28. Maggio. 2021

$$f(x) = \left(x^3 + 3x^2 - 3x - 3\right) e^{-x}$$

$$D(f) = \mathbb{R}$$

$$\lim_{x \to -\infty} \underbrace{\left(x^3 + 3x^2 - 3x - 3\right)}_{\substack{\downarrow \\ -\infty}} \overset{\substack{+\infty \\ \uparrow}}{e^{-x}} = -\infty$$

$$\lim_{x \to +\infty} \underbrace{\left(x^3 + 3x^2 - 3x - 3\right)} \overset{\substack{0 \\ \uparrow}}{e^{-x}} = 0^+$$

$$f'(x) = \left(\cancel{3x^2} + 6x \cancel{- 3}\right) e^{-x} + \left(-x^3 \cancel{- 3x^2} + 3x \cancel{+ 3}\right) e^{-x}$$

$$= e^{-x} \left(-x^3 + 9x\right)$$

$$= \underset{\substack{\vee \\ 0}}{e^{-x}} \cdot x \cdot \left(-x^2 + 9\right)$$

		-3		0		3	
x	$-$		$-$	0	$+$		$+$
$-x^2+9$	$-$	-3	$+$		$+$	3	$-$
f'	$+$		$-$		$+$		$-$
f	↗	-3	↘	0	↗	3	↘

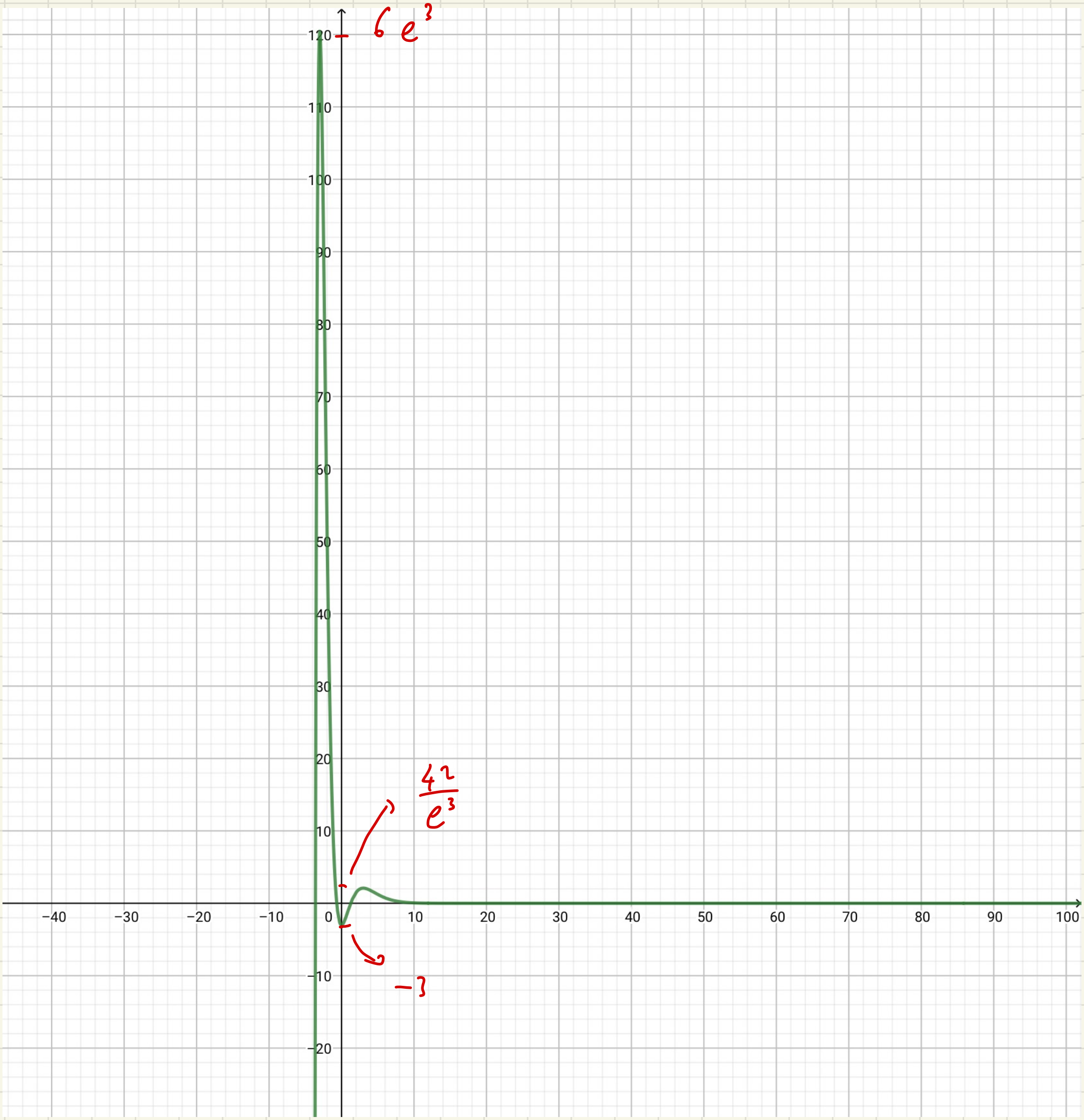
$$f(x) = (x^3 + 3x^2 - 3x - 3)\,e^{-x}$$

$54 - 12$

$f(-3) = 6e^3$ MAX

$f(0) = -3$ MIN

$f(3) = 42\,e^{-3} = \dfrac{42}{e^3}$ MAX



$\operatorname{Im} f = \left]-\infty, 6e^3\right]$

$f(x) = \lambda$ ha 2 soluz se $0 < \lambda < \dfrac{42}{e^3}$

(2)

$$\lim_{x \to 0} \frac{\sin(\sin x) - \sin x + \sqrt{1 + x^3} - 1}{x^3}$$

$$\sin x = x - \frac{x^3}{6} + o(x^3)$$

$$\sin(\sin x) = \sin x - \frac{1}{6} \sin^3 x + o(x^3)$$

$$= x - \frac{x^3}{6} - \frac{1}{6}\left(x + o(x)\right)^3 + o(x^3) =$$

$$= x - \frac{x^3}{6} - \frac{x^3}{6} + o(x^3) =$$

$$= x - \frac{x^3}{3} + o(x^3)$$

$$\sqrt{1 + x^3} = 1 + \frac{1}{2} x^3 + o(x^3)$$

$$\sin(\sin x) - \sin x + \sqrt{1+x^3} - 1 =$$

$$= \cancel{x} - \frac{x^3}{3} \cancel{- x} + \frac{x^3}{6} + o(x^3)$$

$$+ \cancel{1} + \frac{x^3}{2} \cancel{- 1}$$

$$= \left(-\frac{1}{6} + \frac{1}{2}\right) x^3 + o(x^3) =$$

$$= \frac{x^3}{3} + o(x^3)$$

$$\lim_{x \to 0} \frac{\sin(\sin x) - \sin x + \sqrt{1+x^3} - 1}{x^3} =$$

$$= \lim_{x \to 0} \frac{\frac{x^3}{3} + o(x^3)}{x^3} = \frac{1}{3}$$