18. Ottobre. 2021

DEF.:

$$I \subseteq \mathbb{R}$$

$$f : I \longrightarrow \mathbb{R}$$

① f si dice crescente (strettamente crescente) se:

$$\forall x, y \in I : \quad x < y \implies f(x) \leq f(y)$$
$$(f(x) < f(y))$$

② f si dice decrescente (strettamente decrescente) se:

$$\forall x, y \in I : \quad x < y \implies f(x) \geq f(y)$$
$$(f(x) > f(y))$$

FUNZIONE ESPONENZIALE:

$a \in \mathbb{R}: \quad 0 < a \quad , \quad a \neq 1$

$$y = a^x$$

<u>DSJ.</u>: Perché $a > 0$?

<u>Esempio</u>:

$$a = -2$$

$$(-2)^3 = -8 \neq$$

$$(-2)^{\frac{6}{2}} = \sqrt{(-2)^6} = \sqrt{64} = 8$$

L'esponenziale non è ben definito per basi negative.

<u>Es.:</u>

$$(8)^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$$

$$= \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

$$(27)^{-\frac{4}{3}} = \left(\frac{1}{27}\right)^{\frac{4}{3}} =$$

$$= \left(\sqrt[3]{\frac{1}{27}}\right)^4 = \left(\frac{1}{3}\right)^4 = \frac{1}{81}$$

Notazione:

$$\exp_a(x) := a^x$$

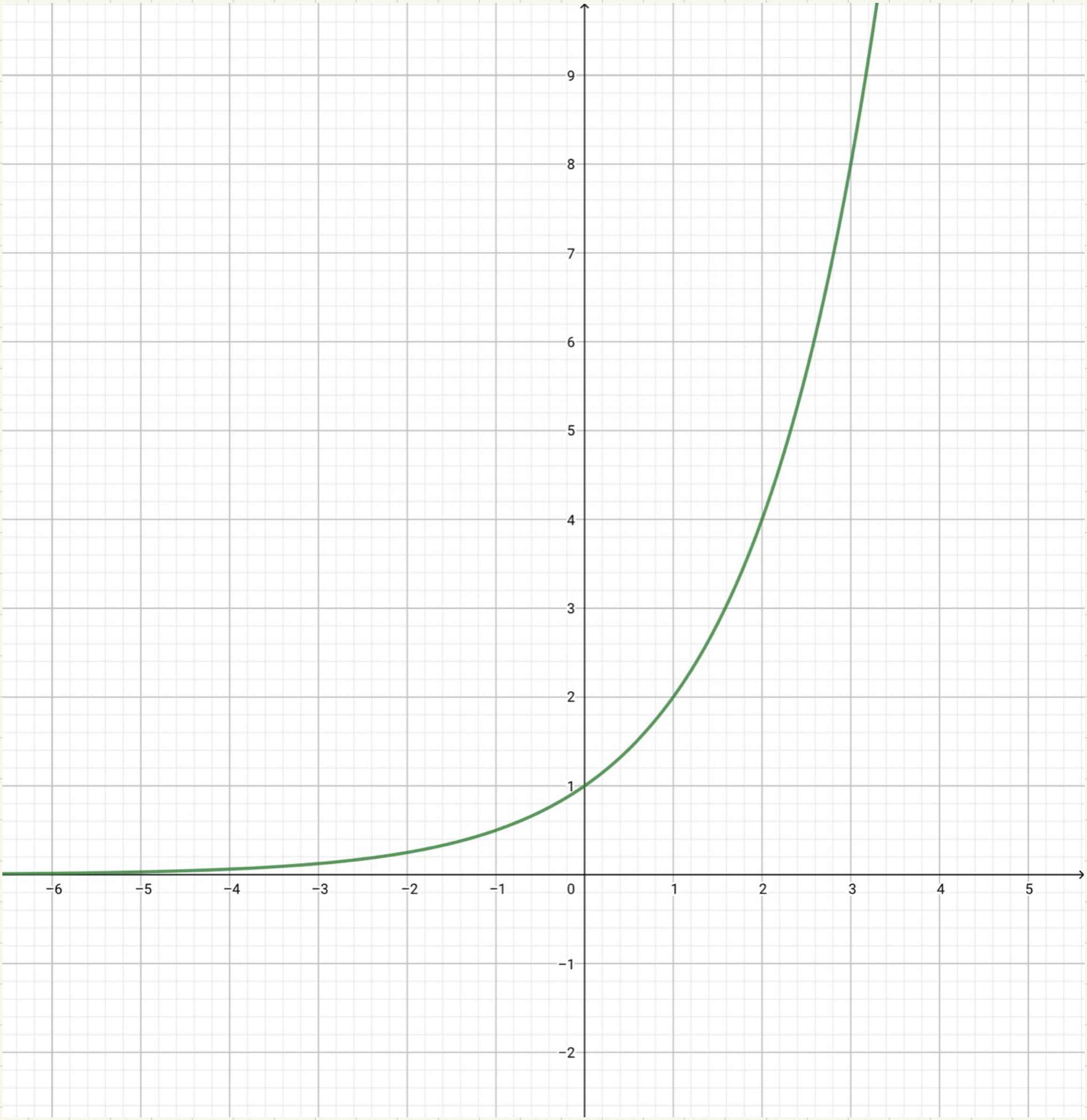
Se si sceglie come base a della funzione esponenziale: il numero e di Euler/Neper

$$e^x$$

$$\exp(x) := e^x$$

esponenziale a base naturale

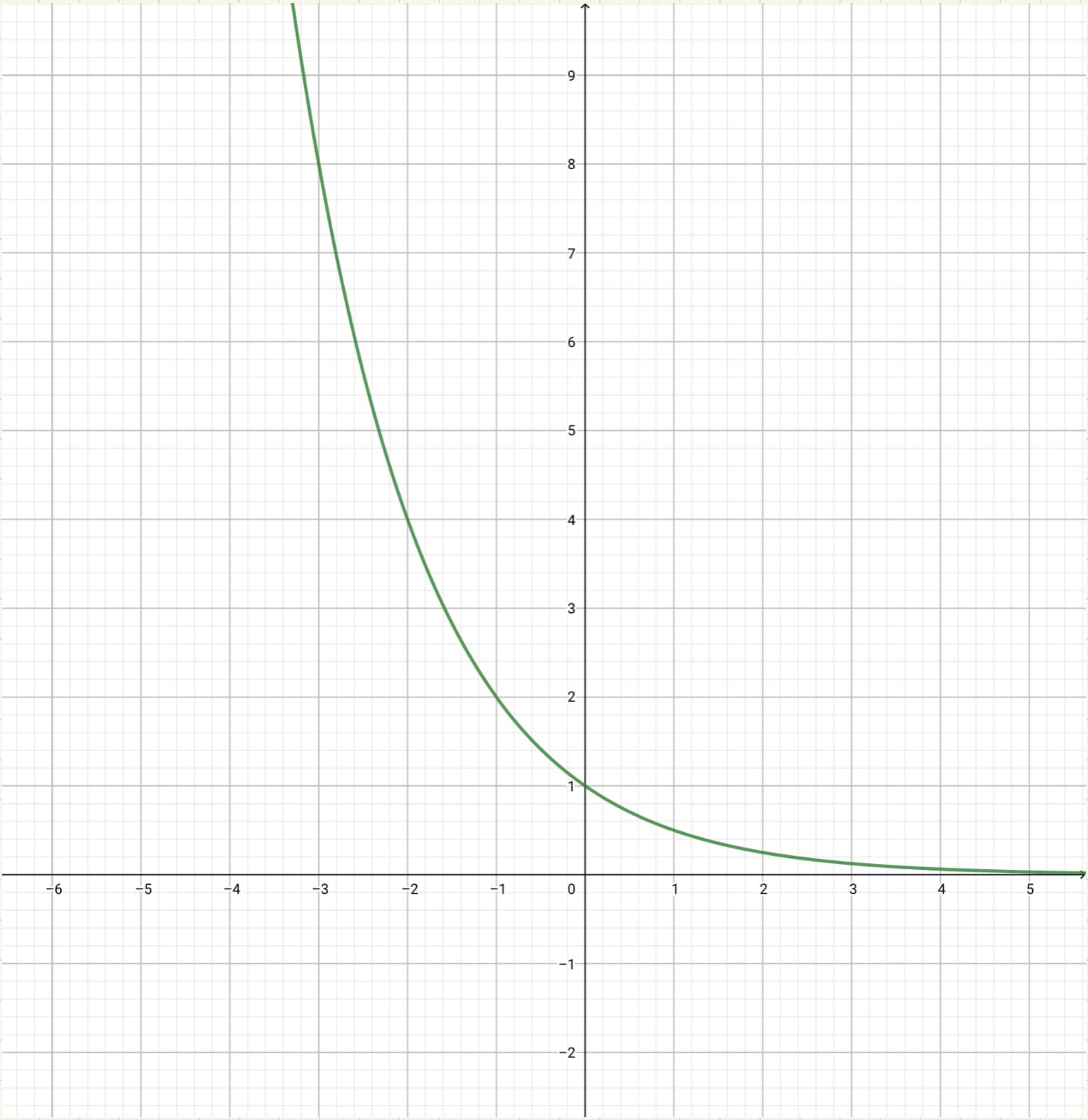
GRAFICO DI a^x $(a > 1)$:



a^x è crescente (strettam.)

(caso $a = e$)

GRAFICO DI a^x $(0 < a < 1)$



a^x è decrescente (strett.)

FUNZIONE LOGARITMICA:

TEOR.:

$a \in \mathbb{R}$: $0 < a$, $a \neq 1$

$\forall y \in \mathbb{R}$: $y > 0$ $\exists !\, x \in \mathbb{R}$:

$$a^x = y$$

$$\log_a y := x$$

$\uparrow$

logaritmo di y in base a

$$\log_a : \mathbb{R}^*_+ \longrightarrow \mathbb{R}$$

è una funzione di dominio

$$\mathbb{R}^*_+ = \{ r \in \mathbb{R} \mid r > 0 \}$$

Esempi:

$$\log_2 16 = 4 \qquad (2^4 = 16)$$

$$\log_2 \frac{1}{32} = -5 \qquad \left(2^{-5} = \frac{1}{32}\right)$$

$$\log_{10} 0{,}001 = -3 \qquad \ldots$$

$$\log_a 1 = 0 \qquad (a^0 = 1)$$

$$\log_{\frac{1}{3}} 81 = -4$$

$$\cancel{\log_a 0}$$

$$\log_2 2^6 = 6$$

$$\log_2 \left(\frac{1}{2}\right)^4 = -4$$

$$3^{\log_3 5} = 5$$

$$3^{\log_3 \left(\frac{1}{4}\right)} = \frac{1}{4}$$

$$\exp_a : \mathbb{R} \longrightarrow \mathbb{R}_+$$

$$\log_a : \mathbb{R}_+^* \longrightarrow \mathbb{R}$$

Dal teorema precedente:

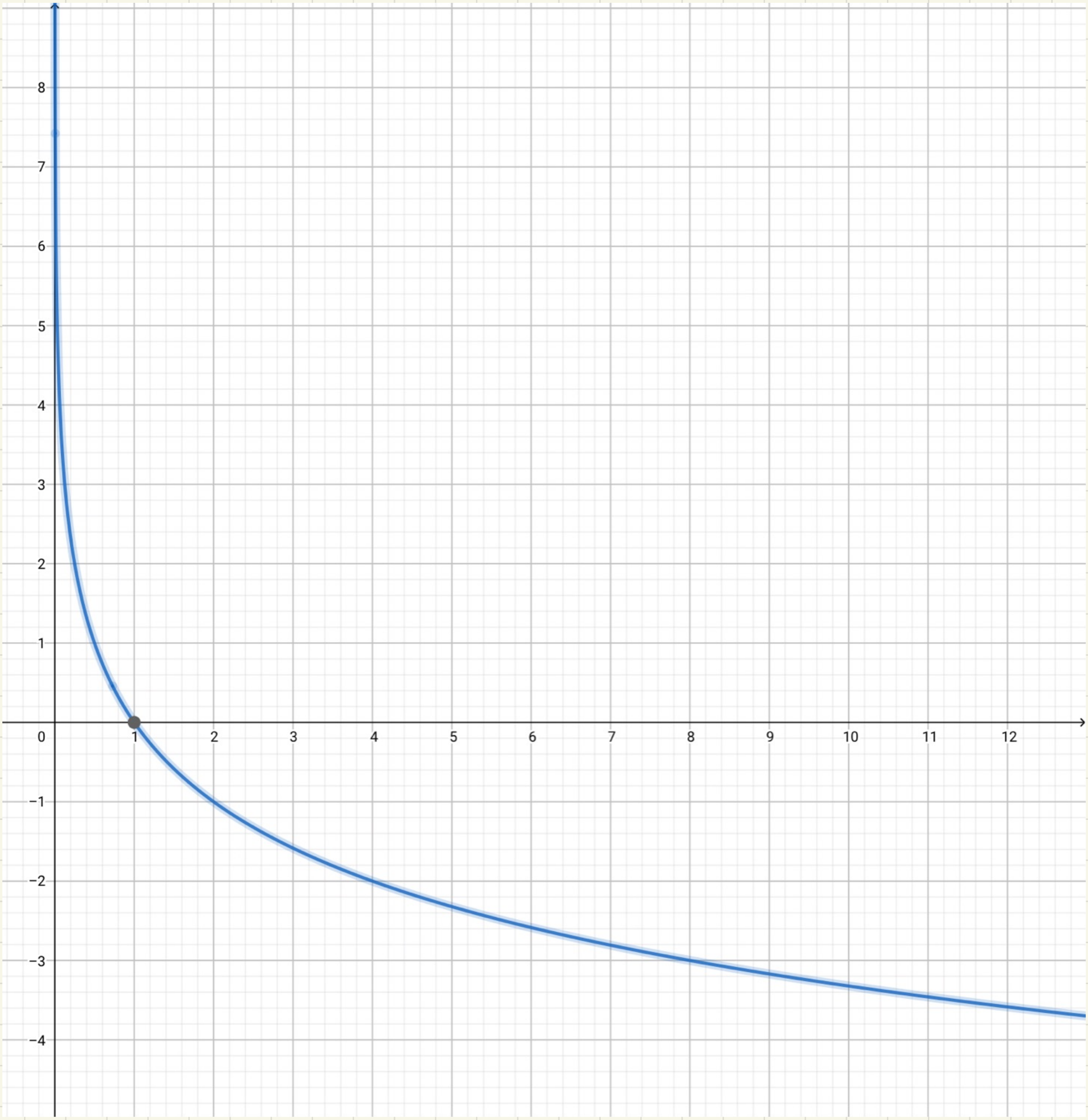
$$a^{\log_a y} = y \qquad \forall y \in \mathbb{R}_+^*$$

$$\log_a (a^x) = x \qquad \forall x \in \mathbb{R}$$

Dunque $\log_a$ è la funzione inversa di $\exp_a$

GRAFICO DI $\log_a x$ $(a > 1)$

GRAFICO DI $\log_a x$ $(0 < a < 1)$



α alfa (alpha)

β beta

γ gamma

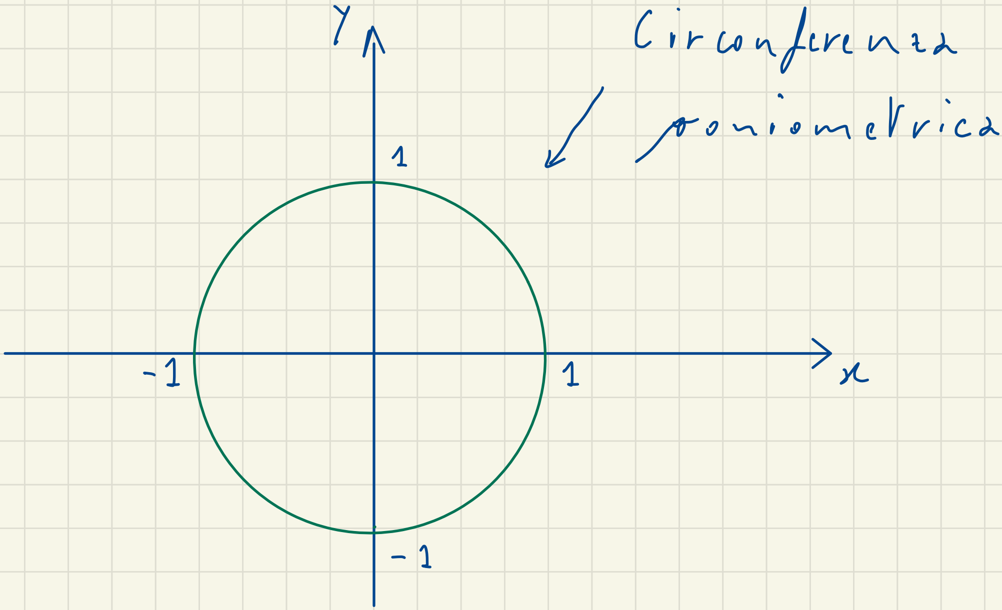
ω omega

θ teta

ρ rho

FUNZIONI GONIOMETRICHE:

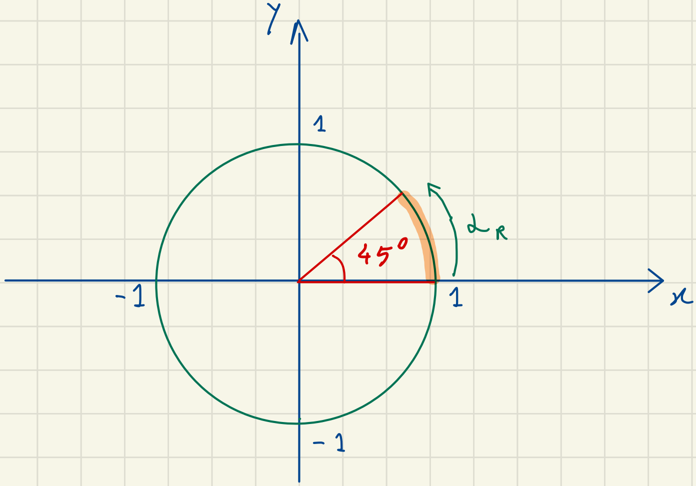
Angoli in radianti:



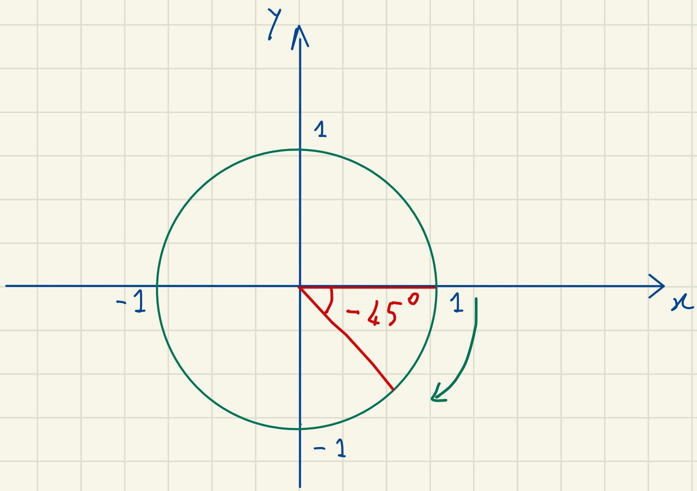
$$x^2 + y^2 = 1$$ Eq. della circonf. goniometrica

Lunghezza della circonferenza gon.

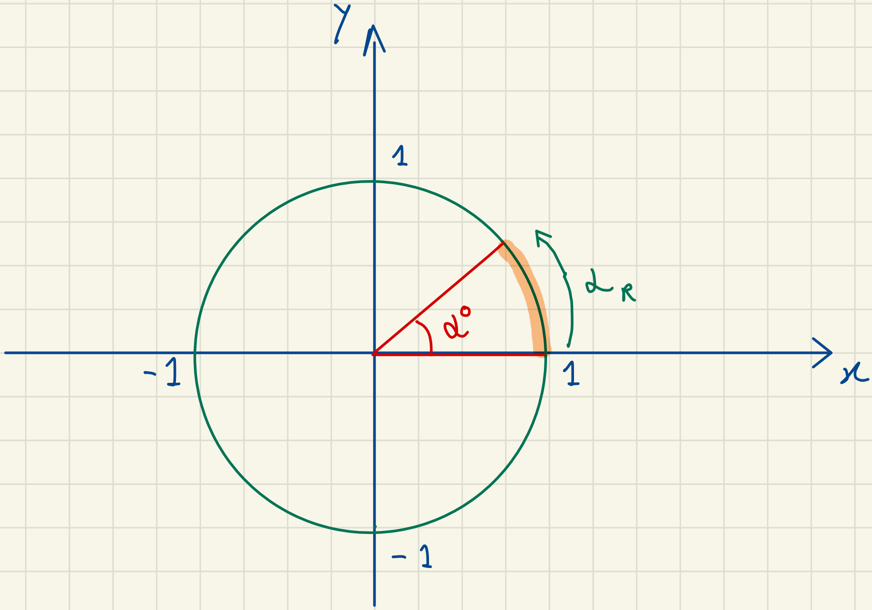
$= 2\pi \cdot 1 = 2\pi$



Orientamento positivo =
= senso antiorario



Orientamento negativo =
= senso negativo



α° = misura dell'angolo in gradi

α_R = misura dell'angolo in radianti

$$\alpha^\circ : 360^\circ = \alpha_R : 2\pi$$

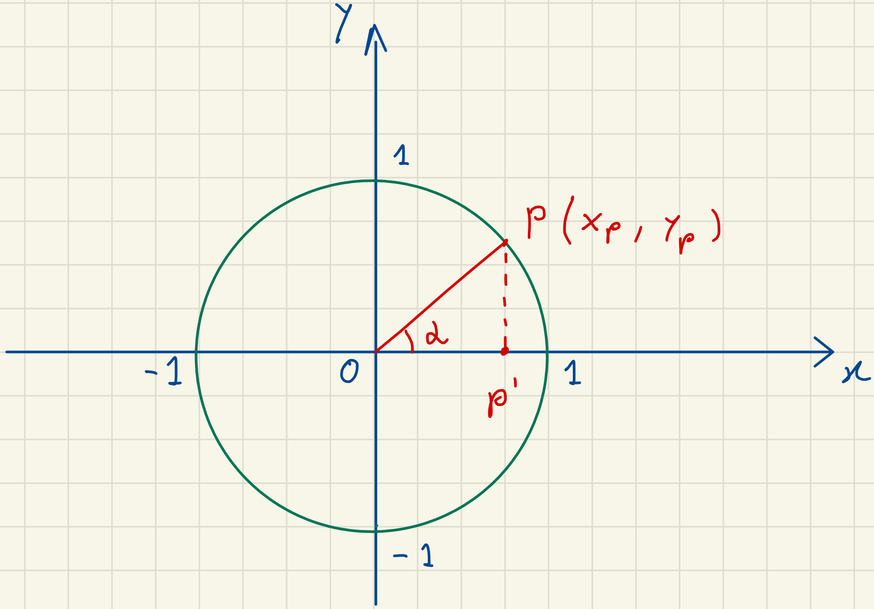
$$\alpha^{\circ} : 360 = \alpha_R : 2\pi$$

$$\Rightarrow \quad \alpha_R = \alpha^{\circ} \cdot \frac{2\pi}{360} = \alpha^{\circ} \cdot \frac{\pi}{180^{\circ}}$$

$$\boxed{\alpha_R = \alpha^{\circ} \cdot \frac{\pi}{180}}$$

α°	α_R
0	0
45°	$\frac{\pi}{4}$
90°	$\frac{\pi}{2}$
180°	π
270°	$\frac{3\pi}{2}$
360°	2π

Funzioni seno e coseno:



Sia P un punto sulla circonf. goniometrica: $P(x_P, y_P)$

Sia α l'angolo che il raggio OP forma con l'asse delle x.

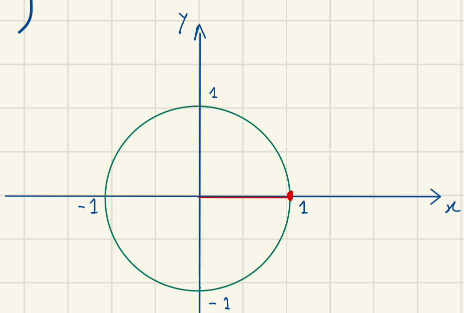
DEF.: Si definiscono:

$$\sin \alpha := y_P$$

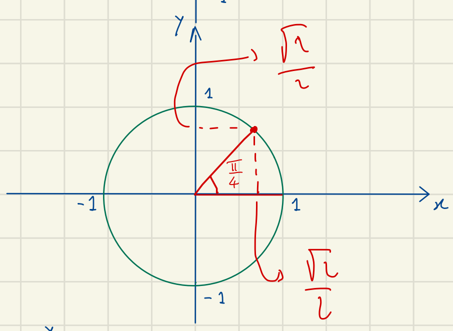
$$\cos \alpha := x_P$$

Esempi: (in radianti)

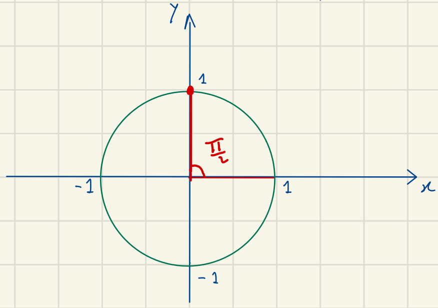
$$\cos 0 = 1$$
$$\sin 0 = 0$$



$$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$



$$\cos \frac{\pi}{2} = 0$$
$$\sin \frac{\pi}{2} = 1$$



Siccome (x_P, y_P) giace sulla circonferenza goniometrica:

$$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$$

Per costruzione, il seno e il coseno sono funzioni periodiche di periodo 2π:

$$\sin(\alpha \pm 2\pi) = \sin\alpha \quad , \quad \forall \alpha \in \mathbb{R}$$

$$\cos(\alpha \pm 2\pi) = \cos\alpha \quad , \quad \forall \alpha \in \mathbb{R}$$

La relazione fondamentale

$$\sin^2\alpha + \cos^2\alpha = 1$$

permette di calcolare $\sin\alpha$ conoscendo $\cos\alpha$ (o viceversa) A PATTO DI SAPERE IN QUALE QUADRANTE SI TROVA α:

$$\sin^2\alpha = 1 - \cos^2\alpha$$

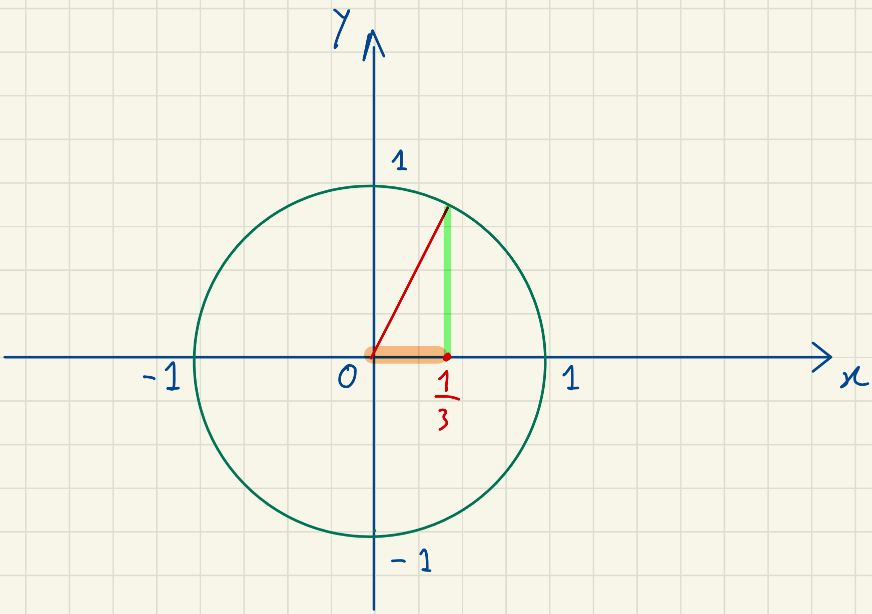
$$\Rightarrow \quad \sin\alpha = \pm\sqrt{1-\cos^2\alpha}$$

(Attenzione! il seno è UNA FUNZIONE)

Esempio:

$$\cos\alpha = \frac{1}{3} \qquad \sin\alpha = ?$$

$$\alpha \in \left[0, \frac{\pi}{2}\right]$$



$$\sin\alpha = +\sqrt{1-\cos^2\alpha} =$$

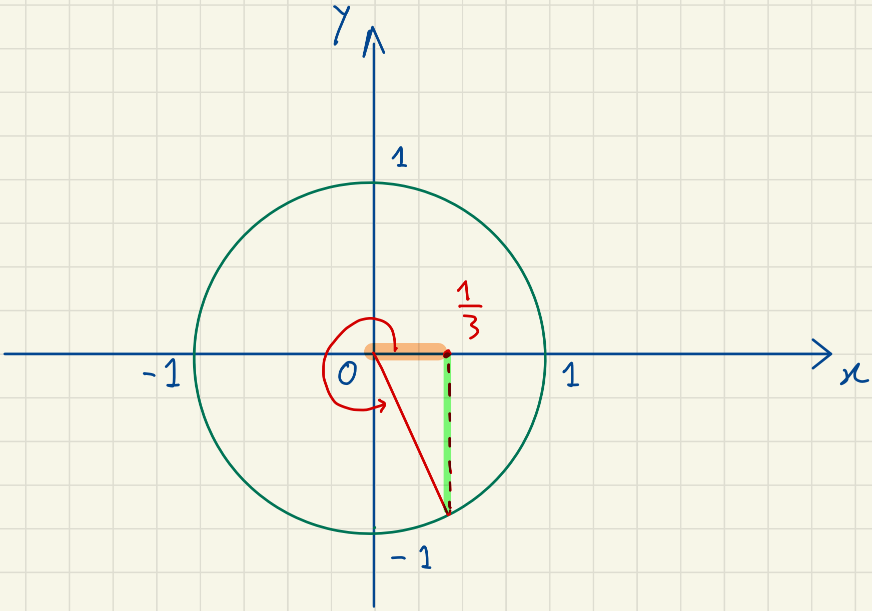
$$= \sqrt{1-\left(\frac{1}{3}\right)^2} = \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

Esempio:

$$\cos\alpha = \frac{1}{3}$$

$$\sin\alpha = ?$$

$$\alpha \in \left[\frac{3\pi}{2}, 2\pi\right]$$



$$\sin\alpha = -\sqrt{1-\cos^2\alpha} =$$

$$= -\sqrt{1-\left(\frac{1}{3}\right)^2} = -\sqrt{1-\frac{1}{9}} = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$$

$$f : A \to \mathbb{R}$$

DEF.:

$$\begin{cases} f(-x) = f(x) & \forall x \in A \\ f \text{ si dice PARI} \end{cases}$$

$$\begin{cases} f(-x) = -f(x) & \forall x \in A \\ f \text{ si dice DISPARI} \end{cases}$$

$$f(x) = x^n \quad , \quad n \in \mathbb{N}$$

se n è pari $\Rightarrow f$ è PARI

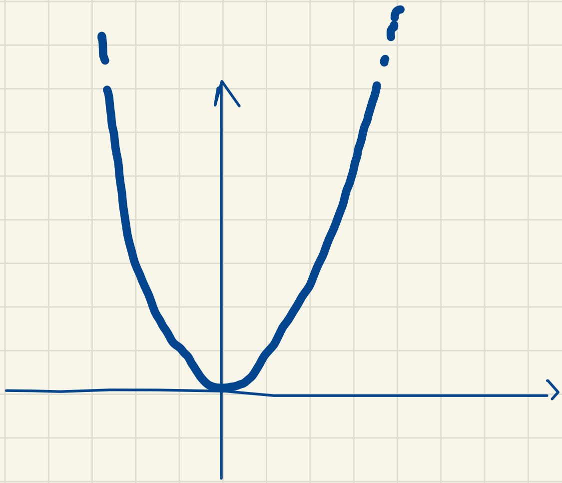
$$f(-x) = (-x)^n = (-1)^n \cdot x^n = x^n = f(x)$$

se n è dispari $\Rightarrow f$ è DISPARI

$$f(-x) = (-x)^n = (-1)^n \cdot x^n = -x^n = -f(x)$$

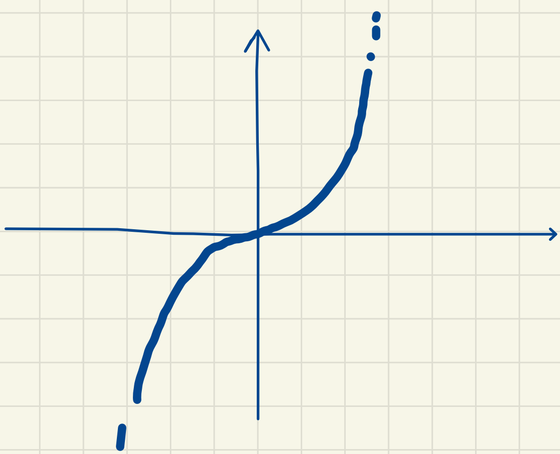
f PARI $\Rightarrow$ il suo grafico è simmetrico rispetto l'asse y

$y = x^2$



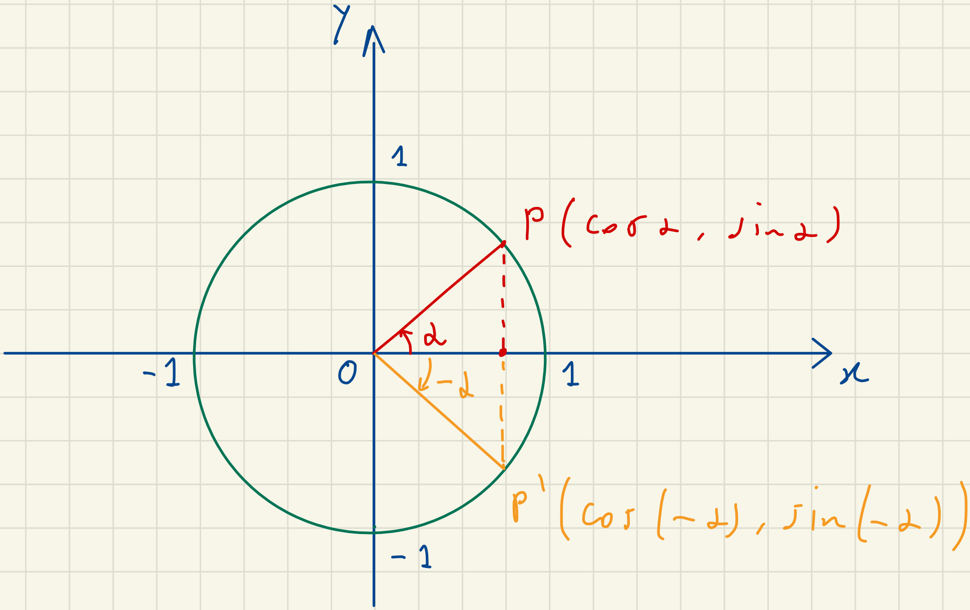
f è DISPARI $\Rightarrow$ il suo grafico è SIMMETRICO l'origine O

$y = x^3$



OSS. :

In definizione :



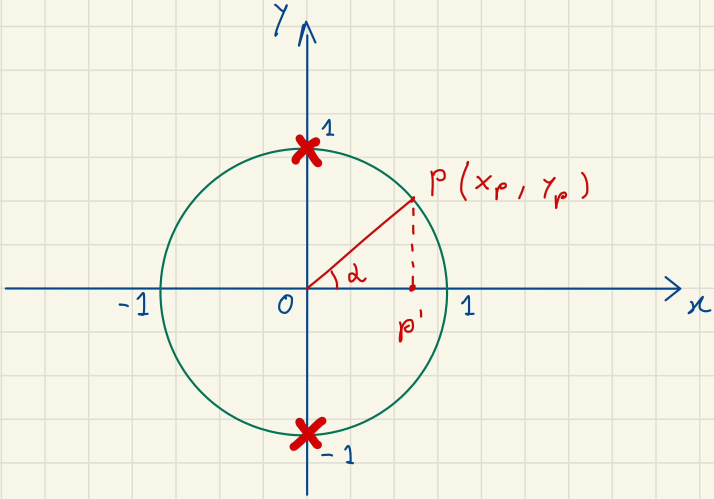
$$\boxed{\begin{aligned} \cos(-\alpha) &= \cos\alpha \\ \sin(-\alpha) &= -\sin\alpha \end{aligned}}$$

$\leftarrow$ funz. PARI

$\leftarrow$ funz. DISPARI

Esercizio: per quali $\alpha \in \mathbb{R}$

$$\cos\alpha = 0 \ ?$$



$$\alpha = \frac{\pi}{2} + 2k\pi \quad , \quad k \in \mathbb{Z}$$

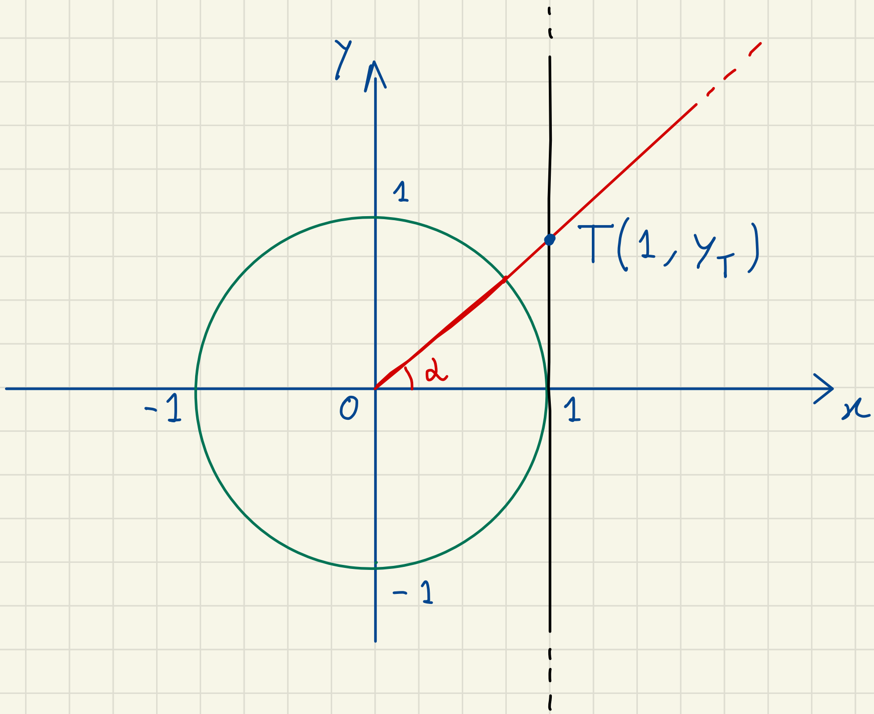
$$\alpha = \frac{3\pi}{2} + 2k\pi \quad , \quad k \in \mathbb{Z}$$

$$= \frac{\pi}{2} + \pi + 2k\pi$$

In sintesi:

$$\alpha = \frac{\pi}{2} + k\pi \quad , \quad k \in \mathbb{Z}$$

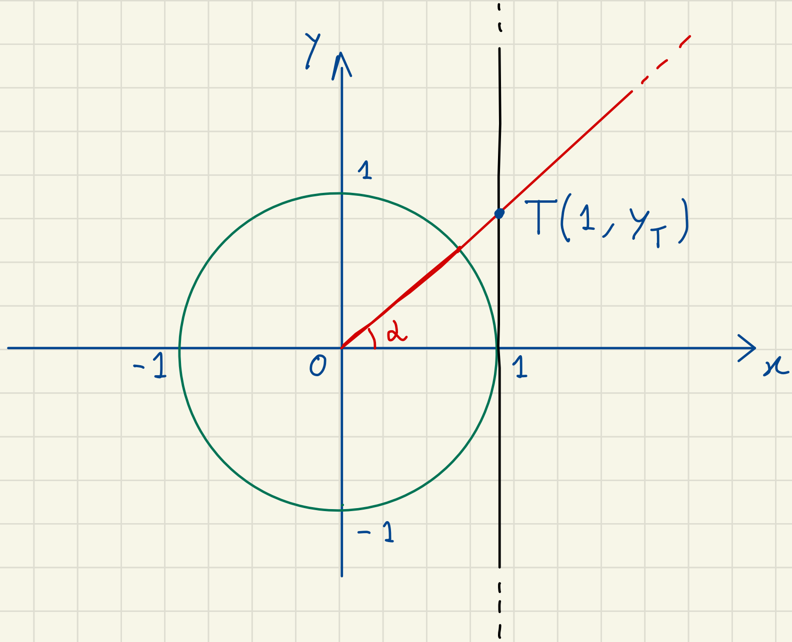
Funzione tangente:



DEF. (tangente di α)

$$tg\ \alpha := y_T$$

tangente di α ($\tan \alpha$)

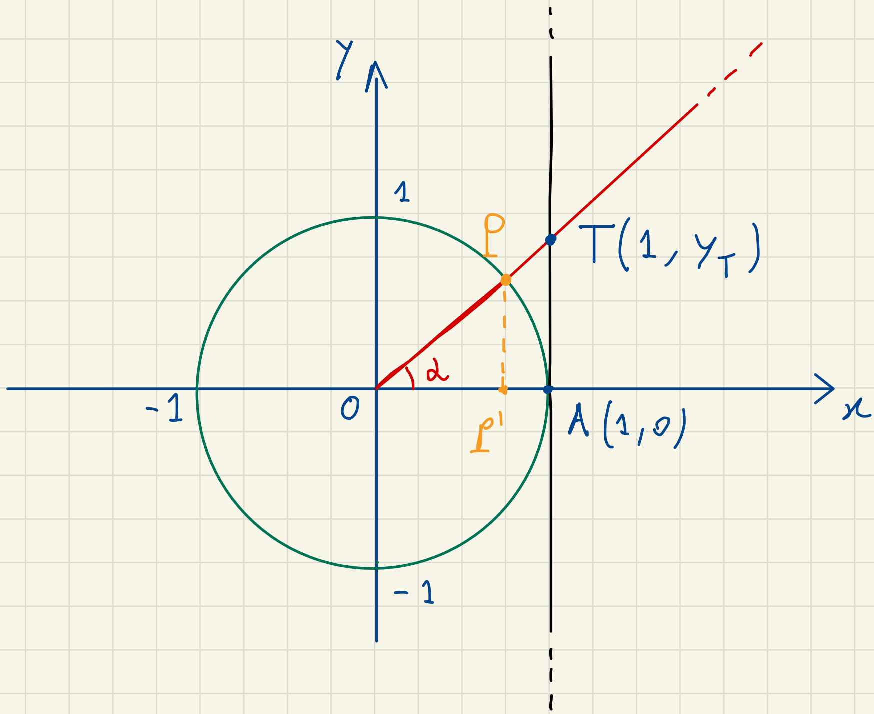


$tg\,\alpha$ non è definita

per $\alpha = \frac{\pi}{2}, \frac{3}{2}\pi, \ldots$

e

$\alpha = -\frac{\pi}{2}, -\frac{3}{2}\pi, \ldots$



I triangoli $O\overset{\Delta}{P}P'$ e $O\overset{\Delta}{T}A$ sono simili, quindi

$$\overline{TA} : \overline{OA} = \overline{PP'} : \overline{OP'}$$

$$\Rightarrow \underset{\substack{\parallel \\ \tan\alpha}}{\overline{TA}} = \frac{\overline{PP'} \cdot \overset{=1}{\overline{OA}}}{\overline{OP'}} = \frac{\sin\alpha}{\cos\alpha}$$

Si è così provato che:

$$\boxed{\tan \alpha = \frac{\sin \alpha}{\cos \alpha}}$$

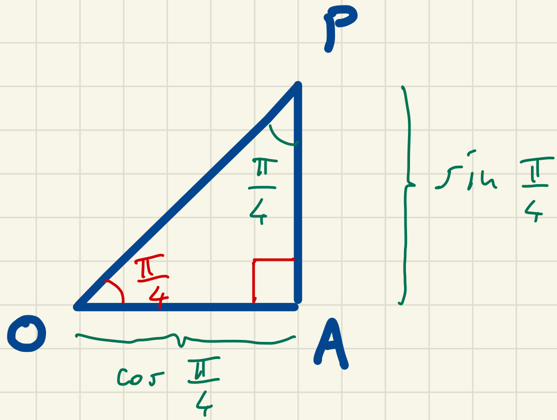
$$D(\tan) = \{\alpha \in \mathbb{R} \mid \cos \alpha \neq 0\}$$

$$= \{\alpha \in \mathbb{R} \mid \alpha \neq \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}\}$$

Angoli speciali:

$$\alpha = \frac{\pi}{4}$$

$$\overline{OP} = 1$$



$$\hat{A} = \frac{\pi}{2} \qquad \hat{O} = \frac{\pi}{4}$$

$$\Rightarrow \quad \hat{P} = \pi - \hat{A} - \hat{O} = \pi - \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$O\hat{P}A$ triangolo rettangolo isoscele

$$\overline{OA} = \overline{PA}$$

Dal T. di Pitagora:

$$\underset{\substack{\shortparallel \\ 1}}{\overline{OP}^2} = \overline{OA}^2 + \overline{PA}^2 = 2\,\overline{OA}^2$$

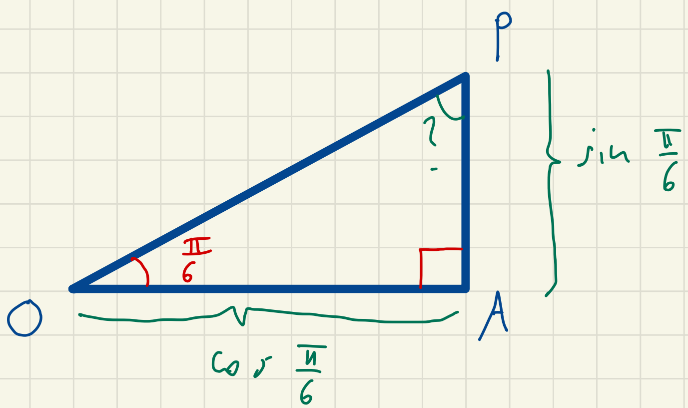
$$\overline{OA}^2 = \frac{1}{2} \implies \overline{PA} = \overline{OA} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} = \cos\frac{\pi}{4}$$

$$\operatorname{tg}\frac{\pi}{4} = \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = 1$$

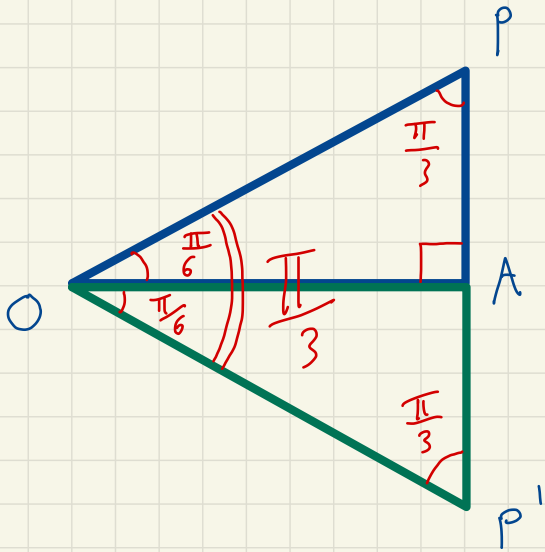
$$\overline{OP} = 1$$

$$\alpha = \frac{\pi}{6}$$



$$\hat{P} = \pi - \hat{O} - \hat{A} = \pi - \frac{\pi}{6} - \frac{\pi}{2} =$$

$$= \frac{\pi}{3}$$



$\overset{\triangle}{OPP'}$ è un triangolo equilatero

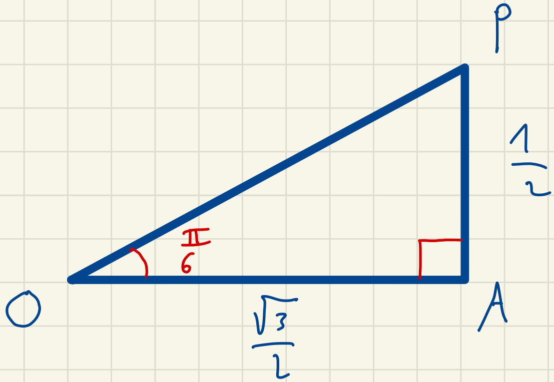
$\Longrightarrow \quad \overline{PP'} = \overline{OP} = 1$

$\overline{PA} = \frac{1}{2}\,\overline{PP'} = \frac{1}{2}$

Dal T. di Pitagora:

$$\overline{OA} = \sqrt{\overline{OP}^2 - \overline{AP}^2} = \sqrt{1 - \left(\frac{1}{2}\right)^2} =$$

$$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

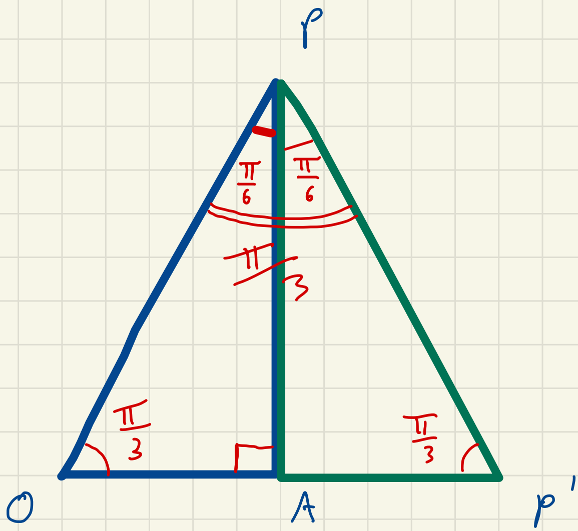


$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\sin \frac{\pi}{6} = \frac{1}{2}$$

$$\operatorname{tg} \frac{\pi}{6} = \frac{\sin \frac{\pi}{6}}{\cos \frac{\pi}{6}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$\alpha = \frac{\pi}{3}$



$\overset{\triangle}{OPP'}$ è un triangolo equilatero

$$\overline{OA} = \frac{1}{2}\,\overline{OP'} = \frac{1}{2}$$

$$\overline{AP} = \sqrt{\overline{OP}^2 - \overline{OA}^2} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$$

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\Longrightarrow \operatorname{tg}\frac{\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$\cos\frac{\pi}{3} = \frac{1}{2}$$

TABELLA RIASSUNTIVA:

α	$\sin\alpha$	$\cos\alpha$	$\operatorname{tg}\alpha$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	X
π	0	-1	0
$\frac{3\pi}{2}$	-1	0	X

GRAFICO DEL SENO

$y = \sin x$

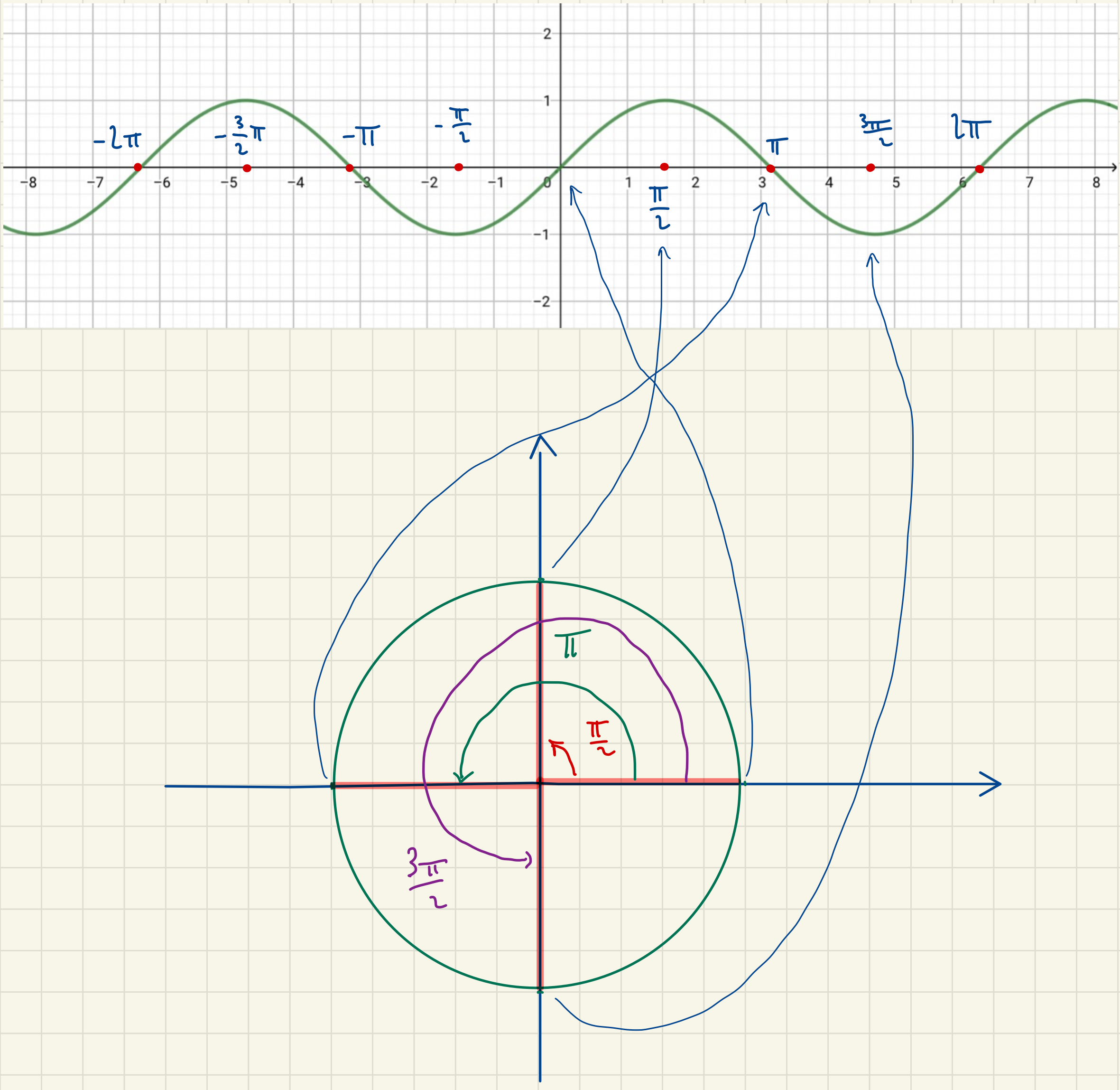
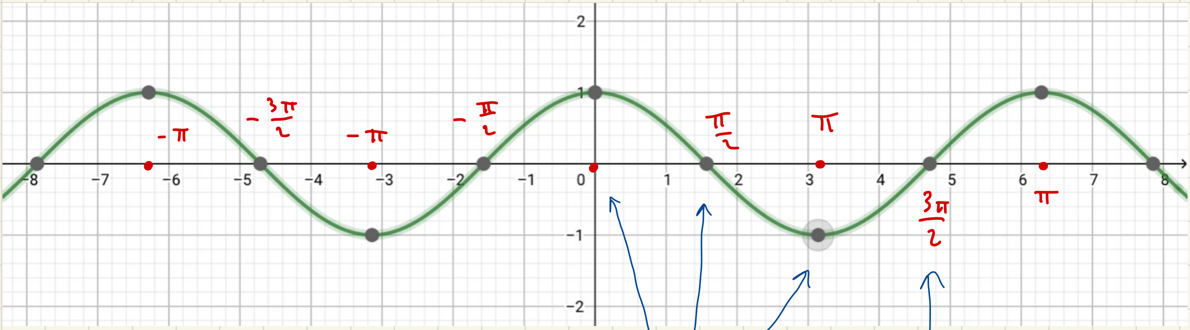


GRAFICO DEL COSENO:

$y = \cos x$



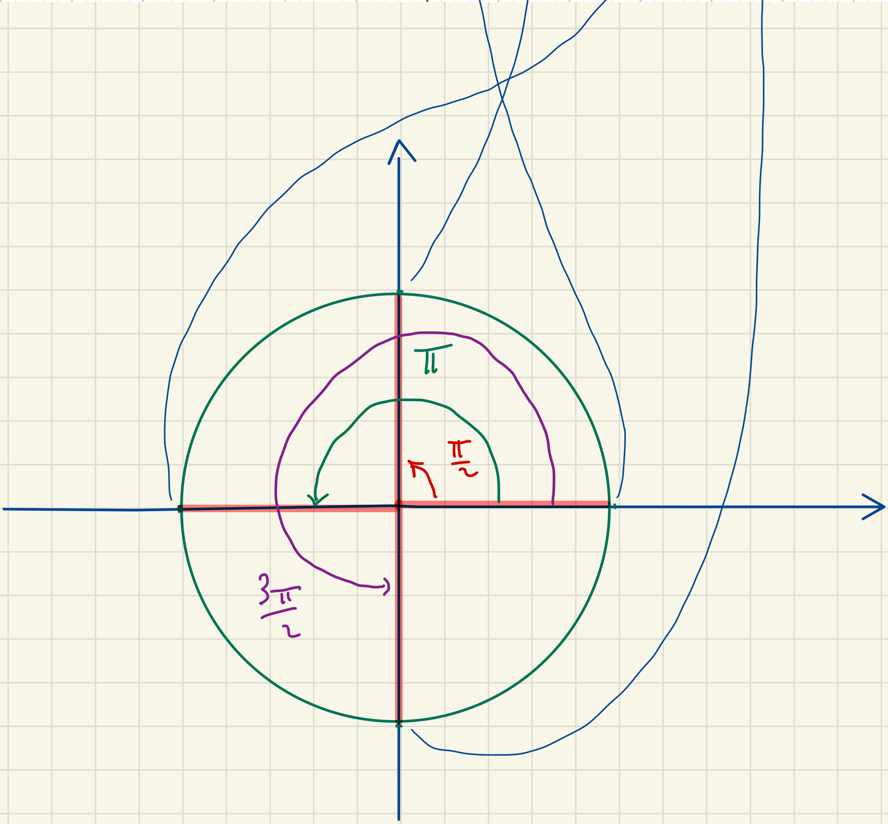
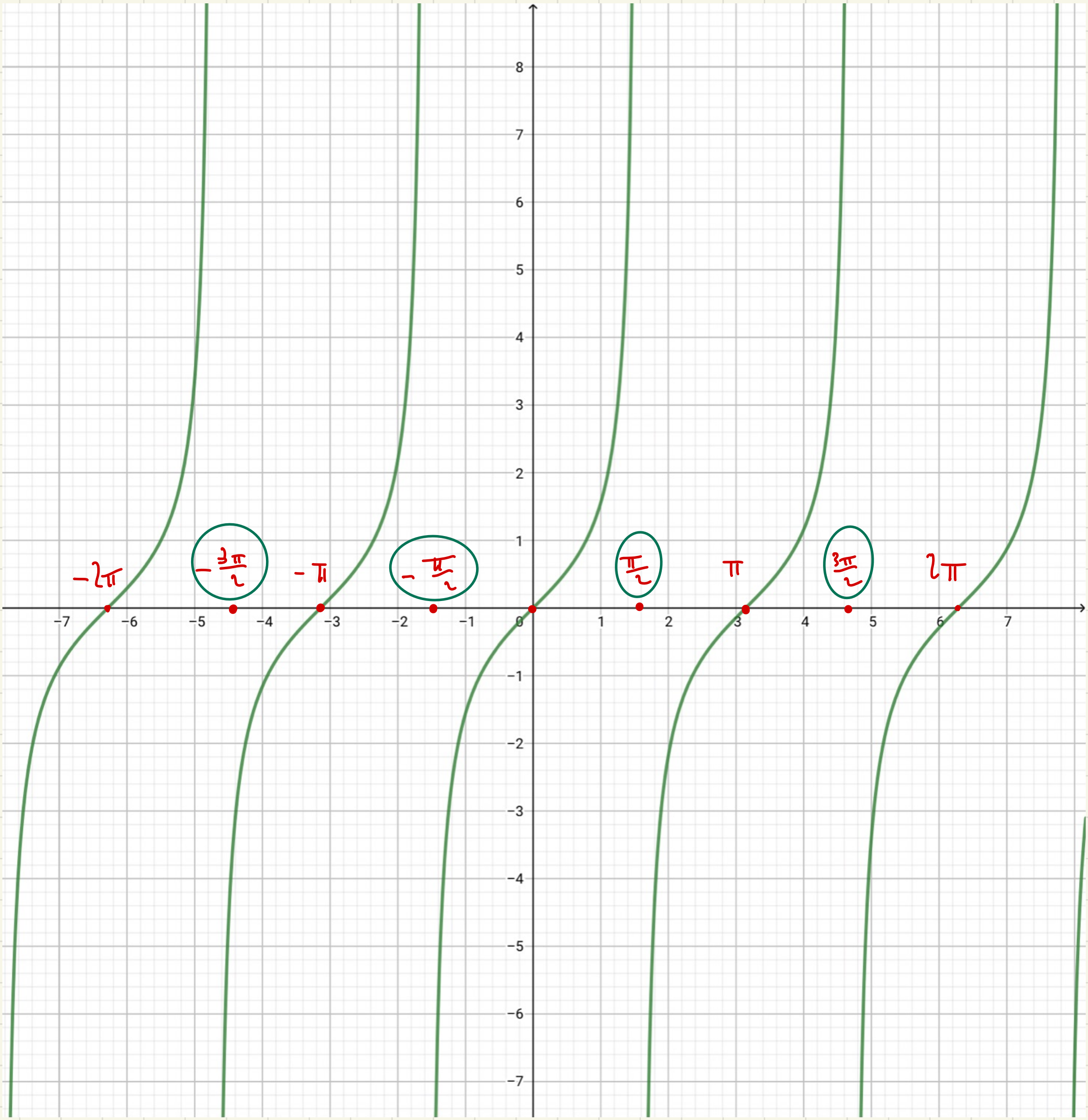


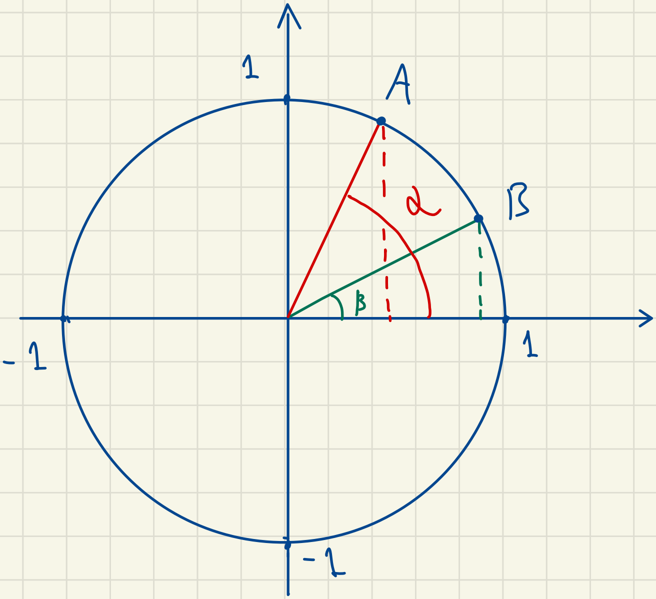
GRAFICO DELLA TANGENTE

$y = \operatorname{tg} x$



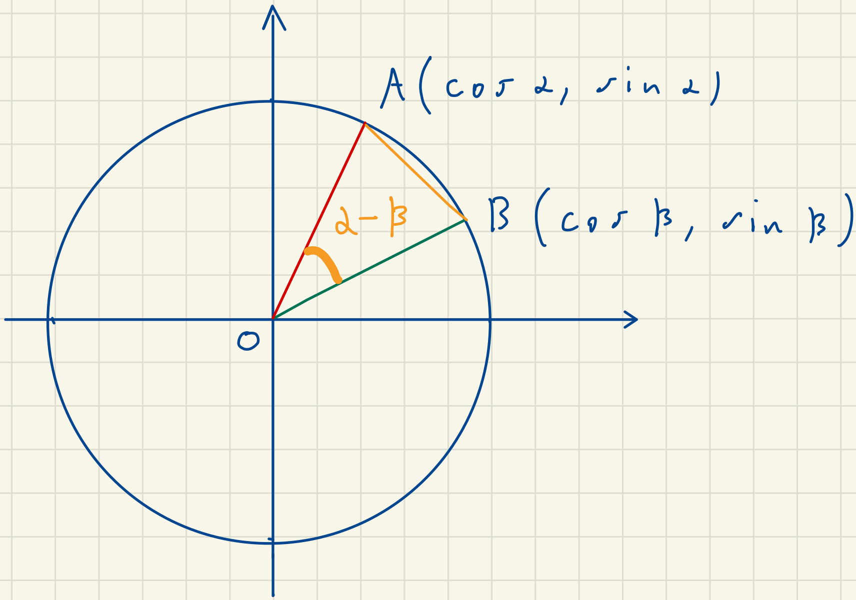
FORMULE DI ADDIZIONE E SOTTRAZIONE

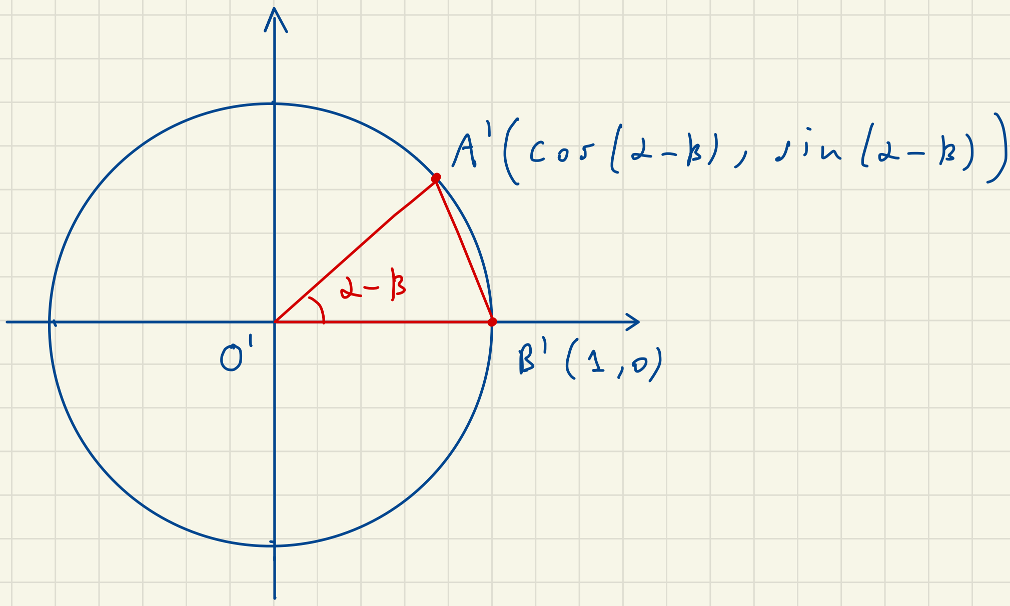
$$\cos(\alpha - \beta) = \, ?$$



$B(\cos\beta, \sin\beta)$

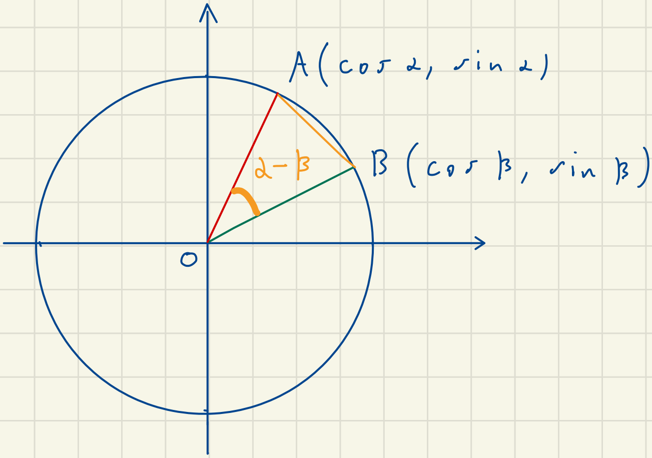
$A(\cos\alpha, \sin\alpha)$





$$\overline{OA} = \overline{O'A'}\,, \quad \overline{OB} = \overline{O'B'}\,, \quad A\hat{O}B \simeq A'\widehat{O'}B'$$

$$\Rightarrow \quad \overset{\triangle}{AOB} \simeq \overset{\triangle}{A'O'B'} \quad \Rightarrow \quad \boxed{\overline{AB} = \overline{A'B'}}$$

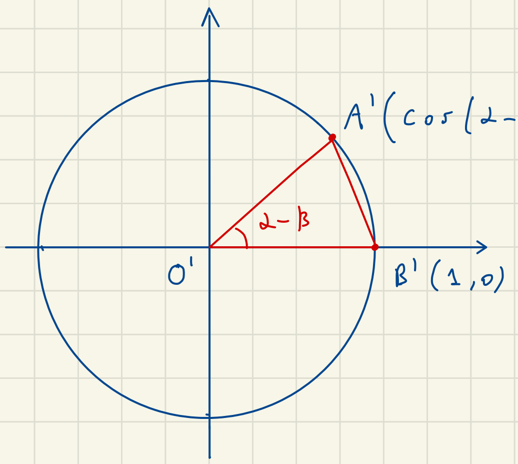


$$\overline{AB}^2 = (\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$$

$$= \cos^2\alpha - 2\cos\alpha\cdot\cos\beta + \cos^2\beta + \sin^2\alpha$$

$$- 2\sin\alpha\cdot\sin\beta + \sin^2\beta =$$

$$= 2 - 2\cos\alpha\cdot\cos\beta - 2\sin\alpha\cdot\sin\beta$$



$$\overline{A'B'}^2 = (\cos(\alpha-\beta)-1)^2 + \sin^2(\alpha-\beta) =$$

$$= \cos^2(\alpha-\beta) + 1 - 2\cos(\alpha-\beta) + \sin^2(\alpha-\beta)$$

$$= 2 - 2\cos(\alpha-\beta)$$

$$\overline{AB}^2 = \overline{A'B'}^2$$

$$\cancel{2} - 2\cos\alpha \cdot \cos\beta - 2\sin\alpha \cdot \sin\beta =$$

$$= \cancel{2} - 2\cos(\alpha - \beta)$$

$$\boxed{\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta}$$

Usando il fatto che coseno è una funzione pari:

$$\cos(\alpha+\beta) = \cos(\alpha-(-\beta)) =$$

$$= \cos\alpha \cdot \underset{\substack{\parallel \\ \cos\beta}}{\cos(-\beta)} + \sin\alpha \cdot \underset{\substack{\parallel \\ -\sin\beta}}{\sin(-\beta)}$$

$$= \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

Quindi:

$$\boxed{\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta}$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\alpha = \frac{\pi}{2}$$

$$\Downarrow$$

$$\cos\left(\frac{\pi}{2} - \beta\right) = \overset{0}{\overset{\parallel}{\cos\frac{\pi}{2}}} \cdot \cos\beta + \overset{1}{\overset{\parallel}{\sin\frac{\pi}{2}}} \cdot \sin\beta$$

$$\Longrightarrow \quad \boxed{\cos\left(\frac{\pi}{2} - \beta\right) = \sin\beta} \qquad (A)$$

Se poniamo: $\gamma := \frac{\pi}{2} - \beta \rightarrow \beta = \frac{\pi}{2} - \gamma$

$$\cos\gamma = \sin\left(\frac{\pi}{2} - \gamma\right)$$

$$\Longrightarrow \quad \boxed{\sin\left(\frac{\pi}{2} - \gamma\right) = \cos\gamma} \qquad (B)$$

Esercizio:

Provare in modo analogo che:

$$\boxed{\sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta}$$

Suggerimento:

Usare la relazione (A) di prima:

$$\sin(\alpha-\beta) = \cos\left(\frac{\pi}{2}-(\alpha-\beta)\right) =$$

$$= \cos\left(\left(\frac{\pi}{2}-\alpha\right)+\beta\right)$$

e analogamente dimostrare che:

$$\boxed{\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\,\sin\beta}$$

$$\sin(\alpha - \beta) = ?$$

$$\sin(\alpha - \beta) = \cos\left(\frac{\pi}{2} - (\alpha - \beta)\right) =$$

$$= \cos\left(\left(\frac{\pi}{2} - \alpha\right) + \beta\right) =$$

$$= \cos\left(\frac{\pi}{2} - \alpha\right) \cdot \cos\beta - \sin\left(\frac{\pi}{2} - \alpha\right) \cdot \sin\beta$$

$$= \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$$

In sintesi:

$$\boxed{\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta}$$

$$\boxed{\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta}$$

$$\boxed{\sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta}$$

$$\boxed{\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta}$$

FORMULE DI DUPLICAZIONE:

Si provano usando le formule di addizione:

$$\cos(2\alpha) = \cos(\alpha+\alpha) =$$
$$= \cos^2\alpha - \sin^2\alpha$$

$$\sin(2\alpha) = \sin(\alpha+\alpha) =$$
$$= \sin\alpha\cdot\cos\alpha + \cos\alpha\cdot\sin\alpha =$$
$$= 2\sin\alpha\cdot\cos\alpha$$

$$\cos(2\alpha) = \cos(\alpha+\alpha) =$$

$$= \cos^2\alpha - \sin^2\alpha \qquad (\mathrm{I})$$

$$= (1-\sin^2\alpha) - \sin^2\alpha =$$

$$= 1 - 2\sin^2\alpha \qquad (\mathrm{II})$$

$$= \cos^2\alpha - (1-\cos^2\alpha) = \qquad (\mathrm{III})$$

$$= 2\cos^2\alpha - 1$$

FUNZIONI GONIOMETRICHE "INVERSE":

La funzione seno:

$$\sin : \mathbb{R} \longrightarrow \mathbb{R}$$

NON è invertibile, poiché non è né SU né 1-1.

Tuttavia se restringiamo il suo codominio:

$$\sin : \mathbb{R} \longrightarrow [-1, 1]$$

è SURIETTIVA (ma non INIETTIVA)

Se però sostituiamo alla funzione seno la sua restrizione all'interv. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$:

$$\sin\Big|_{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]} : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$$

$\Longrightarrow$ $\sin\Big|_{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$ è su e 1-1

$\Longrightarrow$ è invertibile

$$\sin\Big|_{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]} : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$$

funzione inversa

$$\left(\sin\big|_{\left[-\frac{\pi}{2},\frac{\pi}{2}\right]}\right)^{-1}(y) =: \arcsin y$$

arcoseno di y

$$\arcsin : [-1,1] \longrightarrow \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$

$$y \longmapsto \arcsin y$$

ATTENZIONE:

Il fatto che arcsin sia l'inversa di una restrizione del seno ha delle conseguenze:

$\forall y \in [-1, 1]:$

$$\sin(\arcsin y) = y$$

$\forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (e non in $\mathbb{R}$!)

$$\arcsin(\sin x) = x$$

(<u>Esempio</u>:

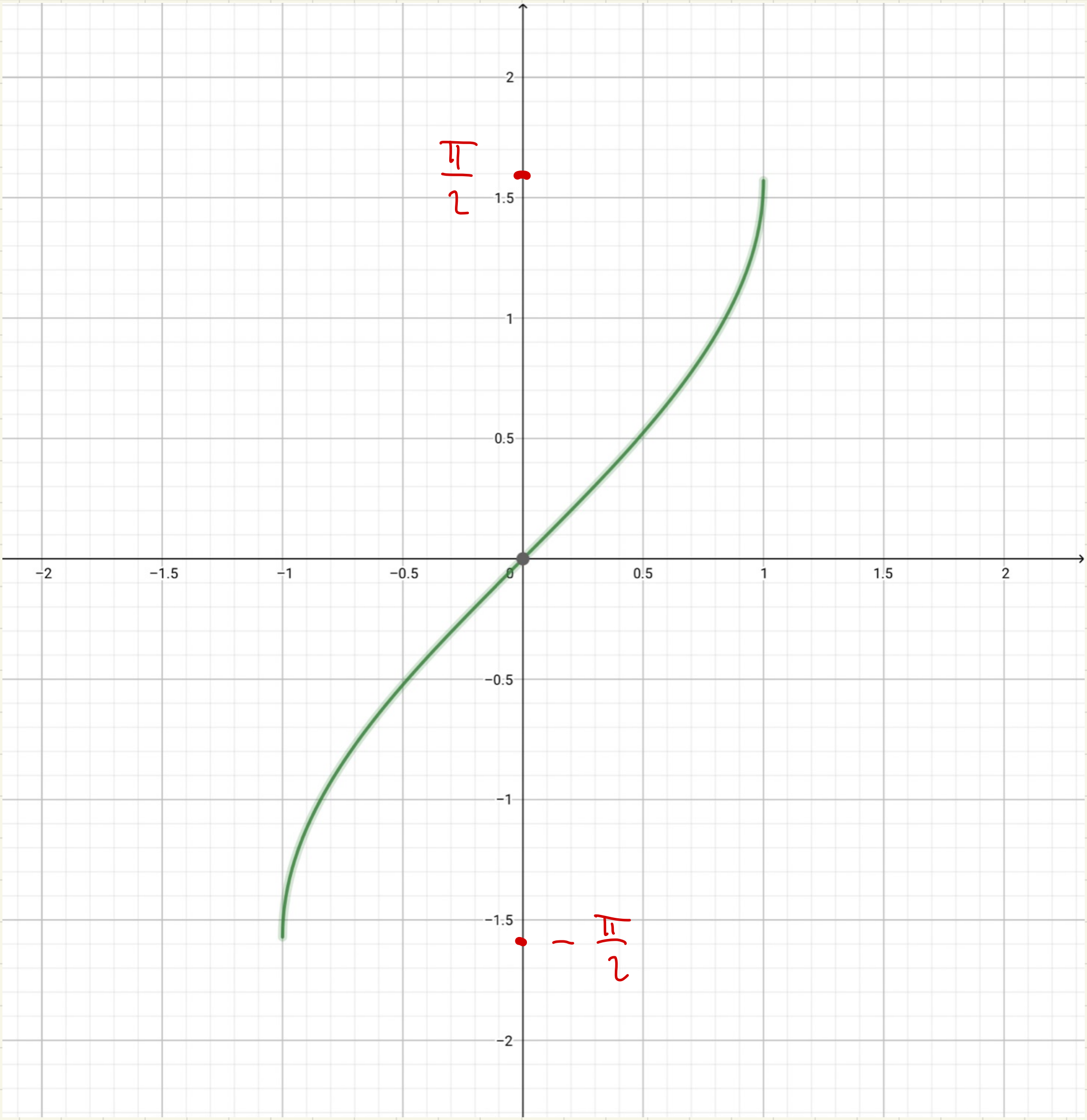
$x = \pi$ (nota che $\pi \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$)

$$\arcsin(\sin \pi) =$$

$$= \arcsin(0) = 0 \neq \pi$$

)

$y = \arcsin x$



Anche la funzione coseno non è invertibile; consideriamo la sua restrizione a $[0, \pi]$:

$$\cos\big|_{[0,\pi]} : [0, \pi] \longrightarrow [-1, 1]$$

$\Longrightarrow$ $\cos\big|_{[0,\pi]}$ è su e 1-1

$\Longrightarrow$ è invertibile

$$\cos\big|_{[0,\pi]} : [0, \pi] \longrightarrow [-1, 1]$$

funzione inversa

$$\left(\cos\big|_{[0,\pi]}\right)^{-1}(y) =: \arccos y$$

arcocoseno di y

$$\arccos : [-1,1] \longrightarrow [0,\pi]$$

$$y \longmapsto \arccos y$$

<u>ATTENZIONE</u>:

Il fatto che arccos sia l'inversa di una restrizione del coseno ha delle conseguenze come prima.

$\forall y \in [-1, 1]$:

$$\cos(\arccos y) = y$$

$\forall x \in [0, \pi]$ (e non in $\mathbb{R}$!)

$$\arccos(\cos x) = x$$

(Esempio:

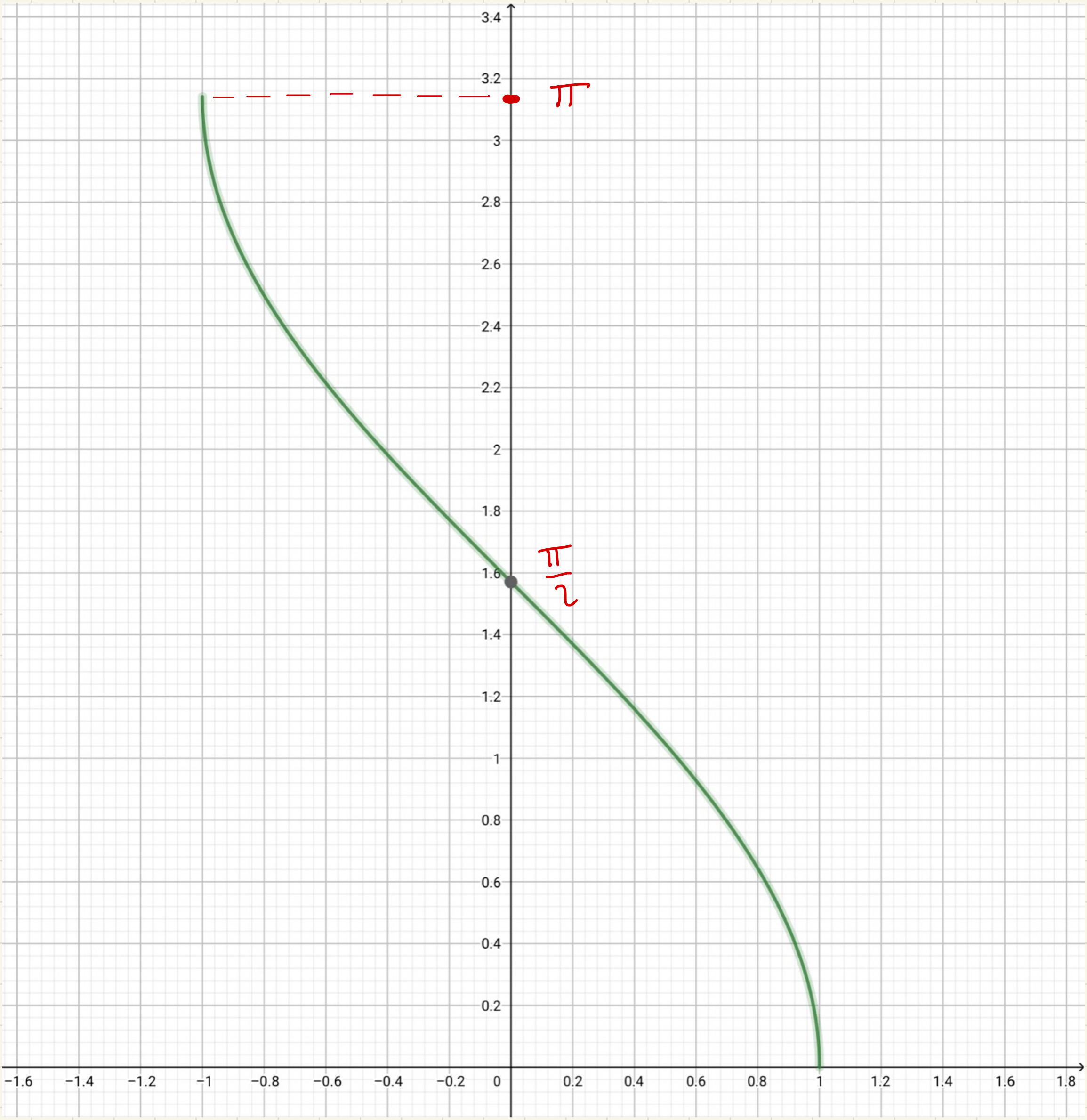
$x = \frac{3\pi}{2}$ (nota che $\frac{3\pi}{2} \notin [0, \pi]$)

$$\arccos\left(\cos \frac{3\pi}{2}\right) =$$

$$= \arccos(0) = \frac{\pi}{2} \neq \frac{3\pi}{2}$$

)

$y = \arccos x$



3.4
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
−1.6
−1.4
−1.2
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
π
π/2

Anche la funzione tangente <u>non</u> è invertibile (non è biunivoca) ma lo è la sua restrizione:

$$tg\big|_{]-\frac{\pi}{2},\frac{\pi}{2}[} : \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[\longrightarrow \mathbb{R}$$

$$\Longrightarrow \quad tg\big|_{]-\frac{\pi}{2},\frac{\pi}{2}[} \quad \text{è su e 1-1}$$

$$\Longrightarrow \quad \text{è invertibile}$$

$$tg\big|_{]-\frac{\pi}{2},\frac{\pi}{2}[} : \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[\longrightarrow \mathbb{R}$$

funzione inversa

$$\left(\operatorname{tg}\big|_{]-\frac{\pi}{2},\frac{\pi}{2}[}\right)^{-1}(y) =: \operatorname{arctg} y$$

(arctan y)

arcotangente di y

$$\operatorname{arctg} : \mathbb{R} \longrightarrow \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$$

$$y \longmapsto \operatorname{arctg} y$$

ATTENZIONE:

Il fatto che arctg sia l'inversa di una restrizione della tangente ha delle conseguenze!

$\forall y \in \mathbb{R}:$

$$\operatorname{tg}(\operatorname{arctg} y) = y$$

$\forall x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$ (e non in $\mathbb{R}$!)

$$\operatorname{arctg}(\operatorname{tg} x) = x$$

$$\sin x = \frac{1}{2}$$

$$\operatorname{tg} x > 1$$

Grafico di $y = \text{arctg}\, x$

$(= \arctan x)$