Correzione scritto 6 luglio2020

(1)

$$f(x) = \frac{e^{-x}}{4x^2+3}$$

$$D(f) = \mathbb{R}$$

$$\lim_{x \to -\infty} \frac{e^{-x}}{4x^2+3} \overset{H}{=} \lim_{x \to -\infty} \frac{-e^{-x}}{8x} \overset{H}{=}$$

$$\overset{H}{=} \lim_{x \to -\infty} \frac{e^{-x}}{8} = +\infty$$

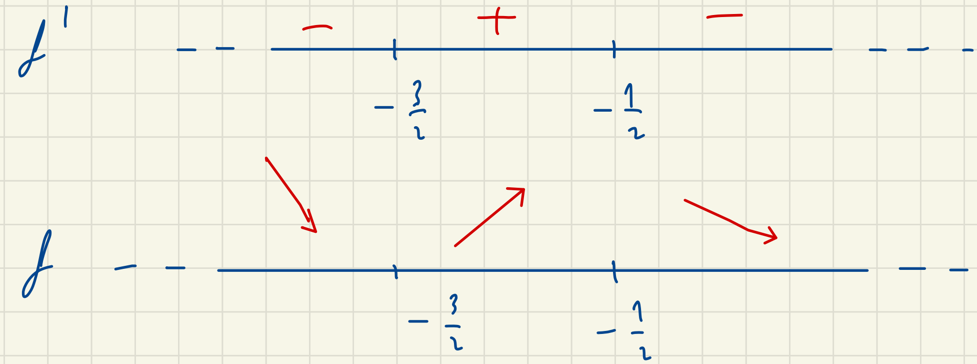
$$\lim_{x \to +\infty} \frac{e^{-x}}{4x^2+3} = 0$$

$$f'(x) = -\frac{1}{4x^2+3} e^{-x} - \frac{8x}{(4x^2+3)^2} e^{-x} =$$

$$= -\frac{e^{-x}}{(4x^2+3)^2} \left(4x^2+3+8x\right)$$

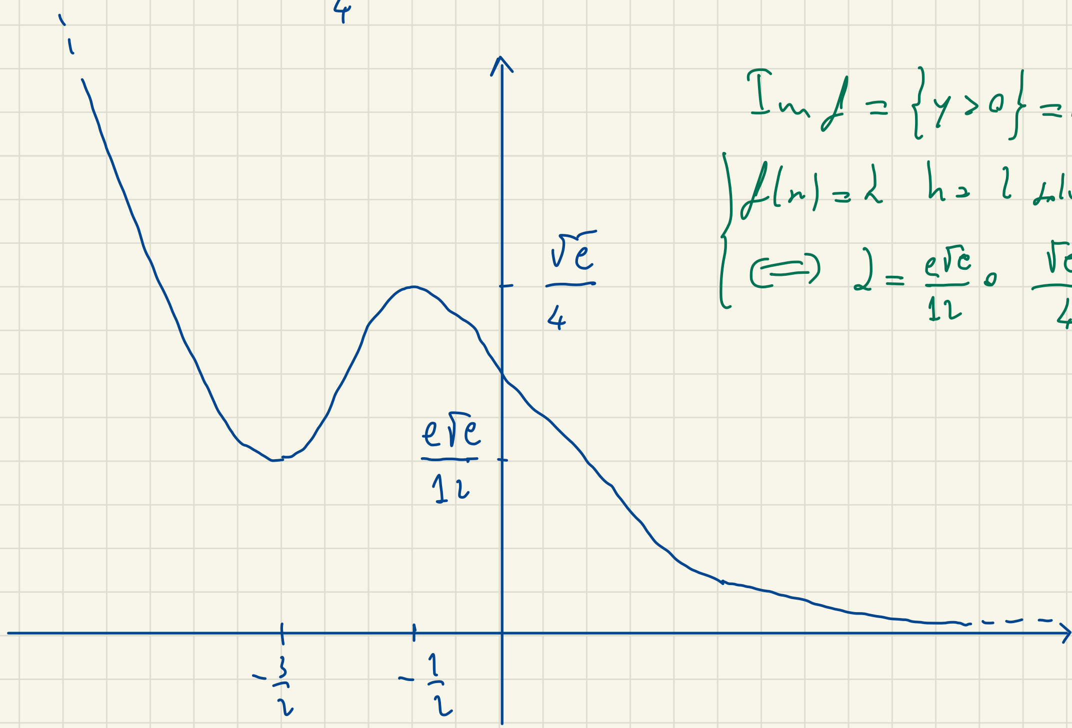
$$\Delta = 64 - 48 = 16$$

$$x_{1,2} = \frac{-8 \pm 4}{8} = \begin{cases} -\frac{12}{8} = -\frac{3}{2} \\ -\frac{4}{8} = -\frac{1}{2} \end{cases}$$



$$f\left(-\frac{3}{2}\right) = \frac{e^{+\frac{3}{2}}}{4\cdot\frac{9}{4}+3} = \frac{e^{+\frac{3}{2}}}{12} = \frac{e\sqrt{e}}{12} \qquad \text{min. rel.}$$

$$f\left(-\frac{1}{2}\right) = \frac{e^{\frac{1}{2}}}{4\cdot\frac{1}{4}+3} = \frac{\sqrt{e}}{4} = \frac{\sqrt{e}}{4} \qquad \text{max. rel.}$$



$\mathrm{Im}\, f = \{y>0\} = \mathbb{R}^+$

$$\begin{cases} f(x) = \lambda \text{ ha 2 soluz.} \\ \iff \lambda = \frac{e\sqrt{e}}{12} \text{ o } \frac{\sqrt{e}}{4} \end{cases}$$

2

$$\lim_{x \to 0} \frac{\cos(\sin x) + \frac{1}{2}\ln(1 + x\sin x) - 1}{x^4}$$

$$\sin x = x + o(x) \qquad x\sin x = x^2 + o(x^2)$$

$$\cos(\sin x) = 1 - \frac{\sin^2 x}{2!} + \frac{\sin^4 x}{4!} + o(x^4)$$

$$= 1 - \frac{\left(x - \frac{x^3}{3!} + o(x^3)\right)^2}{2} + \frac{(x + o(x))^4}{4!} + o(x^4)$$

$$= 1 - \frac{1}{2}x^2 + \frac{1}{6}x^4 + \frac{1}{24}x^4 + o(x^4)$$

$$= 1 - \frac{x^2}{2} + \frac{5}{24}x^4 + o(x^4)$$

$$\ln(1 + x\sin x) = (x\sin x) - \frac{(x\sin x)^2}{2} + o(x^4)$$

$$= x\left(x - \frac{x^3}{6} + o(x^3)\right) - \frac{(x(x + o(x)))^2}{2} + o(x^4)$$

$$= x\left(x - \frac{x^3}{6} + o(x^3)\right) - \frac{(x^2 + o(x^2))^2}{2} + o(x^4)$$

$$= x^2 - \frac{x^4}{6} - \frac{x^4}{2} + o(x^4)$$

$$= x^2 - \frac{2}{3}x^4 + o(x^4)$$

$$\cos(\sin x) - \frac{1}{2}\ln(1 + x\sin x) - 1 =$$

$$= 1 - \frac{x^2}{2} + \frac{5}{24}x^4 + o(x^4)$$

$$+ \frac{1}{2}\left(x^2 - \frac{2}{3}x^4 + o(x^4)\right) - 1$$

$$= \cancel{1} - \cancel{\frac{x^2}{2}} + \frac{5}{24}x^4 + \cancel{\frac{x^2}{2}} - \frac{x^4}{3} + o(x^4) - \cancel{1}$$

$$= -\frac{1}{8}x^4 + o(x^4)$$

$$\lim_{x \to 0} \frac{\cos(\sin x) + \frac{1}{2}\ln(1 + x\sin x) - 1}{x^4} = -\frac{1}{8}$$