Compito Informatica
1 Giugno 2020
I Parte

I VERSIONE

①

$$f(x) = e^{\frac{2x^2+8}{x+2}}(2x+1)$$

$$D(f) = \mathbb{R} \setminus \{-2\}$$

$$\lim_{x\to+\infty} e^{\overbrace{\frac{2x^2+8}{x+2}}^{\to +\infty}} \overbrace{(2x+1)}^{\to +\infty} = +\infty$$

$$\lim_{x\to-\infty} \underbrace{e^{\overbrace{\frac{2x^2+8}{x+2}}^{\to -\infty}}}_{0} \underbrace{(2x+1)}_{-\infty} = 0$$

l'esponenziale decresce molto più velocemente di un polinomio

(oppure, via il T. di De L'Hôpital)

$$\left(= \lim_{x\to-\infty} \frac{2x+1}{e^{-\frac{2x^2+8}{x+2}}} \overset{H}{=} \lim_{x\to-\infty} \frac{2}{\underbrace{-e^{-\frac{2x^2+8}{x+2}} \cdot \frac{2x^2+8x-8}{(x+2)^2}}_{\to -\infty}} = 0 \right)$$

$$\lim_{x \to -2^-} e^{\overset{\to -\infty}{\frac{2x^2+8}{x+2}}} \overset{\to -3}{(2x+1)} = 0^-$$

$$\lim_{x \to -2^+} e^{\overset{\to +\infty}{\frac{2x^2+8}{x+2}}} \overset{\to -3}{(2x+1)} = -\infty$$

$$f(x) = e^{\frac{2x^2+8}{x+2}} (2x+1)$$

$$f' = e^{\frac{2x^2+8}{x+2}} \frac{4x(x+2) - (2x^2+8)}{(x+2)^2} (2x+1) +$$

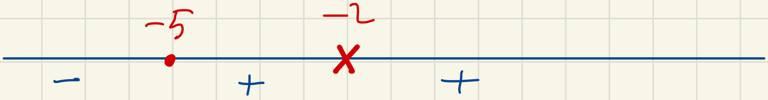
$$+ e^{\frac{2x^2+8}{x+2}} \cdot 2$$

$$= e^{\frac{2x^2+8}{x+2}} \cdot \left(\frac{(\overset{2x^2}{\cancel{4x^2}} + 8x - \cancel{2x^2} - 8)(2x+1)}{(x+2)^2} + 2 \right)$$

$$= e^{\frac{2x^2+8}{x+2}} \frac{4x^3 + 2x^2 + 16x^2 + \cancel{8x} - \cancel{16x} \cancel{- 8} + 2x^2 + \cancel{8x} \cancel{+ 8}}{(x+2)^2}$$

$$= e^{\frac{2x^2+8}{x+2}} \frac{4x^3 + 20x^2}{(x+2)^2} = \underbrace{e^{\frac{2x^2+8}{x+2}} \cdot \frac{4x^2}{(x+2)^2}}_{\geq 0} (x+5)$$

$f'(x)$:



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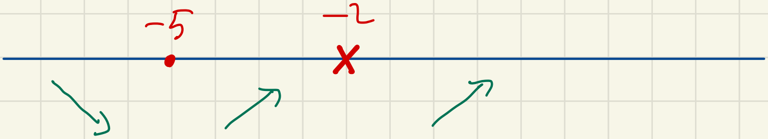
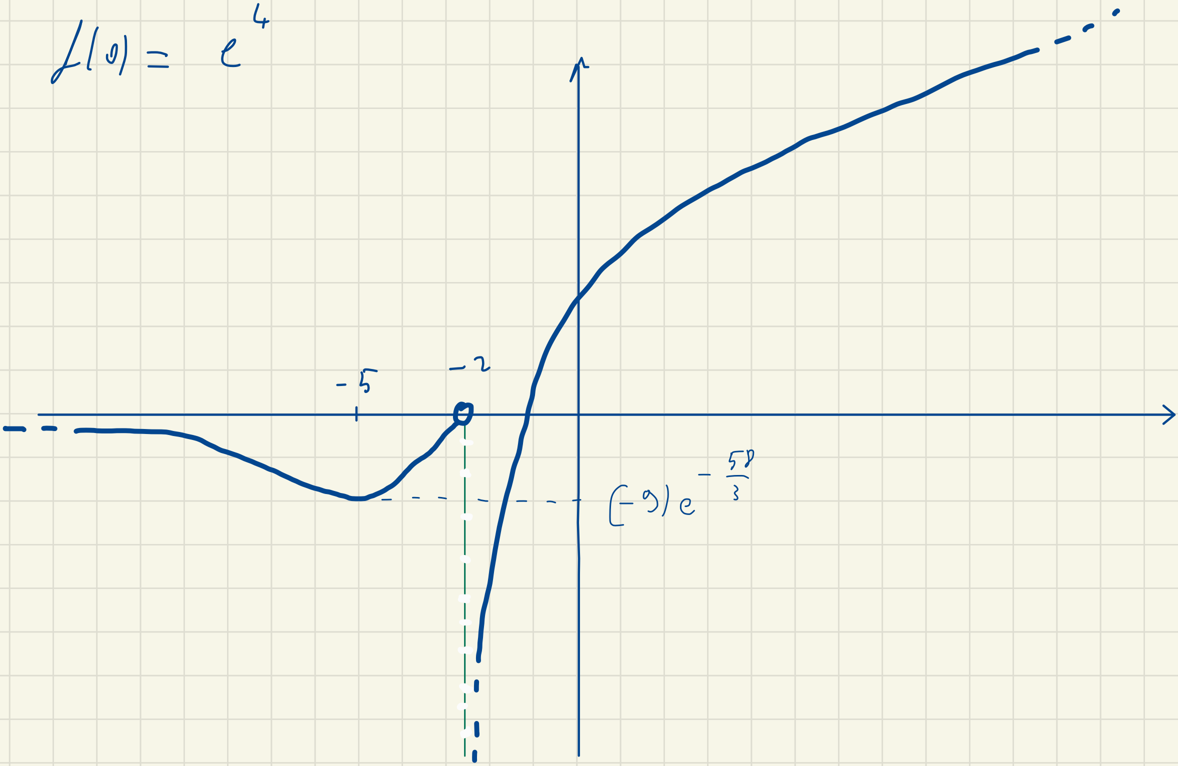


$x = -5$ P. DI MIN. LOCALE

$$f(-5) = e^{\frac{2x^2+8}{x+2}} (2x+1) \Big|_{x=-5} = -9\, e^{-\frac{58}{3}}$$

$$f(0) = e^4$$



$\mathrm{Im}\, f = \mathbb{R}$

$f(x) = \lambda$ ha tre soluzioni (distinte)

se $-9\, e^{-\frac{58}{3}} < \lambda < 0$.

② $$\lim_{x \to 0} \frac{\ln(1+x\cos x) - \sin x - \cos x + 1}{x^4}$$

$$x \cos x = x\left(1 - \frac{x^2}{2} + o(x^3)\right) =$$

$$= x - \frac{x^3}{2} + o(x^4)$$

$$\ln(1+x\cos x) = (x\cos x) - \frac{1}{2}(x\cos x)^2 + \frac{1}{3}(x\cos x)^3$$

$$- \frac{1}{4}(x\cos x)^4 + o(x^4)$$

$$= \left(x - \frac{x^3}{2} + o(x^4)\right) - \frac{1}{2}\left(x - \frac{x^3}{2} + o(x^4)\right)^2 +$$

$$+ \frac{1}{3}\left(x + o(x^2)\right)^3 - \frac{1}{4}\left(x + o(x^2)\right)^4 + o(x^4)$$

$$= \underset{\smile}{x} - \underset{\sim}{\frac{x^3}{2}} - \underset{\wedge}{\frac{x^2}{2}} + \underset{+}{\frac{x^4}{2}} + \underset{\sim}{\frac{x^3}{3}} - \underset{+}{\frac{x^4}{4}} + o(x^4)$$

$$= x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{1}{4}x^4 + o(x^4)$$

$$\sin x = x - \frac{x^3}{6} + o(x^4)$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4)$$

$$\ln(1+x\cos x)-\sin x-\cos x+1=$$

$$=\cancel{x}-\cancel{\frac{x^2}{2}}-\cancel{\frac{x^3}{6}}+\frac{1}{4}x^4-\cancel{x}+\cancel{\frac{x^3}{6}}\cancel{-1}+\cancel{\frac{x^2}{2}}-$$

$$-\frac{x^4}{24}+\cancel{1}+o(x^4)$$

$$=\frac{5}{24}x^4+o(x^4)$$

$$\lim_{x\to 0}\frac{\ln(1+x\cos x)-\sin x-\cos x+1}{x^4}=$$

$$=\lim_{x\to 0}\frac{\frac{5}{4}x^4+o(x^4)}{x^4}=\frac{5}{24}$$

II VERSIONE

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$$f(x) = e^{\frac{2x^2-4x}{x+2}}(2x+1)$$

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$$= e^{\frac{2x^2-4x}{x+2}} \cdot \left(\frac{(\overset{2x^2}{\cancel{4x^2}} + 8x \cancel{-4x} - 8 - \cancel{2x^2} + \cancel{4x}}{(x+2)^2}(2x+1) + 2 \right)$$

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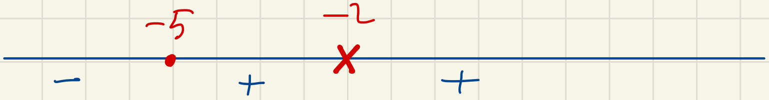
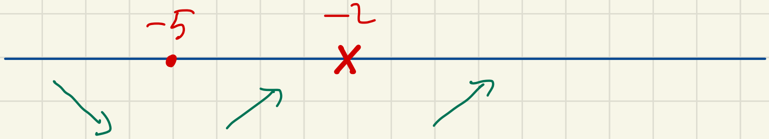


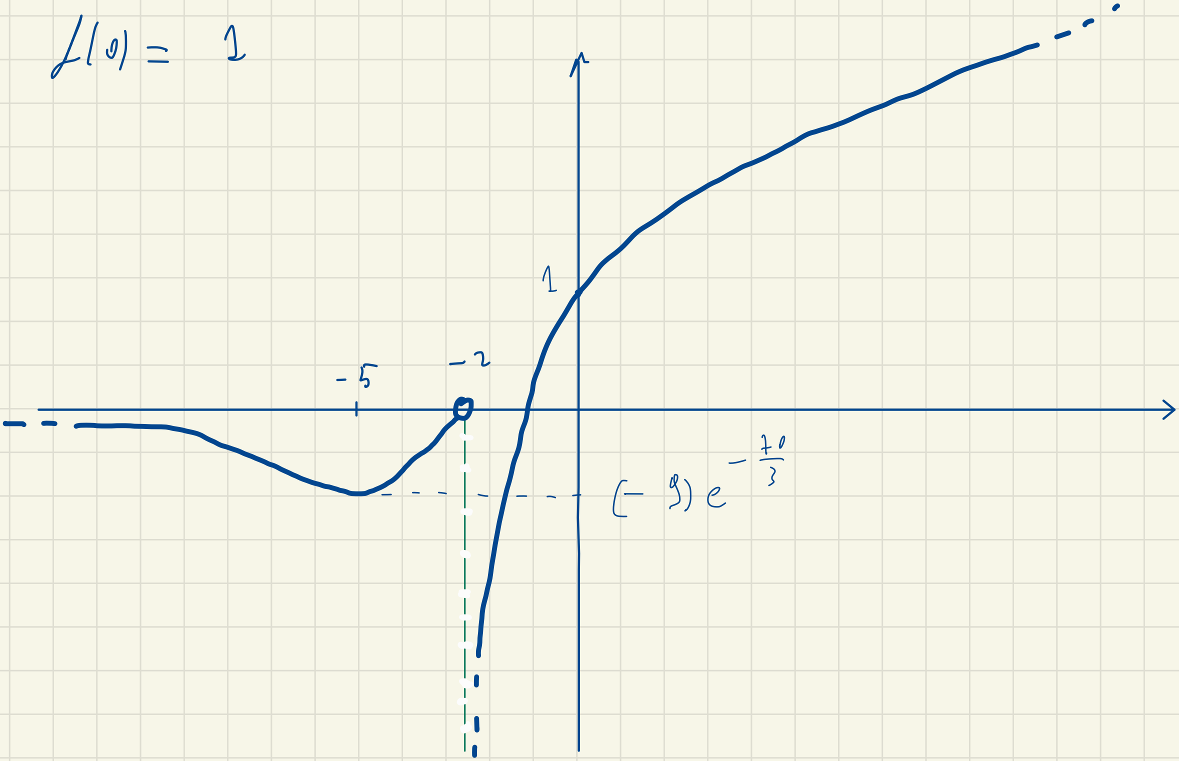
$f(x)$:



$x = -5$ P. DI MIN. LOCALE

$$f(-5) = e^{\frac{2x^2-4x}{x+2}} (2x+1) \Big|_{x=-5} = -9 \cdot e^{-\frac{70}{3}}$$

$f(0) = 1$



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$$= x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{1}{4}x^4 + o(x^4)$$

$$\sin x = x - \frac{x^3}{6} + o(x^4)$$

$$\sin(x^2) = x^2 + o(x^4)$$

$$\ln(1+x\cos x) - \sin x + \frac{1}{2}\sin(x^2) =$$

$$= \cancel{x} - \cancel{\frac{x^2}{2}} - \cancel{\frac{x^3}{6}} + \frac{1}{4}x^4 - \cancel{x} + \cancel{\frac{x^3}{6}} + \cancel{\frac{x^2}{2}} + o(x^4)$$

$$= \frac{x^4}{4} + o(x^4)$$

$$\lim_{x\to 0} \frac{\ln(1+x\cos x) - \sin x + \frac{1}{2}\sin(x^2)}{x^4} =$$

$$= \lim_{x\to 0} \frac{\frac{x^4}{4} + o(x^4)}{x^4} = \frac{1}{4}$$