1

$$\lim_{x \to -2^+} f(x) = -\pi$$

$\forall \varepsilon > 0 : \exists \delta = \delta(-2, \varepsilon) > 0 : \forall x \in \mathbb{R} :$

$-2 < x < -2 + \delta \implies |f(x) + \pi| < \varepsilon$

2

Hp : $f : [a, b] \longrightarrow \mathbb{R}$ continua

$c \in [a, b[$

$$F_c(x) = \int_c^x f(t)\, dt$$

Th : $F_c \in C^1$

$\forall x \in [a, b] :$

$$F_c'(x) = f(x)$$

(3) $f : D(f) \longrightarrow \mathbb{R} \qquad f(x) = x\, e^{-\ln^2(x)}$

$$D(f) = \{ x > 0 \}$$

$$\lim_{x \to 0^+} \underbrace{x}_{\to 0^+}\, e^{\overbrace{-\ln^2(x)}^{\to -\infty}} = 0$$

$$\lim_{x \to +\infty} x\, e^{-\ln^2 x} = \lim_{x \to +\infty} e^{\overbrace{\ln x - \ln^2 x}^{\to -\infty}} = 0$$

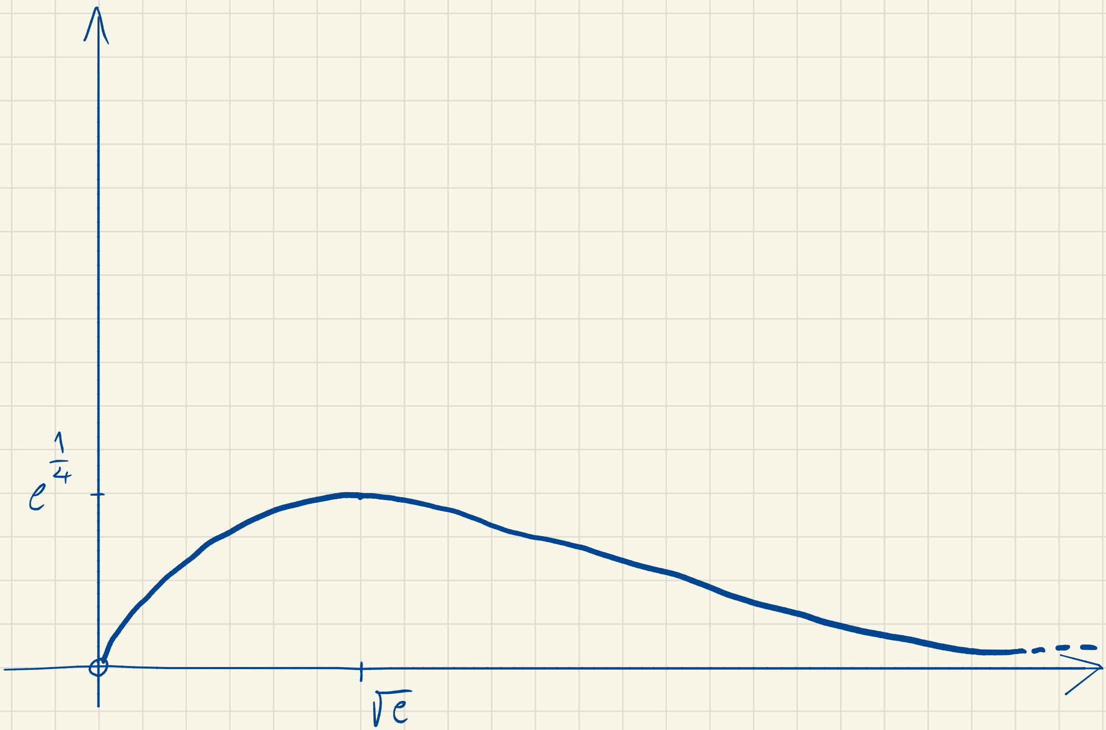
$$f'(x) = e^{-\ln^2 x} + x \cdot \left(-2\ln(x)\right) \cdot \frac{1}{x}\, e^{-\ln^2(x)}$$

$$= \underbrace{e^{-\ln^2 x}}_{>0} \left(1 - 2\ln(x)\right)$$

$$1 - 2\ln(x) > 0 \iff \ln(x) < \frac{1}{2} \iff 0 < x < \sqrt{e}$$

	0		$\sqrt{e}$	
f'	0	$+$	0	$-$
f	0	$\nearrow$	$\sqrt{e}$	$\searrow$

$$f(\sqrt{e}) = \sqrt{e}\, e^{-\ln^2(\sqrt{e})} = \sqrt{e} \cdot e^{-\frac{1}{4}} = e^{\frac{1}{4}}$$



$$\operatorname{Im} f = \left] 0, e^{\frac{1}{4}} \right]$$

$f(x) = \lambda$ ha 2 soluzioni

se $0 < \lambda < e^{\frac{1}{4}}$

4

$$e^{x^2} = 1 + x^2 + \frac{(x^2)^2}{2!} + o(x^4)$$

$$= 1 + x^2 + \frac{x^4}{2} + o(x^4)$$

$$\sqrt[4]{1-4x^2+x^4} = \left(1 + (-4x^2+x^4)\right)^{\frac{1}{4}} =$$

$$= 1 + \frac{1}{4}\left(-4x^2+x^4\right) + \frac{\frac{1}{4}\left(\frac{1}{4}-1\right)}{2!}\left(-4x^2+x^4\right)^2 + o(x^4)$$

$$= 1 - x^2 + \frac{x^4}{4} - \frac{3}{\cancel{32}_{2}} \cdot \cancel{16}^{1} x^4 + o(x^4) =$$

$$= 1 - x^2 - \frac{5}{4}x^4 + o(x^4)$$

$$\lim_{x\to 0} \frac{\sqrt[4]{1-4x^2+x^4} + e^{x^2} - 2}{x^4} =$$

$$= \lim_{x\to 0} \frac{\cancel{1} + \cancel{x^2} + \frac{x^4}{2} + \cancel{1} - \cancel{x^2} - \frac{5}{4}x^4 - \cancel{2} + o(x^4)}{x^4}$$

$$= \lim_{x\to 0} \frac{-\frac{3}{4}x^4 + o(x^4)}{x^4} = -\frac{3}{4}$$

(5) $\displaystyle\int \frac{x^3+4x^2}{x^2+3x+2}\,dx$

$$
\begin{array}{rrrr|l}
x^3 & +4x^2 & +0x & +0 & x^2+3x+2 \\ \cline{5-5}
-x^3 & -3x^2 & -2x & & x+1 \\ \cline{1-3}
// & x^2 & -2x & +0 & \\
 & -x^2 & -3x & -2 & \\ \cline{2-4}
 & // & -5x & -2 &
\end{array}
$$

$$x^2+3x+2=0$$

$$\Delta = 9-8=1 \qquad x_{1,2}=\frac{-3\pm 1}{2} \left< \begin{array}{l} -2 \\ -1 \end{array} \right.$$

$$\frac{-5x-2}{x^2+3x+2} = \frac{A}{x+2} + \frac{B}{x+1} =$$

$$= \frac{(A+B)x + A+2B}{x^2+3x+2}$$

$$\begin{cases} A+B=-5 \\ A+2B=-2 \end{cases} \quad \begin{cases} A=-B-5 \\ -B-5+2B=-2 \end{cases} \quad \begin{cases} A=-8 \\ B=3 \end{cases}$$

5

$$\frac{x^3+4x^2}{x^2+3x+2} = x+1-\frac{8}{x+2}+\frac{3}{x+1}$$

$$\int_0^1 \frac{x^3+4x^2}{x^2+3x+2}\,dx = \int_0^1 \left(x+1-\frac{8}{x+2}+\frac{3}{x+1}\right)dx$$

$$= \left[\frac{x^2}{2}+x-8\ln|x+2|+3\ln|x+1|\right]_0^1$$

$$= \frac{1}{2}+1-8\ln 3+3\ln 2+8\ln 2$$

$$= \frac{3}{2}+11\ln 2-8\ln 3$$

⑥ $$f(x,y) = x^4 + 4x^2y^2 - 2x^2 + 8y^2 - 2$$

$$\begin{cases} f_x = 4x^3 + 8xy^2 - 4x = 0 \\ f_y = 8x^2y + 16y = 0 \end{cases}$$

$$x(x^2-1) \begin{cases} \nearrow x = 0 \\ \searrow x = \pm 1 \end{cases}$$

$$\begin{cases} x^3 + 2xy^2 - x = 0 \\ y(x^2+2) = 0 \end{cases} \longrightarrow \begin{cases} \overset{\shortparallel}{x^3 - x} = 0 \\ y = 0 \end{cases}$$

$A(0,0) \quad B(-1,0) \quad C(1,0)$

$$H_f = \begin{pmatrix} 12x^2 + 8y^2 - 4 & 16xy \\ 16xy & 8x^2 + 16 \end{pmatrix}$$

$$H_f(0,0) = \begin{pmatrix} -4 & 0 \\ 0 & 16 \end{pmatrix} \quad \text{SELLA}$$

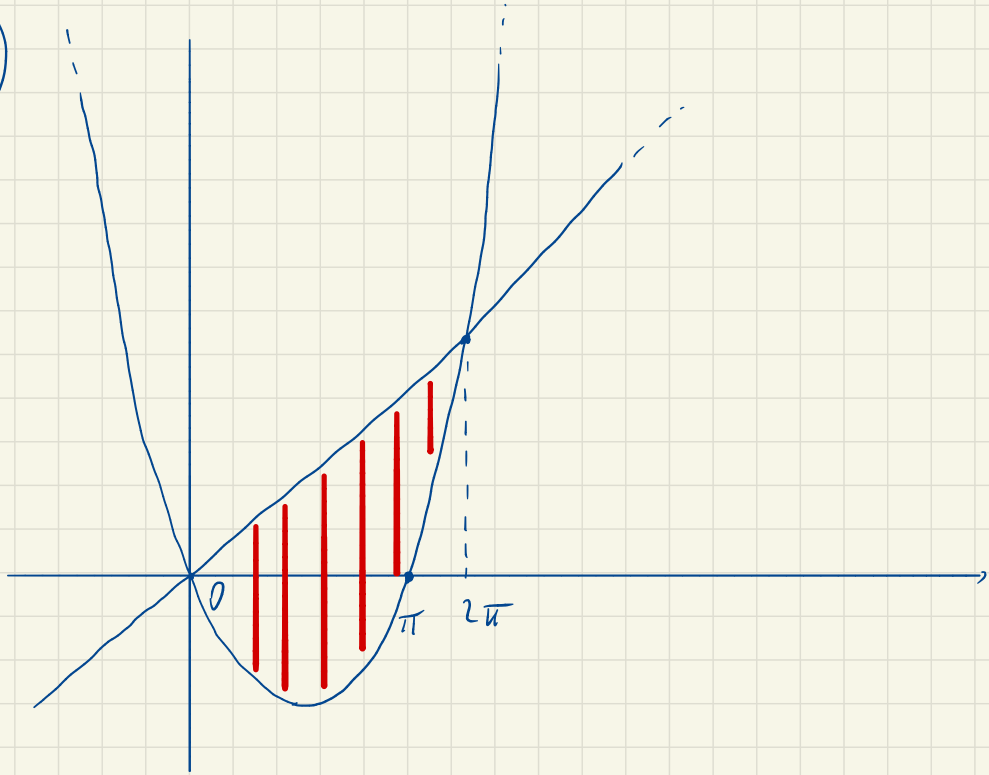
$$H_f(-1,0) = \begin{pmatrix} 8 & 0 \\ 0 & 24 \end{pmatrix} \quad \text{P. DI MIN.}$$

$$H_f(1,0) = \begin{pmatrix} 8 & 0 \\ 0 & 24 \end{pmatrix} \quad \text{P. DI MIN}$$

$$\frac{\partial f}{\partial v}\left(0, -\frac{1}{2}\right) = \left< \nabla f\left(0, -\frac{1}{2}\right), \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix} \right> =$$

$$= \left< \begin{pmatrix} 0 \\ -8 \end{pmatrix}, \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix} \right> = -\frac{24}{5}$$

⑦



$$\begin{cases} y = x^2 - \pi x \\ y = \pi x \end{cases}$$

$$x^2 - \pi x = \pi x \implies x^2 - 2\pi x = 0 \quad \begin{cases} x = 0 \\ x = 2\pi \end{cases}$$

$$\iint_A \sin x \, dx\, dy =$$

$$= \int_0^{2\pi} \left(\int_{x^2-\pi x}^{\pi x} \sin x \cdot dy \right) dx =$$

$$= \int_0^{2\pi} \sin x \cdot \left(\pi x - x^2 + \pi x\right) dx =$$

$$= \int_0^{2\pi} \sin x \cdot \left(2\pi x - x^2\right) dx$$

integrando per parti

$$= \underbrace{\left[-\cos x \cdot \left(2\pi x - x^2\right)\right]_0^{2\pi}}_{0} + \int_0^{2\pi} \cos x \cdot \left(2\pi - 2x\right) dx$$

$$= \underbrace{\left[\sin x \cdot \left(2\pi - 2x\right)\right]_0^{2\pi}}_{0} + \int_0^{2\pi} \sin x \cdot 2 \, dx$$

$$= \left[-2\cos x\right]_0^{2\pi} = 0$$