	$\times$	$\searrow$	0	$\nearrow$

$$f(-1) = \arctan \frac{4}{-2} = \arctan(-2) = -\arctan 2$$

$x = -1$ p. di max rel.

$$f(3) = \arctan \frac{12}{2} = \arctan 6$$

$x = 3$ p. di min. relativo

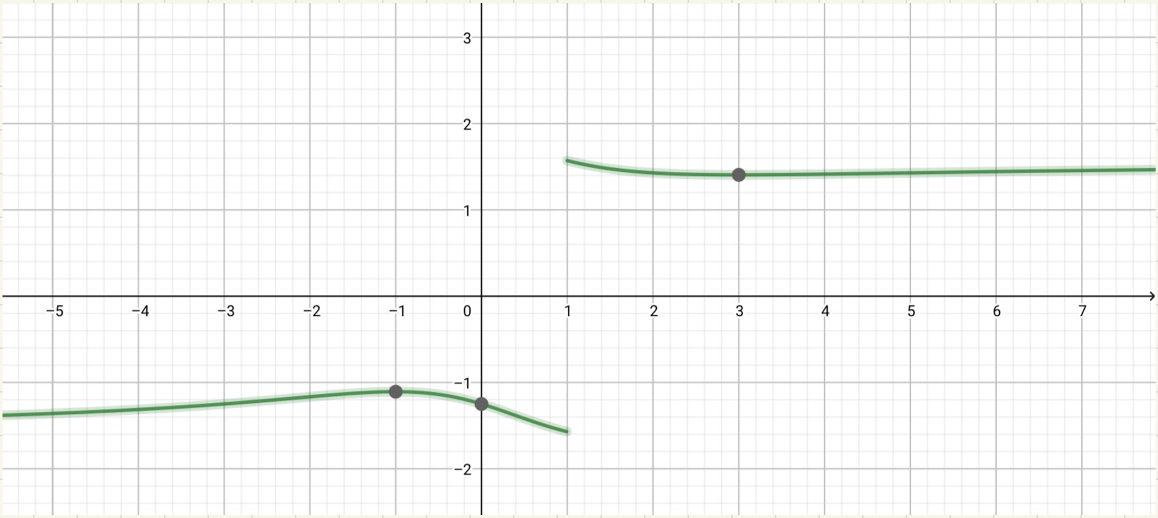


$$\operatorname{Im} f = \left]-\frac{\pi}{2}, \arctan(2)\right] \cup \left[\arctan 6, \frac{\pi}{2}\right[$$

$f(x) = K$ ha 1 soluzione

se $K = \arctan 6$

o $K = -\arctan 2$


3
2
1
−1
−2
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
7

(2)

$$\lim_{x\to 0} \frac{(\ln(1+x))^2 + 2\cos\left(x - \frac{x^2}{2}\right) - 2}{x^4}$$

- $$(\ln(1+x))^2 = \left(x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)\right)^2 =$$

$$= x^2 + \frac{x^4}{4} - x^3 + \frac{2}{3}x^4 + o(x^4)$$

$$= x^2 - x^3 + \frac{11}{12}x^4 + o(x^4)$$

- $$\cos\left(x - \frac{1}{2}x^2\right) = 1 - \frac{1}{2}\left(x - \frac{x^2}{2}\right)^2 +$$

$$+ \frac{1}{4!}x^4 + o(x^4)$$

$$= 1 - \frac{1}{2}x^2 + \frac{1}{2}x^3 \overbrace{- \frac{x^4}{8} + \frac{1}{24}x^4}^{\frac{-2}{24} = -\frac{1}{12}} + o(x^4)$$

$$2\cos\left(x - \frac{x^2}{2}\right) = 2 - x^2 + x^3 - \frac{x^4}{6} + o(x^4)$$

$$\frac{(\ln(1+x))^2 + 2\cos\left(x - \frac{x^2}{2}\right) - 2}{x^4} =$$

$$= \frac{\cancel{x^2} - \cancel{x^3} + \frac{11}{12}x^4 + \cancel{2} - \cancel{x^2} + \cancel{x^3} - \frac{x^4}{6} - \cancel{2} + o(x^4)}{x^4}$$

$$= \frac{\frac{9}{12}x^4 + o(x^4)}{x^4} = \frac{3}{4} + \frac{o(x^4)}{x^4} \longrightarrow \frac{3}{4}$$