1

$$f(x) = x\, e^{-\frac{1}{x+2}}$$

$$D(f) = \mathbb{R} \setminus \{-2\}$$

$$\lim_{x \to +\infty} \underset{\substack{\downarrow \\ +\infty}}{x}\ \underbrace{e^{-\frac{1}{x+2}}}_{\substack{\downarrow \\ 1}} = +\infty$$

$$\lim_{x \to -\infty} \underset{\substack{\downarrow \\ -\infty}}{x}\ \underbrace{e^{-\frac{1}{x+2}}}_{\substack{\downarrow \\ 1}} = -\infty$$

$$\lim_{x \to -2^-} \underset{\substack{\downarrow \\ -2}}{x}\ \underbrace{e^{-\frac{1}{x+2}}}_{\substack{\downarrow \\ +\infty}} = -\infty$$

$$\lim_{x \to -2^+} \underset{\substack{\downarrow \\ -2}}{x}\ \underbrace{e^{-\frac{1}{x+2}}}_{\substack{\downarrow \\ 0}} = 0$$

$$f'(x) = e^{-\frac{1}{2+x}} + x\, e^{-\frac{1}{2+x}} \cdot \frac{1}{(x+2)^2} =$$

$$= e^{-\frac{1}{2+x}} \cdot \frac{x^2+5x+4}{(x+2)^2}$$

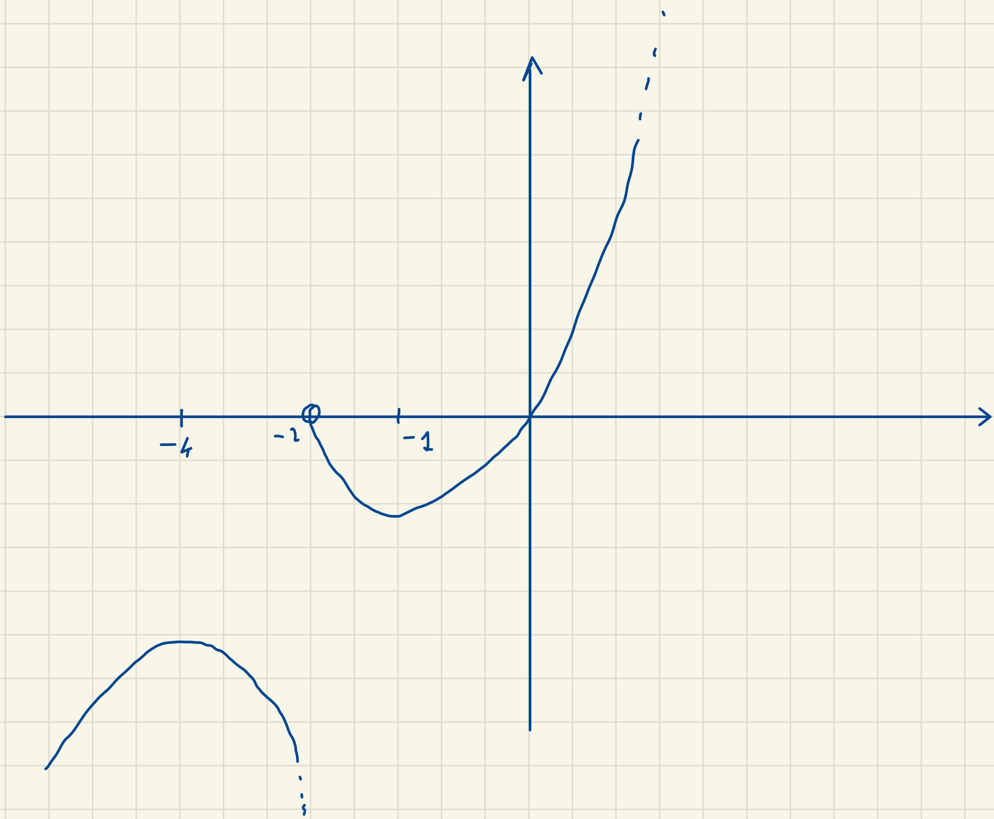
$$x_{1,2} = \begin{cases} -1 \\ -4 \end{cases}$$

f': $+$ $\quad -4 \quad$ $-$ $\quad -2 \quad$ $-$ $\quad -1 \quad$ $+$

f: $\nearrow$ $\quad -4 \quad$ $\searrow$ $\quad -2 \quad$ $\searrow$ $\quad -1 \quad$ $\nearrow$

$$f(-1) = -\frac{1}{e}$$

$$f(-4) = -4\sqrt{e}$$



$$\operatorname{Im} f = \left]-\infty, -4\sqrt{e}\right] \cup \left[-\frac{1}{e}, +\infty\right[$$

$f(x) = \lambda$ L solut $\rightarrow$ $\lambda < -4\sqrt{e}$

$\vee$

$\lambda > -\frac{1}{e}$

②

$$\lim_{x \to +\infty} \frac{(x+1)^{x+1} - e^{x^2} - x}{x^3}$$

$$(x+1)^{x+1} = e^{(x+1)\ln(1+x)}$$

$$(x+1)\ln(x+1) = (x+1)\left(x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)\right)$$

$$= x + \frac{x^2}{2} - \frac{x^3}{6} + o(x^3)$$

$$e^{(x+1)\ln(1+x)} = 1 + x + \frac{x^2}{2} - \frac{x^3}{6} +$$

$$+ \frac{1}{2}\left(x + \frac{x^2}{2} - \frac{x^3}{6}\right)^2 +$$

$$+ \frac{1}{6}x^3 + o(x^3)$$

$$= 1 + x + x^2 + \frac{x^3}{2} + o(x^3)$$

$$e^{x^2} = 1 + x^2 + o(x^3)$$

$$\lim_{x \to +\infty} \frac{(x+1)^{x+1} - e^{x^2} - x}{x^3} = \frac{1}{2}$$